Leptogenesis from reheating after inflation and cosmic string decay

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Abstract

Cosmic strings form at the end of standard supersymmetric hybrid inflation, and both inflation and strings contribute to the CMB anisotropies. If the symmetry which is broken at the end of inflation is gauged $B-L$, there is a mixed scenario for leptogenesis: Right-handed neutrinos can be produced non-thermally during reheating via inflaton decay as well as via cosmic string decay. We show that the parameter space consistent with CMB data can accommodate either or both scenarios depending on the mass of the right-handed neutrinos.

1 Introduction

Supersymmetric theories beyond the standard model which predict light neutrino masses via the see-saw mechanism easily accommodate SUSY hybrid inflation \cite{1, 2}. Reheating can proceed via inflaton decay into right-handed (s)neutrinos and thereby these models also provide an interesting framework for non-thermal leptogenesis \cite{3, 4, 5}. In the simplest version of hybrid inflation cosmic strings form at the end \cite{6, 7}; if the inflaton sector couples to right-handed (RH) (s)neutrinos these are $B-L$ cosmic strings \cite{8, 9}. $B-L$ cosmic strings are not superconducting \cite{10, 11}. Most of the energy lost by the string network goes into gravitational radiation and right-handed neutrinos, and therefore these strings provide a second mechanism of non-thermal leptogenesis \cite{8}. In this paper we calculate the relative contributions to the baryon asymmetry of the universe from reheating at the end of standard $F$-term inflation and from $B-L$ cosmic string decay.

Let $G_{\text{GUT}}$ denote a gauge group which contains the Standard Model gauge group as well as gauged $B-L$. $F$-term inflation requires the existence of a gauge singlet and

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two Higgs superfields which transform in complex conjugate representations of $G_{\text{GUT}}$; we assume that they break $B$-$L$ when acquiring a non-vanishing vacuum expectation value (VEV). In the case of $SO(10)$, they could be a $16 + \overline{16}$ or $126 + \overline{126}$. Note that the transformation properties of the Higgs representation can affect the stability of the strings; we shall not discuss it further here. Inflation takes place as the scalar singlet slowly rolls down a valley of local minima along which the VEV of the Higgs fields vanish. When the singlet falls below a certain critical value, the Higgs mass become tachyonic and inflation ends quickly in a phase transition during which the Higgs fields acquire a non-vanishing VEV breaking $B$-$L$ spontaneously. If $G_{\text{GUT}}$ is semi-simple, the assumption of standard SUSY hybrid inflation then requires that $G_{\text{GUT}}$ breaks down to the Standard Model (SM) gauge group via at least one intermediate step, so that inflation solves the monopole problem; the gauge group broken at the end of inflation is not $G_{\text{GUT}}$ but an intermediate symmetry group $G_{\text{int}} \supset U(1)_{B-L} \rightarrow H \not\supset U(1)_{B-L}$. During this phase transition, $B$-$L$ cosmic strings form. The simplest example is $G_{\text{GUT}} = SO(10), E(6)$ or Pati-Salam, and $G_{\text{int}} = SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ which breaks down to the SM gauge group at the end of inflation.

The GUT Higgs fields which trigger the end of inflation give heavy Majorana masses to the right-handed neutrinos. After inflation the universe is reheated by inflaton decay into RH neutrinos and sneutrinos. If the reheat temperature is less than the neutrino mass their out-of-equilibrium decay into (s)leptons and the SM Higgs(inos) produces a net lepton asymmetry. There is another contribution to the lepton asymmetry, coming from the decay of cosmic string loops. Cosmic string loops decay into $B$-$L$ Higgs and gauge fields, which in turn decay in right-handed neutrinos. Loops can also release right-handed neutrinos which are trapped as zero modes.

Thus both inflation and cosmic strings contribute non-thermally to the baryon asymmetry of the universe. We investigate here which of these scenarios is most efficient using the observed CMB anisotropies as a constraint. The string tension, as well as the inflaton mass are set by the symmetry breaking scale at the end of inflation $\eta$ (the $B$-$L$ breaking scale) and the Higgs self quartic coupling $\kappa$ (the superpotential coupling). The string tension also depends logarithmically on the gauge coupling constant which we set to the unification value for the MSSM. The resulting lepton asymmetry depends on $\eta$, $\kappa$ and the RH neutrino masses. Requiring that the inflaton gives the observed density perturbations fixes $\eta$ as a function of $\kappa$. The string contribution to the density perturbations is
constrained to be less than about 10% [14]. This restricts the value of $\kappa$, as discussed in our previous paper [15].

Most of the lepton asymmetry in $B$-$L$ string decay is generated at the earliest time at which leptogenesis is possible. Refs. [16, 17] assume that the reheat temperature is high enough for the lightest RH neutrino with mass $M_1$ to be initially in thermal equilibrium. Any asymmetry generated is washed-out until the lightest RH neutrino freezes out. Hence, in their analysis the lepton asymmetry is dominated by the contribution generated at the freeze-out temperature $T \sim M_1$. However, there is also the possibility that the RH neutrinos are never in thermal equilibrium after inflation. Then the loops formed immediately at the end of inflation give the dominant contribution to the lepton asymmetry. The final asymmetry is determined by three factors. First, it depends on the initial string density, which can be different from the density during the scaling regime. Second, the universe goes from matter domination to radiation domination during reheating. The earlier this happens, the larger the asymmetry. And the third factor which plays a rôle is the CP asymmetry per decaying (s)neutrino, which depends on the details of the neutrino sector.

The paper is organised as follows. In Sec. 2 we review standard SUSY $F$-inflation coupled to N=1 SUGRA and discuss the CMB constraints [15]. In Sec. 3 we determine the parameter space for successful non-thermal leptogenesis from reheating at the end of inflation. In Sec. 4 we turn to the parameter space for successful non-thermal leptogenesis which results from the decay of $B$-$L$ cosmic strings. We distinguish three different cases. The first possibility is that $M_1 < m_{\chi}/2$ and $M_1 > T_R$, with $M_1$ the lightest RH neutrino mass, $m_{\chi}$ the inflaton mass and $T_R$ the reheat temperature. In this case reheating goes via production of RH neutrinos which are out-of-equilibrium at the end of inflation, and both non-thermal leptogenesis scenarios compete. The second possibility is that $M_1 > m_{\chi}/2$ and reheating is gravitational. Then cosmic strings give the sole contribution to the lepton asymmetry. Finally there is the possibility that $M_1 < T_R$ and the RH neutrinos are in thermal equilibrium at production. In this case both cosmic string decay and standard thermal leptogenesis contribute to the lepton asymmetry. In Sec. 5 we consider the consequences on leptogenesis of generating a dynamical $\mu$-term. We give our conclusions in Sec. 6.
2 Hybrid inflation & CMB constraints

In this section we summarise the bounds on the parameter space implied by current CMB data for standard hybrid inflation with cosmic strings. The details can be found in Ref. [15].

The superpotential for standard hybrid inflation is [1, 2]

$$W_{\text{inf}} = \kappa S (\phi \bar{\phi} - \eta^2),$$

(1)

with $S$ a gauge singlet superfield, and $\phi, \bar{\phi}$ Higgs superfields in $N$-dimensional complex conjugate representations of a gauge group $G_{\text{GUT}}$. Upon acquiring a VEV the Higgs fields break $G_{\text{int}} \supset U(1)_{B-L}$ down to a subgroup $H \not\supset U(1)_{B-L}$. The supersymmetric part of the scalar potential is given by (we represent the scalar components with the same symbols as the superfields)

$$V_{\text{SUSY}} = \kappa^2 |\phi \bar{\phi} - \eta^2|^2 + \kappa^2 |S|^2 (|\phi|^2 + |ar{\phi}|^2) + V_D.$$  

(2)

Vanishing of the $D$-terms enforces $|\bar{\phi}| = |\phi|$. Assuming chaotic initial conditions the fields get trapped in the inflationary valley of local minima at $|S| > S_c = \eta$ and $\bar{\phi} = \phi = 0$. The potential is dominated by a constant term $V_0 = \kappa^2 \eta^4$ which drives inflation. Inflation ends when the inflaton drops below its critical value $S_c$ (or when the second slow-roll parameter $\eta$ equals unity, whatever happens first) and the fields roll toward the global SUSY minima of the potential $|\phi| = |\bar{\phi}| = \eta$ and $S = 0$. During this phase transition $B-L$ cosmic strings form [6, 8]. For a discussion on various GUT models see Ref. [9].

