The light curve of the semiregular variable L₂ Puppis: II. Evidence for solar-like excitation of the oscillations

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ABSTRACT

We analyse visual observations of the pulsations of the red giant variable L₂ Pup. The data cover 77 years between 1927 and 2005, thus providing an extensive empirical base for characterizing properties of the oscillations. The power spectrum of the light curve shows a single mode resolved into multiple peaks under a narrow envelope. We argue that this results from stochastic excitation, as seen in solar oscillations. The random fluctuations in phase also support this idea. A comparison with X Cam, a true Mira star with the same pulsation period, and W Cyg, a true semiregular star, illustrates the basic differences in phase behaviours. The Mira shows very stable phase, consistent with excitation by the κ-mechanism, whereas W Cyg shows large phase fluctuations that imply stochastic excitation. We find L₂ Pup to be intermediate, implying that both mechanisms play a role in its pulsation. Finally, we also checked the presence of low-dimensional chaos and could safely exclude it.

Key words: stars: individual: L₂ Pup – stars: AGB and post-AGB – stars: oscillations – stars: mass-loss – stars: variables: other

1 INTRODUCTION

L₂ Puppis (HR 2748; HIP 34922) is a bright nearby red giant with a pulsation period of about 140 d. In Paper I (Bedding et al. 2002), we showed that the star has undergone a remarkable change in mean visual magnitude over the past century, and is currently undergoing a dramatic decline. We argued that the most likely cause is the recent formation of dust in an extended atmosphere. In this paper, we discuss the pulsation behaviour of the star and suggest that convection makes a significant contribution to the driving mechanism.

The mechanism by which pulsations are excited in long-period variables is a subject of current interest. Mira variables have large amplitudes and are very regular, reflecting the nature of the driving process, which is self-excitation via opacity variations (i.e. the κ-mechanism). Semiregular variables (SRs), by contrast, have smaller amplitudes and less regular light curves. The nature of these irregularities is far from being understood. Multiple periodicity has been found in many cases and is generally interpreted in terms of multimode pulsations (Kiss et al. 1999, Percy et al. 2001). This interpretation was also supported by the multiple period-luminosity relations of SRs in the Magellanic Clouds (Wood et al. 1999, Wood 2000) and by observed mode changes in several semiregular stars (Cadmus et al. 1991, Percy & Desjardins 1996, Bedding et al. 1998, Kiss et al. 2000). However, even the clearest examples of multiply periodic SRs exhibit seemingly irregular additional changes in the light curves (Lebzelter & Kiss 2001, Kerschbaum et al. 2001), so that multimode pulsation with stationary frequency and amplitude content does not fully explain the light variations in SRs. Other explanations suggested in the literature include chaotic phenomena (Icke et al. 1992, Buchler et al. 2004), coupling of rotation and pulsation (Barban et al. 1995, Soszynski et al. 2004) and dust-shell dynamics (Höfner et al. 1995, 2003). Studies of radial velocity variability in SRs (Lebzelter et al. 2000, Lebzelter & Hinkle 2002) suggested that light variations are dominated by stellar pulsations, which implies that the observed irregularities must be largely related to stochastic behaviour of the oscillations.

In semiregulars, it seems plausible that there is a substantial contribution to the excitation and damping from convection. Indeed, Christensen-Dalsgaard et al. (2001) have suggested that the amplitude variability seen in these stars is consistent with the pulsations being solar-like, i.e., stochastically excited by convection (see Bouchy & Carrier 2003 and Bedding & Kjeldsen 2003 for recent reviews of solar-like oscillations). Note that we are following convention by referring to oscillations as ‘solar-like’ if, like those in the Sun, they are stochastically excited and damped. This is in con-
Figure 1. The full light curve of L2 Pup, showing the gradual mean brightness variations as 10-day means (top) and the corresponding amplitude spectrum (bottom). The small insert shows the spectral window (in amplitude), with sidelobes at ±1 cycle per year (0.0027 c/d) reflecting annual gaps in the data.

Figure 2. Oscillations in L2 Pup (after removal of the long-term trend). Each segment shows a 2375-d long subset, which covers roughly 17 cycles. The mean photometric uncertainty is about the same as the symbol size.

