Frequency and surface dependence of the mechanical loss in fused silica

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We have compiled measurements of the mechanical loss in fused silica from samples spanning a wide range of geometries and resonant frequency in order to model the known variation of the loss with frequency and surface-to-volume ratio. This improved understanding of the mechanical loss has contributed significantly to the design of advanced interferometric gravitational wave detectors, which require ultra-low loss materials for their test mass mirrors.

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INTRODUCTION

As part of the research and development for the LIGO and TAMA gravitational wave detectors, we have conducted investigations into the internal friction of fused silica. Displacement of the interferometer’s mirror faces arising from thermal motion of the fused silica test mass mirrors sets a fundamental limit to the detector sensitivity. The frequency distribution of this noise is directly related to the internal friction of the mirror material.

An Advanced LIGO detector has recently been proposed with better sensitivity than initial LIGO. The Advanced LIGO mirror thermal noise must be as low as possible. Two materials have been under consideration for the mirror substrate: fused silica and single crystal sapphire. To its advantage, sapphire has the higher Young’s modulus and a low bulk mechanical loss. However, sapphire also has high thermoelectric noise.

In the advanced detectors, thermal noise in the mirror coatings makes a significant contribution to the total noise budget in the central frequency region of 30-500 Hz. Discussion on the mechanical loss in the mirror coatings can be found elsewhere.

Recent measurements on the mechanical loss in fused silica have revealed a dependence on frequency and on surface-to-volume ratio. This paper combines data from several of these research groups in order to model both of these effects. The frequency dependence of the loss agrees well with results from Weidersich et al. In that work, loss data spanning six decades in frequency is modeled by an asymmetric double-well potential in the bond angle. Together these results provide a more complete picture of the loss in ultrapure glasses and a more physically motivated prediction for the thermal noise in advanced interferometric detectors. It was previously predicted that fused silica’s loss dependence would make it suitable for low frequency detectors (10 – 100 Hz). Indeed, this model’s prediction of a very low mechanical loss in the LIGO frequency regime has motivated the recent selection of fused silica as the Advanced LIGO test mass substrate.

THEORY OF LOSS IN FUSED SILICA

The thermal noise motion of the mirror surface is related to the internal friction of the substrate by the fluctuation-dissipation theorem. The internal friction of very pure fused silica is associated with strained Si-O-Si bonds, where the energy of the bond has minima at two different bond angles, forming an asymmetric double-well potential. Redistribution of the bond angles in response to an applied strain leads to mechanical dissipation, which at audio frequencies has a peak in the cryogenic range 20-60K. Because fused silica is an amorphous material, there is a distribution of potentials which must be inferred from measurements of the dissipation. It can be shown that the frequency dependence of the loss should exhibit a power law spectrum with exponent $k_B T / V_0$ at low temperatures. Both this power law, with $V_0 / k_B = 319 K$, and the distribution of potentials have been measured. The power law exponent of a relaxation process cannot exceed 1, and is expected to saturate near 300 K. At room temperature the exponent is 0.76.

At elevated temperatures there is another loss peak arising from a double-well potential associated with the Si-O-Si bond angles. For this peak the bond angle shift and potential barrier are much larger; the double-well of the cryogenic loss peak is a small feature at the minima...
of this larger potential well. At room temperature, thermal fluctuations allow the bonds to span the cryogenic double-well but not to cross the larger potential barrier, where \( V_0/k_B = 3.54 \times 10^4 K \). The calculated internal friction for this loss peak at audio frequencies and room temperature is utterly negligible compared to other loss mechanisms cited herein.

A separate loss mechanism exists in the surface of the glass. The contribution from the surface loss depends on the mode of the sample. The total energy lost per oscillation in an isotropic sample undergoing slowly decaying vibration, can be described by the integral of the local loss angle, \( \phi(\vec{r}) \) with the energy density \( \rho_E(\vec{r}) \)

\[
\Delta E = 2\pi \int_V \rho_E(\vec{r}) \phi(\vec{r}) \, d^3r
\]

where \( V \) is the sample volume. Assuming that the local loss angle is constant and equal to \( \phi_{\text{bulk}} \) everywhere except within a distance \( h \) of the surface, and that the energy density in that surface layer is approximately the energy density at the surface, then the loss can be expressed as

\[
\phi = \phi_{\text{bulk}} + \mu \alpha_s \frac{S}{V} \quad (2)
\]

where \( S \) is the surface area of the sample and \( \mu \) is a factor of order unity that depends on the mode shape. The surface loss parameter, \( \alpha_s \), is typically several picometers for flame polished or flame drawn fused silica but much higher for abrasively polished surfaces.

**EXPERIMENTAL METHOD**

The measurements at Syracuse University (SU) \([10, 11, 12]\) were performed on fiber/rod samples with diameters ranging from 0.1 – 8 mm over resonant frequencies less than 5 kHz. The samples were drawn from and left suspended in a \( \approx 10^{-6} \) Torr vacuum by a loop of polished stainless steel wire greased with lard. The elastic modes of the mass were excited with an electrostatic actuator and the mode amplitude was monitored using a birefringence sensor. Since friction at the wire could reduce \( Q \), only modes with small motion at the point of wire contact were used in the fit.