The scalar potential in Eq. (2) gets corrections from SUSY breaking by the finite energy density in the universe during inflation (given by the Coleman-Weinberg formula), from SUSY breaking today, and from supergravity. The hidden sector expectation values responsible for low energy SUSY breaking can generically be written as $\langle z \rangle = a m_p$, $\langle W_{\text{hid}} \rangle = \mu m_p^2$, $\langle \frac{\partial W_{\text{hid}}}{\partial z} \rangle = c \mu m_p$, with $z$ a hidden sector field, $a, c$ dimensionless numbers, and $\mu$ a mass parameter related to the gravitino mass via $m_{3/2} = e^{a/2} \mu$. Further $m_p = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The scalar potential along the inflationary valley can be calculated using the SUGRA formula

$$V = e^{K/m_p^2} \left[ \sum_{\alpha} \left| \frac{\partial W}{\partial \phi_\alpha} + \frac{\partial W}{m_p^2} \right|^2 - 3 \left| W \right|^2 \right].$$  

(3)
Assuming a minimal Kähler potential, the scalar potential including all corrections is

$$V = \kappa^2 \eta^4 + \frac{\kappa^4 \eta^4 N}{32 \pi^2} \left[ 2 \ln \left( \frac{2 \kappa^2 \sigma^2}{\Lambda^2} \right) + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right] + \kappa^2 \eta^4 \left[ \frac{\sigma^4}{8 m_p^4} + \frac{|a|^2 \sigma^2}{2 m_p^2} \right] + \kappa A m_{3/2}^2 \kappa^2 \eta^2 \sigma,$$

with $\sigma = |S|/\sqrt{2}$ the normalized real field, $A = 2 \sqrt{2} \cos(\arg \mu - \arg S)$, and we assume that $\arg S$ is constant during inflation. The cosmological constant today vanishes for $|c + a^*|^2 = 3$, and we have dropped subdominant terms. Further we used the notation $z = x^2 = |S|^2/\eta^2 = \sigma^2/(2 \eta^2)$ so that $z = x = 1$ when $\sigma = \sigma_c$. The first line is the tree level potential term, the second line is the one-loop Coleman-Weinberg correction due to SUSY breaking during inflation [2], and the third line are the SUGRA corrections.

The Coleman-Weinberg potential and the non-renormalisable terms are always present, independent of low energy SUSY breaking. The $A$- and mass terms can be made small for a small gravitino mass (as in gauge mediation), or by tuning $A$ and/or $a$. Another possibility is that the hidden sector superfield $z$ only acquires its VEV after inflation, so that these terms are absent during inflation. We note that, assuming gravity mediated SUSY breaking, the generic values $m_{3/2} \sim 10^2 \text{GeV}$, $A \sim 1$, $a \sim 1$, give rise to too large $A$- and mass terms, incompatible with the CMB data [15].

Both strings and the inflaton contribute to the primordial density perturbations [6,15]. Cosmic strings do not predict the measured acoustic peaks in the CMB and hence their contribution to the temperature fluctuations should be small, less than about 10% [14]. The string contribution is proportional to the string tension

$$\mu = 2 \pi \eta^2 \theta(\beta),$$

with $\beta = (m_\phi/m_A)^2$. The Higgs mass is $m_\phi^2 = \kappa^2 \eta^2$ and the vector boson mass is $m_A^2 \simeq g_{\text{GUT}}^2 \eta^2$ with the GUT coupling $g_{\text{GUT}}^2 \approx 4 \pi/25$. When $\beta = 1$, the strings satisfy the Bogomolny bound (this is the case of cosmic strings which form at the end of brane inflation) and $\theta(1) = 1$. However in the case of SUSY GUTs, the strings never satisfy the Bogomolny bound, $\beta < 1$ always, and [18]

$$\theta(\beta) \approx \left\{ \begin{array}{ll} 1.04 \beta^{0.195}, & \beta > 10^{-2}, \\ 2.4 \log(2/\beta), & \beta < 10^{-2}. \end{array} \right. \quad (6)$$
Requiring the string contribution to the quadrupole to be less than 10% gives the bound

\[ G\mu < 6.9 \times 10^{-7} \left(\frac{3}{y}\right) \Rightarrow \eta_{\text{bnd}} < 4.1 \times 10^{15} \sqrt{\frac{3/y}{\theta(3)}}. \]  

(7)

Here \( y \) parameterizes the density of the string network, and should be taken from numerical simulations. Recent work predicts \( y = 9 \pm 2.5 \) \[19\]. Older simulations give \( y = 6 \) \[20\], and semi-analytic approximations give \( y = 3 - 6 \) \[21\].

The density perturbations produced in hybrid inflation can be calculated using the slow roll formalism for the potential in Eq. (4). Setting it equal to the value observed by WMAP gives \( \eta \) as a function of \( \kappa \). We will use the analytic approximations, derived in the limit that the Coleman-Weinberg(CW)-potential respectively the non-renormalisable (NR) terms dominate the potential: \[15\]

\[ \eta_{\text{cw}} = 5 \times 10^{15} \text{ GeV} \mathcal{N}^{1/3} \left(\frac{\kappa}{10^{-3}}\right)^{1/3}, \]  

(8)

\[ \eta_{\text{NR}} = 3 \times 10^{15} \text{ GeV} \left(\frac{\kappa}{10^{-6}}\right). \]  

(9)

The symmetry breaking scale is restricted to the range

\[ \eta_{\text{cw}} \leq \eta \leq \min[\eta_{\text{NR}}, \eta_{\text{bnd}}]. \]  

(10)

If the \( A \)- and mass terms are absent or subdominant during inflation there are two distinct solutions, corresponding to \( \eta_{\text{cw}} \) and \( \eta_{\text{NR}} \) (the upper bound \( \eta_{\text{bnd}} \) comes from the fact that the string contribution to the CMB is limited). If on the other hand these terms do play a role, the whole range is possible. This is illustrated in Fig. 1 which shows \( \eta \) as a function of \( \kappa \) for \( \mathcal{N} = 1, 16, 126 \). In this plot the \( A \)- and mass term are assumed to be negligible; if they are not the solution is somewhere in the range given by Eq. (10). The straight part at relatively large coupling is well approximated by \( \eta_{\text{cw}} \). At low value there is a second branch of solutions given by \( \eta_{\text{NR}} \). Also plotted is \( \eta_{\text{bnd}} \) for \( y = 3 \); above this line the string contribution to the CMB is more than 10%. The CMB constraints are satisfied for the coupling range \( 10^{-6} \lesssim \kappa \lesssim 10^{-2}/\mathcal{N} \) and the SSB scale range \( \eta \sim 10^{15} - 10^{16} \text{ GeV} \).

The CMB bound can be avoided if the strings are semi-local or not topologically stable down to low energy and decay at some later phase transition \[9\].
3 Non-thermal leptogenesis from inflaton decay

In this section we review the non-thermal leptogenesis scenario which happens during reheating as a result of inflaton decay into right-handed (s)neutrinos [4, 5].

The Higgs fields \( \phi \) and \( \bar{\phi} \) break local \( B-L \) spontaneously when developing a VEV at the end of inflation. The right-handed neutrinos acquire super-heavy Majorana masses via their coupling to \( \bar{\phi} \). Some other GUT superfield (this is model dependent) gives a Dirac mass to the neutrinos and the light neutrinos acquire a super light Majorana mass via the see-saw mechanism [22]. The lepton asymmetry, generated as the right-handed (s)neutrinos decay into SM Higgsinos and (s)leptons, is converted into a baryon asymmetry via sphaleron transitions [3].

Depending on the transformation properties of the Higgs representation, the right-handed neutrino masses are generated via normalisable or non-renormalisable superpotential terms

\[
W = \frac{1}{m_p} \gamma_{ij} \bar{\phi} \phi F_i F_j, \tag{11}
\]
\[
W = y_{ij} \bar{\phi} F_i F_j, \tag{12}
\]
where $F$ is the $n$-dimensional spinorial representation of $G$ which contains the right-handed neutrino superfield $N$, and $i, j = 1..3$ for three families. At the end of inflation $S$ and $\phi_+ = (\delta \phi + \delta \bar{\phi})/\sqrt{2}$, with $\phi = \eta + \delta \phi$ and $\bar{\phi} = \eta + \delta \bar{\phi}$, oscillate around the global SUSY minimum of the potential until they decay into right-handed neutrinos and sneutrinos, thereby reheating the universe \[4\]. We work in the basis where the right-handed neutrinos mass matrix is diagonal. The decay rates $\Gamma(\phi_+ \rightarrow N_i N_i)$ and $\Gamma(S \rightarrow \tilde{N}_i \tilde{N}_i)$ are equal and given by

$$\Gamma_N = \frac{1}{8\pi} \left( \frac{M_i}{\eta} \right)^2 m_\chi,$$

(13)

with $\chi = S, \phi_+$ the oscillating fields which have equal mass $m_\chi = \kappa \eta$, and $M_i = y_i \eta, \gamma_i \eta^2/m_p$ the mass of the heaviest right-handed neutrino $N_i$ (sneutrino $\tilde{N}_i$) the inflaton can decay into (i.e., which satisfies $M_i < m_\chi/2$). The reheat temperature is then

$$T_R = \left( \frac{45}{2\pi^2 g^*} \right)^{1/4} (\Gamma_N m_p)^{1/2} \simeq 6 \times 10^{-2} M_i \sqrt{\frac{\kappa m_p}{\eta}}.$$  

(14)

where we have used $g^* = 228.75$ for the MSSM spectrum.