2 DATA ANALYSIS

The updated light curve of L2 Pup is plotted in the top panel of Fig. 1. The observations since JD 2451000 were made by A.J. Jones, P. Williams and L.L. Kiss. The dataset consists of 1981 points between JD 2425249 and 2453487, each one being a 10-day average. This is roughly 1000 days longer than in fig. 1 of Paper I and it clearly shows that the latest dimming event is still in progress, though it may have slowed down a little bit. The frequency spectrum (bottom panel in Fig. 1) is dominated by a set of low-frequency peaks, caused by the slow variations of the mean brightness, and the main group of closely separated peaks at the pulsation frequency ~0.007 c/d. There is also a slight indication of an increased amplitude density around ~0.015 c/d, which may be attributed to either a harmonic of the main pulsation (indicating non-sinusoidal variations) or a second (overtone) pulsation mode.

Details of the light curve can be seen in Fig. 2. Here, we subtracted the mean trend of the light curve by fitting a low-order polynomial to the data. Individual cycles have peak-to-peak amplitudes ranging from practically zero up to over 2 magnitudes, with no cycles having the same shape. This illustrates very well the semiregular nature of L2 Pup. There seems to be no correlation between the actual mean brightness and amplitude or cycle shape, which supports the idea that gradual brightness changes are not coupled to the pulsations.

The data were submitted to the following analysis:

(i) the mode lifetime was determined from a power spectrum fit;
(ii) phase variations were compared to a true Mira star with the same period;
(iii) the presence of low-dimensional chaos was tested with phase space reconstruction.

3 DISCUSSION

3.1 Power spectrum and mode lifetime

The power spectrum of the light curve is shown in Fig. 3. We show a close-up of the region corresponding to the 140-day pulsation period, and see that the power is split into a series of peaks under a narrow envelope. Note that the long term trends in the light curve, which we discussed in detail in Paper I, are irrelevant here because they occur at much lower temporal frequencies. We have verified that removing these long-term variations by subtracting a low-order polynomial has negligible effect on the power spectrum in Fig. 3.

The power distribution in Fig. 3 is typical of a stochastically excited oscillator and is strikingly similar to close-up views of
individual peaks in the power spectrum of solar oscillations. Assuming a stochastically excited damped oscillator, we have fitted a Lorentzian profile assuming $\chi^2$ statistics with two degrees of freedom. This is based on a maximum likelihood fit assuming an exponential distribution of the noise (Anderson et al. 1990, Toutain & Fröhlich 1992). The fit gives a period of 138.3 d with a half-width at half power of $\Gamma = 9.0 \times 10^{-3}$ d$^{-1}$, which implies a mode lifetime of $\tau = (2\pi\Gamma)^{-1} = 4.8$ yr (i.e. approximately 12.5 pulsation cycles).

This represents one of the first measurements of mode lifetime (or damping time) in a star with solar-like oscillations. Apart from the Sun itself, in which the lifetime is a 2–4 days (e.g. Chaplin et al. 1997), there are recent measurement of oscillation lifetimes in four stars. Bedding et al. (2004) and Kjeldsen et al. (2005) reported mode lifetimes of 2–3 d in both $\alpha$ Cen A and B. For the giant star $\xi$ Hyi Stello et al. (2004) suggested a lifetime of $\sim$2 days (about 20 mean cycle lengths), which is much shorter than was calculated theoretically (Houdé & Gough 2002). Finally, in the K giant Arcturus, which is the closest example to $L_2$ Pup, photometric data taken by the star tracker on the WIRE satellite implied strongly damped oscillations with a lifetime comparable to the mean period of 2.8 days (Retter et al. 2003). Since the damping rate depends on the stellar structure and convection properties in a complex way with a number of weakly constrained theoretical parameters (e.g. Balmforth, 1992), and moreover, there are no theoretical calculations directly applicable to $L_2$ Pup, it is currently impossible to qualify the agreement with theoretical expectations. We will address the dependence of mode lifetime on main physical parameters in a subsequent study of bright semiregular variables.