**MODELING METHOD**

Resonant \( Q \) measurements from each of the labs were submitted for generating this model of the loss. The measurements spanned several types of fused silica, \( V/S \) ratios from 0.03 – 28 mm, and frequency up to \( 10^5 \) Hz. The data was first separated by silica type since the loss is known to vary significantly between varieties of fused silica \([4, 11, 20]\). Only Suprasil 2 and 312 had sufficient data to warrant a fit over both frequency and \( V/S \) ratio. Characteristics of these samples are listed in Table I.

We chose a model for the mechanical loss that included terms describing the frequency dependence, the surface loss, and the thermoelastic loss. The loss function took the form:

\[
\phi\left(\frac{V}{S}\right) = \phi_{\text{surf}} + \phi_{\text{bulk}} + \phi_{\text{th}}
\]

\[
= C_1 \left(\frac{V}{S}\right)^{-1} + C_2 (f/1 \text{ Hz})^{C_3} + C_4 \phi_{\text{th}}
\]

where \( C_1 = \mu \alpha_s \) from Eqn. 2. Given that the surface loss term only contributes significantly to the rod (fiber) samples, we have assumed for all samples that \( \mu \approx 2 \) which is appropriate for cylindrical rods. We have also not distinguished the loss angle arising from the Young’s modulus from that due to the shear modulus.

<table>
<thead>
<tr>
<th>Label</th>
<th>Type</th>
<th>D</th>
<th>h</th>
<th>V/S</th>
<th>Surface</th>
<th>Anneal</th>
<th>Lab</th>
</tr>
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<tr>
<td>P1</td>
<td>312, cyl.</td>
<td>254</td>
<td>100</td>
<td>28</td>
<td>SP</td>
<td>None</td>
<td>Caltech</td>
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<tr>
<td>K12</td>
<td>312, cyl.</td>
<td>70</td>
<td>60</td>
<td>11</td>
<td>SP</td>
<td>980°C, vac.</td>
<td>Tokyo</td>
</tr>
<tr>
<td>SU2</td>
<td>312, rod</td>
<td>3</td>
<td>–</td>
<td>0.75</td>
<td>FP</td>
<td>1025°C in Ar</td>
<td>SU</td>
</tr>
<tr>
<td>SV4</td>
<td>312, rod</td>
<td>8</td>
<td>–</td>
<td>2</td>
<td>FP</td>
<td>950°C in Ar</td>
<td>SU</td>
</tr>
<tr>
<td>AG5</td>
<td>2, rod</td>
<td>3.5</td>
<td>188</td>
<td>0.88</td>
<td>FP</td>
<td>None</td>
<td>SU</td>
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<tr>
<td>AH1</td>
<td>2, rod</td>
<td>0.300</td>
<td>108</td>
<td>0.075</td>
<td>FP</td>
<td>None</td>
<td>SU</td>
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<tr>
<td>AN1</td>
<td>2, rod</td>
<td>0.318</td>
<td>160</td>
<td>0.080</td>
<td>FP</td>
<td>None</td>
<td>SU</td>
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<tr>
<td>AB1</td>
<td>2, rod</td>
<td>0.062</td>
<td>175</td>
<td>0.016</td>
<td>FP</td>
<td>None</td>
<td>SU</td>
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<tr>
<td>AC1</td>
<td>2, rod</td>
<td>0.340</td>
<td>310</td>
<td>0.085</td>
<td>FP</td>
<td>None</td>
<td>SU</td>
</tr>
<tr>
<td>AF1</td>
<td>2, rod</td>
<td>0.120</td>
<td>130</td>
<td>0.030</td>
<td>FP</td>
<td>None</td>
<td>SU</td>
</tr>
<tr>
<td>K13</td>
<td>2, cyl.</td>
<td>70</td>
<td>60</td>
<td>11</td>
<td>SP</td>
<td>900°C, vac.</td>
<td>Tokyo</td>
</tr>
</tbody>
</table>

**TABLE I: Sample characteristics.** Type lists Heraeus Suprasil variety and shape. Samples are cylinders with diameter (D), height (h), and volume-to-surface ratio (V/S) given in mm. Surface types are superpolished (SP) and flame polished (FP).
FIG. 1: Suprasil 2 mechanical loss data: Best fit surface (upper) and Deviation in units of sample variance (lower).

![Graph 1](image1)

TABLE II: Fit coefficients for Suprasil 2 and 312.