Non-thermal leptogenesis from inflaton decay takes place if the following constraints are satisfied:

**Kinematic constraint:** Inflaton decay into right-handed (s)neutrinos is kinematically allowed if $M_i \leq m_\chi/2$, i.e.,

$$M_i(\kappa) \leq \frac{1}{2} \kappa \eta$$

(15)

**Gravitino constraint:** Gravitino overproduction is avoided if the reheat temperature $T_R \lesssim 10^{10}$ GeV \[23\]. This gives

$$M_i(\kappa) \lesssim 1.6 \times 10^{11} \text{ GeV} \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \sqrt{\frac{\eta(\kappa)}{\kappa m_p}}.$$  

(16)

The upper bound on $T_R$ is model dependent and can be as low as $10^6$ GeV. We note that the gravitino constraint can be avoided if the gravitino mass is sufficiently large so that it decays before BBN.

**Gravitational decay:** The decay rate into right-handed (s)neutrinos should be larger than the gravitational decay rate into light particles. In a full theory, the superpotential is $W = W_{\text{infl}} + W_{\text{hid}} + W_{\text{GUT}}$, where $W_{\text{GUT}}$ contains GUT superfields, some
of them containing the MSSM fields. The gravitational decay rate of the inflaton into light SM particles can then be computed by considering for example a term of the form \( W_{\text{GUT}} = a H F^2 \), \( F \) containing the standard model fermions and \( H \) some GUT Higgs superfield containing the SM Higgs. In the SUGRA potential Eq. \([3]\) there is a coupling between the inflaton and the SM particles, leading to a decay rate

\[
\Gamma_{\text{grav}} \simeq \frac{1}{8\pi} \frac{m_\chi^3 \eta^2}{m_p^4},
\]

Note that this is parametrically smaller than the standard gravitational decay rate \( \Gamma_{\text{grav}} = (1/8\pi) m_\chi^3 / m_p^2 \) \([24]\). Requiring \( \Gamma_N > \Gamma_{\text{grav}} \) then leads to

\[
M_i(\kappa) \gtrsim \frac{\kappa^2 \eta^3}{m_p^2}.
\]

**The wash-out constraint:** The lepton asymmetry produced by the decay of the RH neutrino \( N_i \) is washed out by the \( L \)-violating processes involving RH neutrinos, unless they are out of thermal equilibrium which is automatic if \( M_j \lesssim T_R \), with \( j = 1, 2, 3 \). The strongest constraint is for the lightest RH neutrino:

\[
\frac{M_1(\kappa)}{M_i(\kappa)} \gtrsim \frac{1}{16} \left( \frac{m_p \kappa}{\eta(\kappa)} \right),
\]

with as before \( M_i \) the mass of the RH neutrino the inflaton decays into. This implies \( M_1 \gtrsim 10^{-1} - 10^{-3} M_i \) for \( (\eta, \kappa) \) allowed by CMB data (see Eqs. \([8, 10]\)). No wash-out is assured if \( M_1 < m_\chi < M_2, M_3 \) and the inflaton decays into the lightest right-handed neutrino. The CP violating parameter \( \epsilon \) can be improved by at most a factor \( (M_2/M_1) \simeq 10^1 - 10^3 \) if the decay is not into the lightest neutrino, but into the next to lightest one (see the Appendix: \( \epsilon_1 \propto M_1 \) and \( \epsilon_2 \propto M_2 \) \([5]\).

**Perturbative couplings:** We require the couplings \( \gamma_i, y_i \) in Eqs. \([11, 12]\) to be less than unity. For a renormalisable mass term this bound cannot compete with the kinematic constraint. For a non-renormalisable mass term this implies

\[
M_i \lesssim \frac{\eta^2}{m_p^2} \sim 1 \times 10^{15} \text{GeV}(N\kappa)^{2/3},
\]

where in the last step we used \( \eta = \eta_{\text{cw}} \), see Eq. \([8]\). If \( \eta/m_p < \kappa \), which only happens for \( k \gtrsim 10^{-2} \), then for non-renormalisable mass terms all neutrino masses are lighter than the inflaton mass.
**Lepton asymmetry:** The lepton asymmetry produced is \[ \frac{n_L}{s} = \frac{3}{2} \frac{T_R}{m_X} \epsilon_i, \quad (21) \]

with \( \epsilon_i \) the CP asymmetry per decaying RH neutrino \( N_i \). For hierarchical RH neutrino masses and hierarchical light neutrinos the CP asymmetry in the decay of the lightest RH neutrino is bounded by \[ |\epsilon_1| \leq 2 \times 10^{-10} \left( \frac{M_1}{10^6 \text{ GeV}} \right) \left( \frac{\Delta m^2_{\text{atm}}}{0.05 \text{ eV}} \right)^{1/2}. \quad (22) \]

As discussed in the appendix, the upper bound on \( \epsilon_2 \) is of the same order of magnitude, but with \( M_1 \to M_2 \) in the above formula. The CP-asymmetry induced by the decay of the heaviest RH neutrino is suppressed by a factor \( M_2/M_3 \). For quasi-degenerate light neutrinos \( (m_1 \approx m_2 \approx m_3 \gg \Delta m^2_{\text{atm}})^{1/2} \) the asymmetry is smaller \[ |\epsilon_1| \leq 2 \times 10^{-10} \left( \frac{M_i}{10^6 \text{ GeV}} \right) \left( \frac{\Delta m^2_{\text{atm}}}{0.05 \text{ eV}} \right)^{1/2} \left( \frac{\Delta m^2_{\text{atm}}}{\bar{m}} \right)^{1/2}, \quad (23) \]

with \( \bar{m} = 1/3 \sqrt{m_1^2 + m_2^2 + m_3^2} \).

If the RH neutrinos are quasi-degenerate and \( M_i - M_j \sim \Gamma_i \) the CP-asymmetry is enhanced. The only constraint is then \( |\epsilon_i| < 1 \). Upper bounds on the CP-asymmetry in type II see-saw models have also been derived for non-degenerate neutrinos, and are of the same magnitude as Eq. (22) \[ \text{[28].} \]

The baryon asymmetry inferred from BBN translates into a primordial lepton asymmetry given by \( n_L/s = 2.4 \times 10^{-10} \) for the MSSM spectrum. For hierarchical light neutrinos, this is obtained for

\[ M_i(k) \gtrsim 5.4 \times 10^4 \left( \frac{\kappa \eta(\kappa)}{m_p} \right)^{1/4} \sqrt{\text{GeV}}. \quad (24) \]

For smaller \( M_i \) the produced asymmetry is too small.

The parameter space compatible with NT leptogenesis from inflaton decay is shown in Figs. 2 and 3. The kinematic, perturbative coupling and gravitino constraints all give an upper bound on \( M_i \). The perturbative coupling constraint is weakest and not shown. The kinematic constraint is strongest for small \( \kappa \), whereas the gravitino constraint dominates
Figure 2: $M_i$ vs. $\kappa$ for $\eta = \eta_{cw}$ and $N = 1$. The parameter space is bounded by the kinematic constraint (left), successful leptogenesis (right), and the gravitino constraint (lines, for $T_R = 10^7, 10^8, 10^9, 10^{10}$ GeV).

for large coupling. The gravitational decay and leptogenesis constraint give a lower bound on $M_i$. The leptogenesis bound is strongest. The $\kappa$ range is bounded by the CMB data, as given by Eqs. (8, 9, 10) and Fig. 1, to $10^{-6} \lesssim \kappa \lesssim 10^{-2}/N$.