### 3.2 Oscillation amplitude and phase

Could the oscillations in $L_2$ Pup be driven entirely by convection, without any input from the $\kappa$-mechanism? This has been suggested for semiregular variables by Christensen-Dalsgaard et al. (2001), who described such oscillations as solar-like. The expected amplitude is highly uncertain, but we may extrapolate from solar-type stars using empirical scaling laws. Visual photometric amplitudes of red giant stars are highly dependent on the mean temperature, whereas velocity measurements, when available, give a more direct measure of the physical pulsation amplitude. Kjeldsen & Bedding (1995) proposed that the velocity amplitude of solar-like oscillations should scale as $L/M$. More recently, Kjeldsen & Bedding (2001) suggested a revised scaling relation (to account for low observed amplitudes in F stars), in which velocity amplitude scales as $1/g$ (which is proportional to $L/(MT_{\text{eff}}^2)$).

Adopting $L = 1500 L_\odot$, $T_{\text{eff}} = 3400$ K and $M = 1 M_\odot$ (Jura et al. 2002), the original and revised scaling relations predict velocity semi-amplitudes for $L_2$ Pup of about 400 m/s and 3 km s$^{-1}$, respectively. From the observational side, there are two published radial velocity measurements for $L_2$ Pup: Cummings et al. (1999) measured a semi-amplitude of about 2.5 km s$^{-1}$, while Lebzelter et al. (2005) reported six data points in a full range of 12 km s$^{-1}$ over almost three pulsation cycles, implying a semi-amplitude of 6 km s$^{-1}$. Given the large uncertainties of the extrapolated relations, the observed amplitudes do not contradict predictions. It therefore seems possible that the oscillations in $L_2$ Pup are excited entirely by convection, with no Mira-like contribution.

We find further evidence for non-Mira behaviour from a comparison of phase changes in $L_2$ Pup with those of a true Mira with a very similar pulsation period. A well-observed northern circumpolar star is X Cam, for which the General Catalogue of Variable Stars (Kholopov et al. 1985-1988) lists $m_H = \text{7m}^4 - \text{14m}^0.2$, $P=143.56$ d. We downloaded its visual data collected by the Association Française des Observateurs d’Étoiles Variables (AFOEV), extending back to the early 20th century, and binned them into 10-day averages. Also, as an example of a “real” semiregular, we took visual data of W Cyg, an SRb type star with two dominant periodicities, 131 d and 235 d (Howarth 1991). Then we calculated the following quantities for each star:

$$C(t) = \sum_{i=1}^{n} w(t, t_i)(m(t_i) - \langle m(t_i) \rangle) \cos(2\pi f t_i)$$

(1)

\[\text{ftp://cdsarc.u-strasbg.fr/pub/afoev/}\]
phase coherent positive feedback from periodic opacity changes. Consistent with the changes in this star. The phase variations of X Cam are fully cur. In contrast, W Cyg shows random phase variations on short phase stability over several thousand days but then some jumps oc-

Figure 5. Phase variations in L₂ Pup, W Cyg and X Cam as function of time.

\[ S(t) = \sum_{i=1}^{n} w(t, t_i)(m(t_i) - \langle m(t_i) \rangle) \sin(2\pi f t_i) \]  

which are closely related to the Fourier transform of the light curve \( \{m(t_i)\} (i = 1...n) \). The only difference is the presence of the Gaussian weight-function \( w(t, t_i) \), which was used as a moving window over the light curve (in other words, \( C(t) \) and \( S(t) \) are the real and the imaginary components of the Gabor transform; for a recent review of related transforms see Buchler & Kolláth 2001). In our case, \( w(t, t_i) \) was centered at \( t \) and ran between \( t_i \) and \( t_{i+1} \) with a time step of 50 days, and the full-width at half maximum was fixed as 700 days (about 5 cycles). This way we could measure the local phase \( \varphi(t) = \arctan(S(t)/C(t)) \) at fixed frequency \( f = 1/P \). For X Cam we determined \( P \approx 143.69 \) d, very close to the GCVS value, while for L₂ Pup we used the mean period given by the center of the fitted Lorentzian (138.3 d). In the case of W Cyg, we took the shorter period (131 d), which is the dominant one. As usual, \( \langle m(t_i) \rangle \) denotes the mean magnitude.