<table>
<thead>
<tr>
<th>Type</th>
<th>C₁ (pm)</th>
<th>C₂ (×10⁻¹¹)</th>
<th>C₃</th>
<th>C₄</th>
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<tr>
<td>2</td>
<td>12.1 ± 0.8</td>
<td>1.18 ± 0.04</td>
<td>0.77 ± 0.02</td>
<td>0.61 ± 0.05</td>
</tr>
<tr>
<td>312</td>
<td>6.5 ± 0.2</td>
<td>0.76 ± 0.02</td>
<td>0.77 ± 0.02</td>
<td></td>
</tr>
</tbody>
</table>

The thermoelastic loss, \( \phi_{th} \), which is negligible in all but the thinnest fiber samples, is described for fibers by:

\[
\phi_{th} = \frac{Y \alpha^2 T}{\rho C_m} \frac{2\pi f \tau}{1+(2\pi f \tau)^2}
\]

\[
\tau = \frac{(d^2 \rho C_m)}{(13.55 \kappa)}
\]

where \( Y \) is the Young’s modulus, \( \alpha \) is the coefficient of thermal expansion, \( T \) is temperature, \( \rho \) is the density, \( C_m \) is the mass specific heat capacity, \( d \) is the diameter, and \( \kappa \) is the thermal conductivity. We fit the amplitude of \( \phi_{th} \) to account for small changes in the coefficient of thermal expansion among samples. Variations in the fiber diameter can also slightly alter the shape of the thermoelastic peak. Neither of these effects significantly affect the frequency or surface loss terms.

FIG. 2: Suprasil 312 mechanical loss data: Best fit surface (upper) and Deviation in units of sample variance (lower).

Measurements of large resonant \( Q \)'s are subject to numerous mechanisms that can greatly reduce the \( Q \) and few processes that can increase it. These effects produce a distribution in the systematic error that is asymmetric, heavily skewed toward lower \( Q \), and unique for each experiment. Standard data analysis techniques based on normally distributed error, such as linear least squares (LLS) fitting, are therefore inappropriate for analyzing our full data set. We circumvent this problem by first limiting our data to the best measurement at each \( (f, V/S) \) point for each sample. A LLS fitting routine is applied with the sample variance approximating the actual variance of the data. This method is commonly used in analyzing mechanical loss measurements where the lowest loss measurement closely approximates actual mechanical loss for a sufficiently large set of measurements [12, 13]. The results of the method are displayed in Figure 1 for Suprasil 2 and in Figure 2 for Suprasil 312. The fit coefficients are listed in Table II. The frequency dependence, \( C₃ \), agrees well with results from Weidersich et al. [14]. The thermoelastic amplitude, \( C₄ \), is similar to earlier measurements [10]. Assuming no unforeseen loss mechanisms, the Advanced LIGO test masses \((V/S \approx 40 \text{ pm})\).
mm) have a predicted loss ($\phi(100 \text{ Hz}) \approx 4 \times 10^{-10}$) that is a several-fold improvement over previous estimates.

![Graph showing estimated Advanced LIGO thermal noise for a Suprasil 312 test mass substrate and for two mirror coatings: the best measured and the research goal. Laser quantum noise provided for comparison.](image)

**FIG. 3:** Estimated Advanced LIGO thermal noise for a Suprasil 312 test mass substrate and for two mirror coatings: the best measured and the research goal. Laser quantum noise provided for comparison.

**TABLE III:** The distance a single Advanced LIGO interferometer could detect a neutron star or 10 $M_\odot$ black hole binary inspiral, assuming a Suprasil 312 test mass and two possible mirror coatings: the best measured and the research goal.

<table>
<thead>
<tr>
<th>Coating Loss</th>
<th>Loss Angle $\phi_1$</th>
<th>BNSI Range</th>
<th>BH/BH Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>$1.6 \times 10^{-4}$</td>
<td>190 MPc</td>
<td>840 MPc</td>
</tr>
<tr>
<td>Goal</td>
<td>$5.0 \times 10^{-5}$</td>
<td>230 MPc</td>
<td>1060 MPc</td>
</tr>
</tbody>
</table>

**IMPLICATIONS FOR ADVANCED DETECTORS**

The low mechanical loss of silica in the 10 – 1000 Hz bandwidth, coupled with its optical and thermal properties, makes it an attractive material for the optics of next generation interferometric gravitational wave detectors. Fused silica has recently been chosen as the test mass substrate for Advanced LIGO [3], which has been approved and recommended for funding by the US National Science Foundation.

If the bulk and surface loss predicted herein can be achieved, the mirror thermal noise in Advanced LIGO with fused silica mirrors will likely be dominated by the coating [4, 5, 21]. The mirror thermal noise contributions to the total Advanced LIGO noise budget are shown in Figure 8. Table III shows the predicted sensitivity of Advanced LIGO with silica optics to two possible sources of gravitational waves: binary neutron star inspirals (BNSI) and binary 10 $M_\odot$ black hole inspirals. Two different scenarios of coating thermal noise are shown: the best measurements to date [23] and the research goal. The sensitivity goal for a single Advanced LIGO interferometer is to observe BNSI, averaged over sky position and polarization, to a distance of $\approx 200$ Mpc. (See Harry [22] for a description of a LIGO range calculation.)

**CONCLUSIONS**

We have shown that the mechanical loss of fused silica can be described by a model that includes surface loss and a frequency dependent bulk loss. The frequency dependent loss, thought to arise from an asymmetric double-well potential of the bond angle, agrees well with earlier measurements [14] that spanned six decades in frequency. This improved understanding of the loss indicates that at large geometries and low frequency, fused silica is an excellent material for test masses in advanced interferometric gravitational wave detectors.

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