Fig. 2 shows the parameter space for $\eta = \eta_{cw}$ and $N = 1$. The colored regions are excluded. The upper bound is fixed by the reheating temperature; the bounds for $T_R = 10^7, 10^8, 10^9, 10^{10}$ GeV are shown. Leptogenesis is only possible for $T_R > 10^6$ GeV. For $T_R \sim 10^9$ GeV, successful leptogenesis requires $M_i = 10^9 - 10^{11}$ GeV and $\kappa = 10^{-5} - 10^{-3}$. One should remember that in addition to the constraints shown in the plot, it should be checked that $M_1 > T_R$ and there is no wash-out of asymmetry. This is the case for $M_1 \gtrsim 10^{-2} M_i$, with leptogenesis dominated by inflaton decay into $N_i$, in agreement with Eq. (19).

For a fixed coupling value, the possible range of neutrino mass $M_i$ is about a decade. Therefore the bounds are all close to saturation. E.g. the leptogenesis constraint gives $M_i \propto (\epsilon_i/\epsilon_{i,\text{max}})^{-1/2}$, and thus $(\epsilon_i/\epsilon_{i,\text{max}}) \gtrsim 10^{-2}$ is required. Hence, degenerate light neutrinos with $\bar{m} \sim \text{eV}$ are marginally excluded.
Figure 3: $M_i$ vs. $\kappa$ for $\max[\eta_{\text{cw}}, \eta_{\text{NR}}] < \eta < \eta_{\text{had}}$, $T_R = 10^{10}$ GeV and $N = 1, 16, 126$. The parameter space is bounded by the kinematic constraint (left), successful leptogenesis (bottom), the gravitino constraint (top), and for $N = 16, 126$ by the CMB data (right).

Fig. 3 shows the parameter space for $\eta$ in the whole range of Eq. (10) for $T_R = 10^{10}$ GeV and $N = 1, 16, 126$. The parameter space is enhanced compared to Fig. 2 about twice as big. The main consequence of increasing $N$ is that the small $\kappa$ range is excluded by the CMB data, and that leptogenesis requires a slightly larger neutrino mass.

4 Leptogenesis from string decay

Cosmic strings form in SUSY GUT models with standard hybrid inflation \[6, 7\]. The string mass per unit length is then constrained by CMB data, see Eqs. (5), (8) and (9). When the symmetry broken is gauged $B-L$, they also provide a non-thermal scenario for leptogenesis \[8\]. We first describe various possible NT leptogenesis scenarios with $B-L$ strings forming at the end of inflation. We then discuss the evolution of the string network and analyse the various scenarios in details.
4.1 Leptogenesis scenarios

Depending on whether the mass of the lightest RH neutrino $M_1$ is larger or smaller than the reheat temperature $T_R$, and on whether the inflaton decays into right handed neutrinos or not, there will also be a contribution to the lepton asymmetry of the universe from inflaton decay or standard thermal leptogenesis. We distinguish the following cases:

**Case 1:** The reheat temperature is lower than the lightest right-handed neutrino mass, $T_R < M_1$, and there is no wash-out at any time. The lepton asymmetry is set by the earliest time that string loops form, which is right at the end of inflation. Reheating of the universe takes place at a later time, via inflaton decay into right-handed neutrinos. Apart from NT leptogenesis from $B-L$ strings, there is also a contribution from non-thermal leptogenesis from inflaton decay as discussed in section 3.

Constraints: gravitino constraint $T_R < 10^{10}\text{GeV}$, gravitational decay constraint $\Gamma_{\text{grav}} < \Gamma_N$, kinematical constraint $M_i < m_\chi/2$.

**Case 2:** Same as case 1 but now the inflaton does not decay into RH neutrinos. For example, if $M_i > m_\chi/2 \forall i$ and there is no other superpotential term involving the singlet or the Higgs fields, decay is through gravitational interactions: $\Gamma_N < \Gamma_{\text{grav}}$. Gravitational reheating can alleviate the gravitino constraint. Leptogenesis comes from the decay of the string forming gauge field in right handed neutrinos. This is the only contribution to the lepton asymmetry.

Constraint: gravitino $T_R < 10^{10}\text{GeV}$.

**Case 3:** The reheat temperature is higher than the lightest neutrino mass $T_R > M_1$. The asymmetry will be washed out at high temperatures by $L$-violating processes mediated by $N_1$, and can only be created for $T_R < M_1$. The asymmetry is then dominated by the loop formation rate at $T \sim M_1$. There are two contributions to the lepton asymmetry: NT leptogenesis from $B-L$ strings and standard thermal leptogenesis. This is the case considered before [16, 17].

Constraints: gravitino constraint $T_R < 10^{10}\text{GeV}$, kinematical constraint $M_1 < m_\chi/2$.

In order to analyse the various scenarios, we study analytically the evolution of the string network, both in the scaling regime and at the initial times.
4.2 String network and neutrino density

The evolution of a cosmic string network has been extensively studied over the years [29]. Numerical simulations and analytical studies agree that the string network reaches a scaling regime, in which the energy-density carried by the network remains a constant fraction of the total energy density in the universe. The scaling solution is an attractor solution, and is independent of the initial string density. This is one of the reasons that string network at formation has not been discussed much in the literature. However, the lepton asymmetry is dominated by the initial time, and thus depends sensitively on the initial density. We first discuss the familiar scaling regime, before turning to a discussion of the initial string density.

4.2.1 The scaling regime

To describe the approach to the scaling regime, we introduce the characteristic length-scale $L$ which sets the correlation length and the average distance between long strings [29]. The energy density in long strings, $\rho_\infty \sim \mu/L^2$, evolves as

$$\dot{\rho}_\infty = -2H \rho_\infty - f(p) \frac{\rho_\infty}{L}$$

where the terms on the right hand side describe the energy loss due to expansion of universe, and due to production of loops respectively. The function $f(p)$ depends on the reconnection probability $p$ as $f(p) \sim \sqrt{p}$. For gauge field theory cosmic strings $p = 1$. Introducing $\gamma(t)$ such that

$$L = \gamma(t)t,$$

one finds that the above equation has a stable attractor solution, the scaling solution. It does not depend on the initial string density:

$$L = \gamma_s(t)t \equiv \frac{f(P)}{2(1 - \beta)}t$$

where we wrote $\beta = Ht$. Since $\gamma_s = \sqrt{p}/(2(1 - \beta)) = \mathcal{O}(1)$ is constant, the long strings scale with the horizon.

The scale $L$ characterizes the network on macroscopic scales, but does not say anything about what happens on the smallest scales. Simulations show that the long strings have small-scale wiggles, whose characteristic length also scales with time [29]. These wiggles
set the typical loop size, which we parametrise as

\[ l_{\text{loop}} \sim \alpha t \]  

(28)

with \( \alpha \sim (\Gamma G \mu)^n \), and \( \Gamma \sim 50 \). The “standard” value is \( n = 1 \) giving \( \alpha = \alpha_1 \equiv (\Gamma G \mu) \)  

(29). More recent simulations suggest \( n = 3/2 (5/2) \) in the radiation (matter) dominated era  

(30). The loop formation rate \( \dot{n}_{\text{loop}} \) is set by the requirement that the string network keeps its scaling solution. The loops loose energy by emitting gravitational radiation and contract until the loop radius becomes of the order of the string width, at which point it decays emitting \( X \)-particles (with \( X = \phi, A, N \), i.e., the string Higgs or gauge fields, or RH neutrino zero modes). For \( \alpha \lesssim \alpha_1 \), the loop lifetime is less than a Hubble time and we can then neglect the red shifting between birth and death. The injection rate of right-handed neutrinos during scaling is then simply

\[ \dot{n}_N = x_N \dot{n}_{\text{loop}} \sim \frac{x_N}{\gamma^2 \alpha p} t^{-4}, \]  

(29)

where \( x_N \) is the number of right-handed (s)neutrinos produced per decaying loop.

The minimal number of RH neutrinos released per loop is \( x_N = 1 \). However, we expect the loop to decay when its radius becomes of the order of the string width \( m_\phi^{-1} \) with a burst of the strings Higgs and gauge particles  

(29). These in turn (mostly) decay into RH neutrinos. The number of neutrinos emitted per loop is then of the order

\[ x_N \lesssim \frac{E_{\text{loop}}|_{R \sim m_\phi^{-1}}}{m_X} = \frac{(2\pi)^2 \theta(\beta)}{\kappa \eta} \frac{m_\phi}{m_X}, \]  

(30)

with \( \theta(\beta) \sim 0.1 - 1 \) given in Eq. (29). If the loops decay mainly into Higgs fields \( m_X = m_\phi \) and \( x_N \sim \kappa^{-2} \), whereas if decay is mainly into gauge bosons \( m_X = m_A \) and \( x_N \sim \kappa^{-1} \). If the string width at which the loop decays is smaller than the the inverse Higgs mass \( m_\phi^{-1} \), then \( x_N \) is correspondingly smaller.