First we show the phase variations by plotting \( S(t) \) versus \( C(t) \) in Fig. The fact that L₂ Pup and W Cyg are closer to the origin than X Cam is a simple consequence of their smaller light curve amplitudes (proportional to \( \sqrt{C(t)^2 + S(t)^2} \)). The interesting thing is the phase range covered by the stars. While X Cam was meandering over a fairly narrow range of about 20–30°, drawing a loosely defined arc in the phase plane, L₂ Pup went almost all around the full circle. We interpret this difference as evidence that in L₂ Pup, the changes in both amplitude and phase are close to random, almost as randomly as in W Cyg, which is centered on the origin.

The intermediate nature of L₂ Pup is also apparent from the phase variations as functions of time (Fig. 5). X Cam’s phase is very stable in time, with no sudden changes; L₂ Pup has good phase stability over several thousand days but then some jumps occur. In contrast, W Cyg shows random phase variations on short time scales (see Howarth 1991 for further discussion of the phase changes in this star). The phase variations of X Cam are fully consistent with the \( \kappa \)-mechanism, which by definition includes a phase coherent positive feedback from periodic opacity changes.

3.3 A test for low-dimensional chaos

The phase of W Cyg varies continuously, supporting the view that the oscillations are stochastically excited, presumably by convection. Finally, L₂ Pup is an intermediate case. The phase fluctuations imply stochastic behaviour, but there may also be some driving from the \( \kappa \)-mechanism.

Could the seemingly complex pulsational behaviour of L₂ Pup be caused by a simple low-dimensional chaotic system? Recently, Buchler et al. (2004) concluded that the irregular pulsations in some semiregulars are the result of the non-linear interaction of two strongly nonadiabatic pulsation modes, although they admitted that stochastic procedures may also affect the light curve shape to some extent. The first clear detection of chaos in a Mira star was presented by Kiss & Szatmáry (2002), who demonstrated the chaotic origin of the amplitude modulation of R Cygni. Here we employ the same non-linear tools of light curve analysis in order to yield insights into the dynamics of the pulsations in L₂ Pup. Instead of using the whole dataset, we restricted the nonlinear analysis to the better-sampled two-thirds of the data, from JD 2435000 onwards.

We made the following pre-processing steps: 1. long-term trend removal by subtracting a polynomial that was fitted to the binned light curve; 2. smoothing and interpolating the resulting light curve. The first step was made to remove the long-term mean brightness variations that are caused by the increased dust extinction events and are not connected with pulsation (Paper I). We experimented with different orders of polynomials and accepted 10 as the best one which followed quite well the sudden changes of the mean level. After subtracting the polynomial, the residual light curve clearly showed the complex amplitude changes. Further inter-

Figure 6. A typical light curve segment with the smoothed and interpolated signal.

The pre-processed light curve was submitted to our nonlinear analysis. The applied numerical tools and their basics have been described by Hegger et al. (1999). Briefly, we used time-delay embedding to reconstruct the phase space of the system. First, we estimated the embedding parameters, the optimal time-delay embedding dimension. We then calculated various projections to visualize the trajectories in the phase space. The existence of an attractor would produce regular structures, and finding these was the aim of the analysis.

As for R Cygni, the optimal delay was found to be between 10–30% of the formal period. Since the reconstructed images did not show strong dependence on the time delay, we adopted 50 days,
Solar-like oscillations in $L_2$ Pup

3.4 Pulsation mode

Jura et al. (2002) cited the difference between 12 $\mu$m light curve and those at $H$ and $K$ for $L_2$ Pup as evidence that the pulsations are non-radial. Smith (2003) also noted the intriguing difference between the shorter and longer wavelength infrared observations of the DIRBE instrument, which was exceptional in a sample of 207 infrared sources. In particular, the maxima at 1.25 $\mu$m preceded those at 4.9 $\mu$m by 10–20 days, while at 4.9 $\mu$m, there was a secondary peak between the two 1.25 $\mu$m maxima. Also, the 2.2 $\mu$m and 3.5 $\mu$m light curves resembled the 1.25 $\mu$m curve, while the 12 $\mu$m curve was similar to that at 4.9 $\mu$m. The Jura et al. hypothesis was based on the assumption that, while the 12 $\mu$m flux tracks the light emitted by the circumstellar dust, and hence measures the luminosity of the entire star, the near-infrared fluxes measure only emission from the hemisphere of the star that faces the Earth. If

![Broomhead-King projections](image)

**Figure 7.** Broomhead-King projections. See text for further details.
there are differences then the photosphere must vary in time non-spherically, implying non-radial pulsation, which would also naturally explain the marked time variation of the position angle of the net polarization of the star (Magalhaes et al. 1986).