The lepton density is obtained by integrating Eq. (29) with a red shift factor \( (a(t_{\text{in}})/a(t))^{3/2} = (t_{\text{in}}/t)^{3/2} \) to account for the expansion of the universe. Here it is assumed that both \( t \) and the initial time \( t_{\text{in}} \) are in the radiation dominated era following inflaton decay. This gives (using \( t \sim H^{-1} \))

\[ n_L(H) \sim \frac{x_N \epsilon_i}{\gamma_s^2 \alpha} H_{\text{in}}^{3/2} H^{3/2}, \]  

(31)

independent of the reheating temperature.
Some simulations find that loops form on the smallest possible scale (given by the resolution of the simulation). These results suggest that scaling is maintained mostly by particle emission rather than via loop formation and subsequent gravitational decay. This is called the VHS scenario, after the authors of Ref. [31]. The emitted X-particles are the RH neutrinos themselves, due to the existence of zero mode solutions, and the string forming Higgs and gauge fields which then (mostly) decay into RH neutrinos. This gives a RH neutrino injection rate \[ \dot{n}_N \simeq f_X \frac{\mu}{m_X \gamma^2} t^{-3}. \] \[ (32) \]

with \( X = \phi, A, N \), and \( f_X \) the fraction of the energy in loops that goes into X-particles. Not all of the loop energy can go into high-energy particles, as this would give too large a diffuse \( \gamma \)-ray background, in conflict with the EGRET data [33]. Combining the EGRET bound with the CMB bound gives\(^1\) [32]

\[ f_X \lesssim 10^{-5} \left( \frac{7 \times 10^{-7}}{G\mu} \right) \] \[ (33) \]

We note, however, that the EGRET flux is dominated by late times. It is therefore not impossible that \( f_X \) is time-dependent, and much larger than the bound above at early times.

When leptogenesis takes place after reheating, the lepton number density is obtained by integrating (32) taking into account the expansion of the universe. This gives

\[ n_L(H) \simeq f_X \epsilon_i \frac{\mu H^{1/2}_m H^{3/2}}{\gamma_s^2 m_X}. \] \[ (34) \]

### 4.2.2 The initial string density

Cosmic strings are formed during the \( B-L \) breaking phase transition. The string density is set by the correlation length at the time of the phase transition \( \xi \) [34, 35]. The universe is cold at the end of inflation, and the equilibrium correlation length is set by the mass of the symmetry breaking Higgs fields \( \xi(t)^{-1} = m_\phi(t) = \kappa^2(S^2(t) - \eta^2) \). Both the correlation length and the relaxation time \( \tau = \xi \) diverge during the phase transition, and eventually

\(^1\)A similar injection rate and EGRET constraint apply for large loops which undergo 'quick death', i.e., loops that decay through many self-intersections into small loops which in turn decay emitting heavy X-particles.
\( \phi \) must fall out of equilibrium. The correlation length at freeze out \( \hat{\xi} \) is thus in the range

\[
(\kappa \eta)^{-1} < \hat{\xi} < H_*^{-1}.
\]

with \( H_* \) the Hubble constant at the end of inflation. The lower bound is set by the maximum Higgs mass \( m_\phi = k \eta \) obtained in the vacuum. The upper bound is set by causality, as fluctuations cannot exceed the horizon. A more careful estimate of \( \hat{\xi} \) is given in Ref. [36].

Writing \( S(t) = S_c - \dot{S}t \) near the phase transition, the inverse Higgs mass, which sets the correlation length, is \( m_\phi^2(t) = -(\kappa^2 \eta \dot{S})t \). Freeze out happens approximately at the time when the relaxation time is equal to \( |t| \), and thus

\[
\hat{\xi} \approx \xi(-\tau) = (\kappa^2 \eta \dot{S})^{-1/3}.
\]

During slow roll inflation \( \dot{S} \sim (60H^2) \). If the velocity does not change much between time observable scales leave the horizon and the time of the phase transition, then this is a good estimate. The freeze-out correlation length \( \hat{\xi} \) determines the typical distance between cosmic strings.

At the time of string formation (quantities will be denoted by subscript \( * \)), the correlation length \( \hat{\xi} = L = \gamma_* t_* \) (see Eq. (26))

\[
t_*^{-1} \sim H_* \sim \frac{\kappa \eta^2}{m_p}.
\]

Using Eqs. (35, 36), we get the range and “best” value for \( \gamma_* \):

\[
\gamma_{\text{min}} \equiv \eta/m_p < \gamma_* < 1 \equiv \gamma_{\text{max}}, \quad \gamma_{\text{best}} \equiv \gamma_* \sim 0.1 \kappa^{-1/3}(\eta/m_p)^{1/3}.
\]

Since \( \gamma_* < \gamma_s \) and \( \rho_\infty \propto \gamma^{-2} \), the energy density in the network is initially larger than during the scaling regime. Solving Eq. (26) with the initial conditions above, it can be seen that the scaling regime is typically reached in only a couple of Hubble times. We will use the approximation that initial network reaches the scaling regime instantaneously, and an amount of energy \( \rho_* \sim \mu/(\gamma_* t_*)^2 \) is dumped into loops and/or particles at the initial time given by Eq. (37).

Eq. (28) for the loop size breaks down at the initial time. The reason is that the loop radius is smaller than \( \sim m_\phi^{-1}, (m_A^{-1}) \), i.e., the width of the profile function of the Higgs

\[\text{We neglect friction in the thermal bath which is absent for } T < G \mu m_p. \text{ Note that the strings form before reheating has completed. Taking for } T \text{ the reheat temperature Eq. (14), this gives } M_i < 4\theta\eta^{5/2}/(\kappa^{1/2}m_p^{3/2}) \sim 6 \times 10^{14} \text{GeV} \theta \kappa^{1/3}N^{5/6}, \text{ where in the last step we used } \eta = \eta_{\text{ew}}.\]
(gauge) field, and various parts of the loop overlap. It is not possible to speak of cosmic string loops anymore, which are well defined only for $l_{\text{loop}} \sim \alpha t > 2\pi m_{\phi}^{-1}, (2\pi m_{A}^{-1})$, which requires $t \gg t_*$. Therefore it is expected that initially the energy loss in gravitational radiation is small, and the network mainly decays directly into RH neutrinos and into gauge and Higgs fields which (mostly) decay into RH neutrinos. The lepton number density at $H < T_R$ is then

$$n_L(H) \simeq f_X \epsilon_i \rho_s \left( \frac{a(t_s)}{a(t)} \right)^3 \simeq f_X \epsilon_i \mu \frac{\Gamma^{1/2} H^{3/2}}{m_X}.$$  \hspace{1cm} (39)

where $f_X$ is the fraction of the energy going into X-particles and $\epsilon_i$ is the CP asymmetry per decaying RH neutrino, see Eq. (22). Before inflaton decay the universe is matter dominated and $a \propto t^{2/3}$, afterwards the universe is radiation dominated and $a \propto t^{1/2}$. This is used in the second step to write $(a(t_s)/a(t))^3 = H^{3/2} H_s^{-2} \Gamma^{1/2}$, where the time $t \sim H^{-1}$ is after reheating of the universe $H < \Gamma$. This factor takes the red shift due to the expansion of the universe into account. The earlier the transition from matter to radiation domination, i.e., the larger the decay rate, the larger $T_R$ and the larger is the final number density. The initial time $t_{\text{in}}$ is right at the end of inflation, see Eq. (37).

Eq. (39) is the same as the lepton asymmetry produced during the scaling regime as given by Eq. (34) under the replacement $\gamma_s \rightarrow \gamma_s$ (difference in energy densities stored in long strings), and $\Gamma \rightarrow H_{\text{in}}$ (difference in whether leptogenesis takes place before or after reheating).

4.3 Results

For the parameters at hand $\Gamma < H_s$ and the inflaton decays some time after inflation. The lepton asymmetry $n_L/s$ is to be evaluated after reheating, when entropy is defined. The entropy is $s = (2\pi^2/45) g_s T^3 \sim 10^2 T^3$ with $T \simeq 0.4\sqrt{m_p H}$.

4.3.1 Case 1

The string contribution is dominated by the asymmetry produced at the initial time. The lepton number density is given by Eq. (39). Dividing by the entropy gives

$$\frac{n_L}{s} \simeq \frac{\epsilon_i f_X \mu \Gamma^{1/2}}{\gamma_s^2 m_X m_p^{3/2}}.$$  \hspace{1cm} (40)
Using Eqs. (13) and (22), we see that \( n_L/s \) is proportional to \( M_i M_j \) the mass of the heaviest RH neutrino the inflaton can decay into and the mass of the RH neutrino which is mostly produced by strings. For \( M_i = M_j \), which is automatic if the strings decay mostly into \( \phi \)-particles, successful leptogenesis with \( n_L/s = 2.4 \times 10^{-10} \) requires

\[
M_i \simeq 9 \times 10^3 C \left( \frac{m_0^3 \text{GeV}^2}{\eta \kappa} \right)^{1/4} = \frac{4 \times 10^{12} C \text{GeV}}{\kappa^{1/3} \mathcal{N}^{1/12}},
\]

(41)

where in the second step we used \( \eta = \eta_{\text{CW}} \), and we introduced

\[
C = \gamma_s \sqrt{m_X/\eta} \left( \frac{\theta(\beta)}{f_X} \right).
\]

(42)

\( C \) is minimized for \( \gamma_s \to \gamma_{\text{min}} \) and \( f_X \to 1 \). In this limit the energy density in the string network is of the same order as the energy density in the oscillating inflaton field. If both the inflaton and the strings decay into \( \phi \) particles \( (X = \phi) \) this gives a contribution to the lepton asymmetry of similar magnitude. If however \( \gamma_s > \gamma_{\text{min}} \) or \( f_X < 1 \), the energy stored in the string network is subdominant, and thus also its contribution to the asymmetry:

\[
\frac{(n_L)_{\text{inflaton}}}{(n_L)_{\text{strings}}} \sim f_X \left( \frac{\gamma_s}{\gamma_{\text{min}}} \right)^2.
\]

(43)

The string contribution can dominate over the inflaton contribution to the asymmetry if the strings decay mostly into RH (s)neutrinos. Another possibility is that \( M_1 < m_\chi < M_2 \) and the string decays mostly into gauge particles. The inflaton decays into lightest RH (s)neutrino, whereas the gauge field can also decay in the next to lightest one. Since \( \epsilon_2/\epsilon_1 \sim M_2/M_1 \) the CP-asymmetry is then larger per decaying \( A \)-particle than per decaying \( \phi \)-particle. However, the number of gauge particles produced by string decay can be of the same order as the number of Higgs particles from inflaton decay only in the limit \( \kappa \to 1 \) (so that \( m_\phi \sim m_A \)) and \( f_X \to 1 \).

For degenerate light neutrinos the CP asymmetry \( \epsilon_i \) is smaller by a factor \( \Delta m_{\text{atm}}/m_3 \), and \( M_i \propto \epsilon_i^{1/2} \) is larger. The \( \mathcal{N} \)-dependence of the neutrino mass \( M_i \) is weak; the main effect of considering a larger Higgs representation is the stronger constraint on the coupling coming from CMB data: \( \kappa \lesssim 10^{-2}/\mathcal{N} \). The bound on the neutrino mass is weakest in the limit \( \kappa \to 1 \). This is only possible if the strings do not contribute to the CMB anisotropies, i.e., if they are semi-local or are not topologically stable [9].
4.3.2 Case 2

If $m_\chi < M_i < \eta$ inflaton decay into RH neutrinos is kinematically forbidden, and in the absence of other direct couplings the inflaton decays gravitationally. This implies a low reheat temperature which alleviates the gravitino constraint. Note however that the lepton asymmetry $n_L \propto \Gamma_{1/2}$ is less efficient and large neutrino masses are needed. The string loops decay into RH neutrinos and gauge quanta which subsequently decay into RH neutrinos.

To get the RH neutrino mass required for leptogenesis in case 2, we use Eq. (40) with $\Gamma_N$ replaced by the gravitational decay rate, to yield

$$M_i \simeq 9 \times 10^5 \text{GeV} \frac{C^2}{\kappa^{3/2}} \left(\frac{m_p}{\eta}\right)^{7/2} = \frac{7 \times 10^{11} \text{GeV} C^2}{\kappa^{8/3} N^{7/6}}, \quad (44)$$

with $C$ given by Eq. (12). The neutrino mass is quadratic in $C$, and thus the dependence on the uncertain parameters grouped in $C$ is larger than in case 1. Minimizing $C$ gives
\( M_i \gtrsim 2 \times 10^{13} \text{GeV} \).

The lower bound on the neutrino mass Eq. (44) is shown in Fig. (1) as well as the kinematic constraint \( m_\chi/2 < M_i < m_A/2 \sim \eta/2 \), and the gravitino constraint. The reheat temperature is independent of the neutrino mass, but does depend on the coupling \( \kappa \). For \( \kappa < 10^{-2} \) one has \( T_R < 3 \times 10^9 \text{GeV} \) and there is no gravitino problem. Leptogenesis is possible for \( M_i \sim 10^{14} - 10^{16} \text{GeV} \) and \( \kappa \sim 10^{-2} \). The RH neutrino mass increases rapidly with small \( \kappa \), and much smaller couplings are excluded. The large neutrino masses needed are incompatible with a non-renormalisable mass term as in Eq. (11) and perturbative couplings, see Eq. (20).

4.3.3 Case 3

Consider now the case that the lightest RH neutrino reaches thermal equilibrium after inflation \( (M_1 < T_R) \). As follows from Eq. (19) thermal equilibrium can only occur if the inflaton decays in \( N_2 \) or \( N_3 \), and not in the lightest RH neutrino. The lepton asymmetry is thus produced via string decay in one of the heavier RH neutrinos and \( i = 2, 3 \). Any produced lepton asymmetry will be washed out until \( L \)-violating reactions fall out of equilibrium at \( T \sim M_1 \). We assume that this occurs in the scaling regime.

**Loop scenario** The lepton number density is now given by Eq. (31). Diving by the entropy we get

\[
\frac{n_L}{s} \simeq \frac{\epsilon_i f_N}{\gamma^2 \alpha} \left( \frac{H_{\text{in}}}{m_p} \right)^{3/2},
\]

with \( H_{\text{in}} \simeq M_1^2/m_p \). Setting it equal to the observed value gives the RH neutrino mass needed for successful leptogenesis:

\[
M_i \simeq 10^{14} \text{GeV} C^{1/2} \left( \frac{M_i}{M_1} \right)^{3/4} \gtrsim 2 \times 10^{14} \text{GeV} \left( \frac{C' N^{1/2}}{\kappa} \right)^{1/4}.
\]

In the second step we used the equilibrium condition Eq. (19) and \( \eta = \eta_{\text{GW}} \); further we defined

\[
C' = \sqrt{\frac{(\alpha/\alpha_1)}{f_N}}
\]

The number of RH neutrinos released per loop is bounded by \( x_N \lesssim \kappa^{-2} \) (see Eq. (30)). Further \( \alpha \) gives the loop size at birth (see Eq. (28)); taking \( \alpha = (\Gamma G \mu)^n \) with \( n = 3/2 \).
Figure 5: $M_i$ vs. $\kappa$ for case 3 with $\eta = \eta_{\text{CW}}, N = 1$, and $\alpha = \alpha_1$. The lila part corresponds to leptogenesis in the loop scenario with $1 < x_N < \kappa^{-2}$, and the lines A-B to the VHS scenario with $A = (\gamma_{\text{in}}, X, f_{\text{VHS}}) = (\gamma_s, \phi, 1)$, and $B = (\gamma_s, A, 1)$. The parameter space is bounded by the kinematic constraint (top), and the gravitino constraint (parallel lines, for $T_R = 10^{11}, ..., 10^{14}$ GeV).

instead of $n = 1$ lowers $C'$ by about a factor 10. Thus $C' \gtrsim 0.1\kappa$. The lower bound on the RH neutrino mass is then

$$M_i \gtrsim 6 \times 10^{12} \text{GeV} N^{1/8} \left(\frac{\kappa}{10^{-2}}\right)^{1/4},$$

(48)

together with the CMB constraint $10^{-6} \lesssim \kappa \lesssim 10^{-2}/N$.

The bound on the neutrino mass Eq. (48) is shown in Fig. (5) for $1 < f_N < \kappa^{-2}$, together with the kinetic and gravitino constraint. Large couplings $\kappa \gtrsim 10^{-2}$ are needed, which is marginally excluded by the CMB data. The reheat temperature has to be large $T_R \gtrsim 10^{13}$ GeV.