However, as we have shown in Paper I, the extinction-corrected $K$ band magnitude of $L_2$ Pup is very close to the value expected from the Mira P–L relations. The good agreement tends to argue against the hypothetical nonradial pulsations (unless one allows that Miras also pulsate non-radially). The full velocity amplitude of 12 km s$^{-1}$ (Lebzelter et al. 2005) is also hardly compatible with non-radial oscillations, which are not expected to result in large radial velocity amplitudes (Wood et al. 2004). Looking at DIRBE light curves of Mira stars, it is apparent that not only are there similar phase lags in Mira stars (Smith et al. 2002), but also that small differences do exist between the near- and mid-infrared data. As discussed by Smith et al. (2002), there are no theoretical models that predict these phase lags, because of the complexity of the problem. In addition to the pulsation-induced luminosity variations, the 12 $\mu$m light curve also reflects the dynamics of dust formation process which is governed by a time scale of its own (Winters et al. 2000 and references therein). Furthermore, the DIRBE observations were taken a few pulsation cycles before the beginning of the major dimming event in 1994. It seems to be likely that at the time of the DIRBE observations, there had already been some activity in dust formation, which can explain the extra features in the mid-infrared data without invoking non-radial pulsations. For example, Soker (2000) examined the effects of magnetic cool spots, above which large quantities of dust are expected to form. Alternatively, interactions with a close binary component (but outside the AGB envelope) may yield a strong density contrast in the equatorial plane, with similar outcomes. While currently there is no evidence for binarity in $L_2$ Pup, Ireland et al. (2004), using optical interferometry, found evidence for highly clumpy structures in the circumstellar dust shell as close as the dust condensation radius. Very recently, Wood et al. (2004) presented observations that suggested large scale star spot activity in some semirregulars with long secondary periods. Although $L_2$ Pup does not belong to that subclass of SRS, we think that the Soker (2000) model offers a more realistic mechanism to produce inhomogeneous dust and therefore different mid-infrared light curves in $L_2$ Pup than non-radial pulsations.

4 CONCLUSIONS

$L_2$ Pup is a very interesting semiregular star. Its light curve is dominated by two completely different mechanisms: gradual dimmings caused by circumstellar dust and pulsations. These two are completely independent, as we did not find any evidence for coupling between them. The present paper discussed properties of the pulsations, outlining the physical implications.

The frequency spectrum of the long-term light curve shows a peak of power that is resolved into multiple peaks under a narrow envelope. This is consistent with stochastic excitation, as seen in solar oscillations. The mode life-time can be derived from the width of the envelope: for $L_2$ Pup this is found to be about 5 yr. The oscillation amplitude agrees roughly with the predictions of simple scaling laws, while the phase behaviour is markedly different from a Mira star with the same pulsation period. The seemingly random amplitude and phase changes argue against excitation by the $\kappa$-mechanism alone, although it might make some contribution.

A test for low-dimensional chaos ruled out the possibility of non-linear interactions of a few pulsation modes as the reason for seemingly irregular light variations. We also argued against the hypothesis of non-radial pulsations in $L_2$ Pup: the difference between the near- and mid-infrared data may only be related to clumpy dust production, possibly driven by spot activity.

Bedding (2003) has recently given examples of other semirregulars whose power spectra display similar evidence for solar-like oscillations. The inferred mode lifetimes range from a few years, as found in $L_2$ Pup, down to less than a year. It seems plausible that stochastic excitation, presumably from convection, plays an important – perhaps dominant – role in the behaviour of semiregular variables.

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Solar-like oscillations in \textit{L\_2 Pup}