**VHS Scenario** The lepton number density is now given by Eq. (34), leading to

$$\frac{n_L}{s} = \frac{\epsilon_i f_X}{\gamma_{\text{in}}^2} \frac{\mu H_{\text{in}}^{1/2}}{m_X m_{\phi}^{3/2}}$$

(49)
with $H_{in} \simeq M_1^2/m_p$, $\gamma_{in} = \gamma_s \sim 1$, and with $i = 2, 3$. Leptogenesis requires

$$M_i \simeq 4 \times 10^2 C \frac{m_p \text{GeV}^{1/2}}{\eta^{1/2}} \left( \frac{M_i}{M_1} \right)^{1/2} \gtrsim 7 \times 10^{12} \text{GeV} C \kappa^{1/2} N^{1/12}$$

(50)

where in the second step we used the equilibrium condition Eq. (19) and $\eta = \eta_{CW}$.

The EGRET data requires the fraction of the total energy that goes into $X$-particles to be small $f_X \lesssim 10^{-5}$; as a result leptogenesis is not efficient enough and too large neutrino masses are needed, incompatible with the kinematic constraint. If however in the early scaling regime $f_X \sim 1$ also possible — remember that the EGRET bound is determined by late times — smaller neutrino masses are possible, as shown in Fig. (5).

Our results Eq. (46, 50) for $C = 1$, $C' = 1$ agree with those found in Ref. [16]. Ref. [17] assumes degenerate light neutrino masses, and finds stronger bounds.

5 $\mu$-term

The $\mu$-problem can naturally be resolved in SUSY hybrid inflation with the introduction of a superpotential term [37]

$$W = \lambda S H H'$$

(51)

where $H, H'$ contains the two Higgs doublets of the MSSM. After inflation, $S$ gets a VEV due to low energy SUSY breaking which is of order $\langle S \rangle \sim m_{3/2}/\kappa$ provided $\lambda > \kappa$ (otherwise $S$ ends up in wrong minimum), and the $\mu$-term is generated. However, the superpotential term above opens up a new decay channel for the inflaton and jeopardizes non-thermal leptogenesis.

The kinematic constraint $M_i < m_\chi/2$ together with the constraint $\lambda > \kappa$ assures that decay rate for inflaton decay into SM higgses and higgsinos

$$\Gamma_H = \frac{\lambda^2}{16\pi} m_\chi$$

(52)

is larger than the decay rate into RH neutrinos. Inflaton decay is predominantly into SM Higgs fields (unless all the couplings are tuned $2M_i/\eta \sim \kappa \sim \lambda$), and NT leptogenesis via inflaton decay does not occur. Likewise, string decay into Higgs fields does not contribute to the lepton asymmetry, since the Higgs decays into SM Higgses. On the other hand, if the string decays into gauge fields, or if RH zero modes are released during decay, leptogenesis is still possible. We will consider this possibility in some detail.
As a side remark, we note that there are other ways in these models to generate a \(\mu\)-term which do not alter the inflaton decay rate, and are thus compatible with NT leptogenesis from reheating \[38\].

Either case 2 or case 3 is realized, depending on whether the reheat temperature

\[ T_R \simeq 4 \times 10^{-2} \kappa^{3/2} \sqrt{\eta} m_p \left( \frac{\lambda}{\kappa} \right) \]

(53)

is smaller or larger than the mass of the lightest RH neutrino. Note that the reheat temperature is minimized in the limit \(\lambda \to \kappa\).

5.1 Results

5.1.1 Case 2

Leptogenesis is a result of string decay into RH neutrinos and into gauge fields which in turn decay into RH neutrinos. The RH neutrinos are out-of-equilibrium at all times, and the asymmetry is dominated by the initial time just at the end of inflation. There is no contribution to the asymmetry from inflaton decay nor from thermal leptogenesis.

The lepton asymmetry is given by Eq. (39) with the replacement \(\Gamma_N \to \Gamma_H\) and \(X = A\). This gives

\[ M_i \simeq 3 \times 10^8 \text{GeV} C^2 \left( \frac{\kappa}{\lambda} \right) \]

(54)

where we used \(\eta = \eta_{cw}\). The reheat temperature is a function of \(\kappa\) only: \(T_R(\kappa/\lambda) = 10^8, 10^{10}, 10^{12}\) GeV for \(\kappa = 10^{-5}, 2 \times 10^{-4}, 3 \times 10^{-3}\). The neutrino mass can be lowered by lowering the ratio \((\kappa/\lambda) \leq 1\), but at the cost of increasing the reheat temperature with the inverse ratio. This makes it harder to satisfy the out-of-equilibrium condition \(M_i > T_R\).

The results are shown in Fig. 6 for various values of \(C\) together with the kinematic constraint \(M_i < m_A \sim \eta, \) and the out-of-equilibrium condition \(M_i > T_R\). Note that the lower bound on the neutrino mass is proportional to \(f_X^{-1}\), and the results for different values of \(f_X\) can be obtained by multiplying with the appropriate factor. Masses as low as \(M_i \sim 10^8\) GeV are compatible with leptogenesis.

5.1.2 Case 3

The lightest RH neutrino reaches thermal equilibrium, and all asymmetry is erased until it falls out of equilibrium at \(T \sim M_1\).
Figure 6: $M_i$ vs. $\kappa$ for case 2 with $\mu$-term and $\eta = \eta_{CW}, N = 1$. The lines A-B correspond to $A = (\gamma_{in}, X, f_X) = (\gamma_{min}, A, 1)$, and $B = (\gamma_{best}, A, 1)$. The parameter space is bounded by the kinematic constraint (top), and the wash-out constraint $T_R < M_1$ (bottom).

**Loop scenario**  The lepton asymmetry is independent of the decay rate, in particular, on whether the inflaton decays into RH neutrinos or SM Higgses. Hence, the results of case 3 without a $\mu$-term apply and

$$M_i \simeq 10^{14} \text{GeV} C^{\alpha/2} \left( \frac{M_i}{M_1} \right)^{3/4}.$$  \hspace{1cm} (55)

The only differences are that now $i = 1$ is possible and still the lightest RH neutrino reaches equilibrium, and $f_N < \kappa^{-1}$ so that $C > \kappa^{1/2}$. The results do not depend on the decay rate and the symmetry breaking scale $\eta$; the $\kappa$-dependence enters only via $x_N$.

The lower bound on $M_i$ is shown in Fig. 7 for $1 < x_N < \kappa^{-1}$, together with the kinetic and equilibrium $M_1 < T_R$ constraint. Here it is assumed that $(\kappa/\lambda) = 1$, which minimizes the reheat temperature. Hence, the equilibrium constraint can be relaxed by taking $(\kappa/\lambda) < 1$, but at the cost of increasing the reheat temperature, and thereby aggravating the gravitino problem.
VHS scenario  The lepton asymmetry is the same as in case 3 without a $\mu$-term, and requires

$$M_i \simeq 4 \times 10^2 \frac{m_p GeV^{1/2}}{\eta^{1/2}} \left( \frac{M_i}{M_1} \right)^{1/2} = 4 \times 10^{12} \frac{GeV C'}{(\kappa N)^{1/6}} \left( \frac{M_i}{M_1} \right)^{1/2}$$

(56)

where in the second step we used $\eta = \eta_{cw}$. The result is shown in Fig. 7. The lower bound on the neutrino mass is proportional $C \propto f_X^{-1/2}$, and the result for different values of $f_X$ can be obtained by multiplying with the appropriate factor.

For couplings $\kappa < 10^{-4}$ the reheat temperature is $T_R < 10^{10} \text{ GeV}$, and this is a scenario with can accommodate inflation and leptogenesis, and in which both the gravitino problem and the $\mu$-problem are solved. The lightest RH neutrino can be in equilibrium or not: both case 2 and 3 can work.
6 Conclusions

In this paper, we have investigated various possibilities for leptogenesis after hybrid inflation when gauged $B-L$ is spontaneously broken at the end. One of the Higgs fields gives heavy Majorana mass to the RH neutrinos and NT leptogenesis can take place during reheating via inflaton decay into RH (s)neutrinos. Cosmic strings form at the end of inflation $[6,7]$. If stable, they also contribute to primordial fluctuations. Interestingly enough, since the string Higgs field breaks $B-L$, these are the so-called $B-L$ strings whose decay gives a second NT contribution to the lepton asymmetry of the universe $[8]$. In this paper we investigated which of these two mechanisms is most efficient, taking into account the CMB constraints $[15]$.

Leptogenesis via inflaton decay can account for the observed asymmetry for neutrino masses in the range $M_i = 10^9 - 10^{11}$ GeV, and quartic Higgs couplings $\kappa = 10^{-5} - 10^{-2}$. The minimal reheat temperature required for getting enough lepton asymmetry is $T_R \sim 7 \times 10^6$ GeV. To assure that all are out-of-equilibrium at $T_R$, and there is no wash-out of lepton number, the mass of the RH neutrino the inflaton decays into has to satisfy $M_i \lesssim 10^2 \times T_R$.

The calculation of the lepton asymmetry created by string decay is hampered by our poor knowledge of the properties of the string network. The initial string density, the loop formation and decay mechanisms all introduce uncertainties. In general, only the 'best case' scenarios give a large contribution to the lepton asymmetry, in which it is assumed that the initial string density is high and/or that the fraction of string energy going into $X$-particles is appreciable. We argued that both of these assumptions are not far-fetched, as they fit well with our knowledge of cosmic strings.

We distinguished three different cases, depending on whether the inflaton field decays into RH (s)neutrinos, and whether the lightest neutrino is out-of-equilibrium at reheating. In case 1, the inflaton decays into RH neutrinos and all RH neutrinos are out-of-equilibrium at $T_R$. The lepton asymmetry is determined by the energy density in the string network right at the end of inflation. If the strings mostly decay into Higgs particles, the contribution to the asymmetry is subdominant with respect to the contribution from inflaton decay. The strings can give a dominant contribution if it decays mostly into RH neutrinos or into gauge particles, but for the latter only in the 'best case' scenario.

If inflaton decay is not into RH neutrinos and the RH neutrinos never attain ther-
mal equilibrium (case 2), the only contribution to the baryon asymmetry of the Universe is from cosmic strings decay. String decay into RH neutrinos and vector particles, which subsequently decay into RH neutrinos, can still produce a lepton asymmetry. The minimal reheat temperature required for getting enough lepton asymmetry $T_R \sim 10^6$ GeV. Gravitational inflaton decay can ameliorate the gravitino constraint. However, large $M_i \sim 10^{13} - 10^{15}$ GeV RH neutrino masses and couplings $\kappa \gtrsim 10^{-3}$ are required. Such large couplings favor small Higgs representations.

The most unfavorable scenario is case 3, in which the lightest RH neutrino attains thermal equilibrium. The mass of the heaviest RH neutrino the inflaton can decay into (which cannot be $M_1$) must be large, $M_i > 10^{13}$ GeV, the coupling must also be large, and the reheat temperature must be high $T_R \gtrsim 10^{13}$ GeV. We note that in this case there is also a contribution to the lepton asymmetry from thermal leptogenesis.

Finally, we looked at the possibility of generating the MSSM $\mu$-term dynamically. Inflaton decay is now into SM Higgses and Higgsinos, and only case 2 and 3 can occur. For a wide range of RH neutrino masses and for couplings $\kappa < 10^{-4}$ this is a scenario which can accommodate inflation and leptogenesis, and in which both the gravitino problem and the $\mu$-problem are solved.

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A CP-asymmetry

In this appendix we derive the typical values for the CP-asymmetry factor $\epsilon_i$, extending the usual analysis to $i = 2, 3$. We use the formalism and notation of [39].

The CP-asymmetry is

$$\epsilon_i = -\frac{1}{8\pi} \frac{1}{(hh)^{1i}_{ii}} \sum_{j \neq i} \text{Im} \left[ \{(hh)^{1j}_{ij}\}^2 \right] f\left(\frac{M_j^2}{M_i^2}\right)$$

(57)

with in SUSY theories

$$f(x) = \sqrt{x} \left[ \frac{2}{x-1} + \ln(1 + x^{-1}) \right].$$

(58)

For hierarchical RH neutrino masses $M_1 \ll M_2 \ll M_3$, we need the limits $x \ll 1$ (where $f \rightarrow 3/\sqrt{x}$) and $x \gg 1$ (where $f \rightarrow -\sqrt{x}(2 + \ln(x))$). The function $f$ is maximized for $f \sim 1$, but this does not occur for hierarchical RH masses. Then [39]

$$\epsilon_1 \approx -\frac{3}{8\pi} \frac{m_u^2}{v^2} \frac{M_1}{M_2}.$$

(59)

Here $I \sim \mathcal{O}(1)$ a phase factor, $v = 174 \text{ GeV} \sin\beta$ the Higgs VEV (note that $\sin\beta \approx 1$ for $\beta \gtrsim 3$). In the second step we have used

$$(hh)^{22}_{22} \approx (m_c/v^2)I_{22}, \quad (hh)^{12}_{12} \approx (m_u m_c/v^2)I_{12}, \quad (hh)^{23}_{23} \approx (m_t m_c/v^2)I_{23},$$

(60)

with $I$ order one constants, and $m_u, m_c, m_t$ be Dirac neutrino masses which are labeled in analogy with the quark masses. One can express the Dirac masses in terms of the RH neutrino masses:

$$|M_1| = m_u^2/A_1, \quad |M_2| = m_c^2/A_2, \quad |M_3| = m_t^2/A_3,$$

(61)

with $A_1 = s^2_{12} \sqrt{\Delta m^2_{\text{sol}}}$, $A_2 = \sqrt{\Delta m^2_{\text{atm}}}/2$ and $A_3 = 2|m_1|/s^2_{12}$. The numerical values for $A_i$ depend on the spectrum of light neutrino masses. For example $A_1 = 2 \times 10^{-12} \text{ GeV}$, $3 \times 10^{-11} \text{ GeV}$ for normal hierarchy ($m_3 \approx (\Delta m^2_{\text{atm}})^{1/2} \gg m_2 \approx (\Delta m^2_{\text{sol}})^{1/2} \gg m_1$) respectively inverted hierarchy($m_1 \approx m_2 \approx (\Delta m^2_{\text{atm}})^{1/2} \gg m_3 \approx (\Delta m^2_{\text{sol}})^{1/2}$) [39]. Here it is approximated that $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. The CP-asymmetry is

$$\epsilon_1 \approx -\frac{3A_1 M_1}{8\pi v^2} = 10^{-11} - 10^{-10} \left(\frac{M_1}{10^6 \text{ GeV}}\right).$$

(62)
The bound in Eq. (22) is obtained for inverted hierarchy and order one phases. For quasi-degenerate neutrinos \( m_1 \approx m_2 \approx m_3 \gg (\Delta m^2_{\text{ATM}})^{1/2} \) the asymmetry is suppressed by a factor \( \Delta \sqrt{m^2_{\text{ATM}}/\bar{m}} \) with \( \bar{m} = 1/3\sqrt{m_1^2 + m_2^2 + m_3^2} \).

A similar calculation for decay of the next to lightest RH neutrino gives

\[
\epsilon_2 = -\frac{1}{8\pi} \frac{1}{(hh^\dagger)_{22}} \left[ \text{Im} \left[ \{(hh^\dagger)_{21}\}^2 \right] \frac{M_1}{M_2} + \text{Im} \left[ \{(hh^\dagger)_{23}\}^2 \right] \frac{M_2}{M_3} \right] \\
\approx -\frac{1}{8\pi} \left[ -\frac{m_2^2}{v^2} \frac{M_1}{M_2} + 3 \frac{A_3 v^2}{v^2} M_2 \right] \\
\approx 2 \times 10^{-10} \left( \frac{M_2}{10^6 \text{GeV}} \right) \tag{63}
\]

where in the last step we have neglected the subdominant term proportional to \( M_1 \), and we have used \( A_3 < 6 \times 10^{-11} \text{GeV} \) valid for hierarchical (light) neutrinos. The bound is of the same order of magnitude as the bound on \( \epsilon_1 \), but with \( M_1 \leftrightarrow M_2 \). Finally, for the CP-asymmetry of the heaviest RH neutrino we get

\[
\epsilon_3 = \frac{1}{8\pi v^2} \left[ \frac{M_1^2 A_1}{M_3} + \frac{M_2^2 A_2}{M_3} \right] \approx \frac{A_2 M_2}{8\pi v^2} \frac{M_2}{M_3} \\
= 3 - 7 \times 10^{-11} \left( \frac{M_2}{10^6 \text{GeV}} \right) \frac{M_2}{M_3} \tag{64}
\]

where we have used \( A_2 = 3 - 5 \times 10^{-11} \) valid for hierarchical light neutrinos. Note that \( \epsilon_3 \) is suppressed by a factor \( M_2/M_3 \) and is smaller than \( \epsilon_1, \epsilon_2 \).

References


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