Unambiguous spin-gauge formulation of canonical general relativity with conformorphism invariance

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We present a parameter-free gauge formulation of general relativity in terms of a new set of real spin connection variables. The theory is constructed by extending the phase space of the recently formulated conformal geometrodynamics for canonical gravity to accommodate a spin gauge description. This leads to a further enlarged set of first class gravitational constraints consisting of a reduced Hamiltonian constraint and the canonical generators for spin gauge and conformorphism transformations. Owing to the incorporated conformal symmetry, the new theory is shown to be free from an ambiguity of the Barbero-Immirzi type.

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The success of the gauge field theory has long inspired efforts to reformulate Einstein’s general relativity (GR) into a canonically quantizable form resembling that of Yang-Mills. A major milestone in this regard is Ashtekar’s introduction of a set of “new” variables for GR\textsuperscript{1,2}. In essence, they consist of a spin connection on the spatial hypersurface with the densitized triad as conjugate momentum. The extrinsic curvature enters into the spin connection as a torsion term, so that the spin connection is independent of its momentum. This is the case since the densitized triad uniquely specifies the intrinsic geometry which in turn determines the torsion-free, i.e. Levi-Civita (LC), spin connection.

Following Sen’s pioneering work\textsuperscript{2}, the coefficient of this torsion term is chosen to be an imaginary number corresponding to the use of SL(2, C) as the gauge group. This choice has the advantage of simplifying the gravitational constraints to a polynomial form that permits a close analogy with the Hamiltonian of the Yang-Mills theory, and has been considered as a crucial rationale behind Ashtekar’s original complex approach to the gauge formulation of GR. This approach has grown into the active research area of “loop quantum gravity” using non-perturbative quantization techniques, as envisaged by Gambini, Jacobson, Rovelli, Smolin et al\textsuperscript{1,2,3,6,7,8}. However, the utility of such an approach has so far been limited mainly to solving the momentum constraints. Problems such as the physical inner product and the meaning of time\textsuperscript{3,6} are left unaddressed. Furthermore, the imposition of certain reality conditions for Ashtekar’s complex variables to yield real observables has proven to be problematic. This has prompted Barbero to consider an alternative set of spin gauge variables\textsuperscript{10} based on the real spin gauge group SO(3). A spin connection is also employed, with an arbitrary real positive parameter entering as the coefficient of the torsion term. Immirzi noted that the appearance of this free (Barbero-Immirzi) parameter in the resulting Hamiltonian constraint makes the quantized theory ambiguous\textsuperscript{11}. Although ongoing efforts are being made to fix this ambiguity by tuning the Barbero-Immirzi parameter to match certain quantum black-hole entropy predictions with the classical results using the Hawking-Bekenstein formula\textsuperscript{12}, such a state of the affair seems unsatisfactory from a theoretical viewpoint.

Following a previous paper\textsuperscript{13}, we show in this work how a real and parameter-free gauge formulation of GR can arise from an extended phase space of canonical gravity that incorporates conformal as well as spin symmetry. In what follows, the canonical analysis is rooted in the Arnowitt-Deser-Misner (ADM) formulation with metric signature (−,+,+,+) and a compact spatial sector. Units of $16\pi G = c = \hbar = 1$ are adopted.

Conformal geometrodynamics. The standard geometrodynamics is based on the ADM variables of GR consisting of the spatial metric $g_{ab}$ with conjugate momentum

$$p_{ab} = \mu(K_{ab} - g_{ab}K)$$

in terms of the (volume) scale factor $\mu := \sqrt{\text{det} g_{ab}}$ and the extrinsic curvature tensor is $K_{ab} = (2N)^{-1}(\dot{g}_{ab} - 2\nabla_a N_b)$ where the overdot denotes a time-derivative, $N$ is the lapse function and $N^b$ the shift vector. The trace $K = g_{ab}K_{ab}$ is the mean (extrinsic) curvature. We denote the LC connection of $g_{ab}$ by $\nabla$ and will also use a subscript “\perp” for the associated covariant differentiation. The ADM momentum and Hamiltonian constraints are given respectively by

$$\mathcal{H}_a = -2p_{ab}^b$$
$$\mathcal{H}_\perp = G_{abcd}p_{ab}^c p_{cd}^\perp - \mu R$$

where $G_{abcd} := (2\mu)^{-1}(g_{ac}g_{bd} + g_{ad}g_{bc} - g_{ab}g_{cd})$ and $R := R_{ab}^a$ is the Ricci scalar curvature of $g_{ab}$ in terms of the Riemann curvature tensor $R_{cde}^a$ satisfying $[\nabla_c, \nabla_d]V^a = R_{cde}^a V^b$ for any vector $V^a$. We shall analyze a sequence of canonical transformations of the gravitational variables. To keep track of different variables being used at each stage, we shall attribute them to a capital letter. We start by referring to $(g_{ab}, p_{ab})$ as the “G-variables”, w.r.t. which the Poisson bracket (PB) is denoted by $\{,\}$\textsuperscript{13}.
In \[13\], the problem of time and true dynamical degrees of freedom of GR has been reexamined in the canonical framework by extending the ADM phase space to that of York’s mean curvature time \( \tau := (4/3)K \) with \( \mu \) as momentum and conformal metric \( \gamma_{ab} \) with momentum \( \pi^{ab} \). Based on York’s decomposition of tensors \[14\], a canonical transformation has been found to relate the G-variables to the “\( \Gamma \)-variables” \( (\gamma_{ab}, \pi^{ab}; \tau, \mu) \) via
\[
g_{ab} = \phi^4 \gamma_{ab}, \quad p^{ab} = \phi^{-4} \pi^{ab} - \frac{1}{2} \phi^2 \bar{\mu} \gamma^{ab} \tau. \tag{4}
\]
Here \( \bar{\mu} := \sqrt{\det \gamma_{ab}} \) is the conformal scale factor and \( \phi := (\mu/\bar{\mu})^{1/6} \) is the conformal factor. The transformation using \( \phi \) is canonical since the original canonical PB relations for the G-variables are strongly preserved by the PBs w.r.t. the \( \Gamma \)-variables denoted by \( \{\ldots\}^\Gamma \). Using an arbitrary function \( f \), the local rescaling \( \gamma_{ab} \to f^2 \gamma_{ab}, \pi^{ab} \to f^{-4} \pi^{ab} \), while holding \( \tau \) and \( \mu \) unchanged, leave \( g_{ab}, p^{ab} \) invariant. This redundancy of the \( \Gamma \)-variables is offset by introducing the “conformal constraint”:
\[
C^\Gamma := \gamma_{ab} \pi^{ab} \tag{5}
\]
which generates local rescaling transformations through its PBs with \( \gamma_{ab}, \pi^{ab}, \tau \) and \( \mu \). In terms of \( C^\Gamma \), we have
\[
\begin{align*}
\mathcal{H} &= C^\Gamma_a + 4(\ln \phi)_a C^\Gamma, \tag{6} \\
\bar{C}^\Gamma_i &= \tau_{ab} \mu - 2 \pi_{ab;}^b, \tag{7} \\
\mathcal{H}_\perp &= \bar{C}^\Gamma_{\perp} + \frac{\tau}{2} C^\Gamma - \frac{1}{2\mu} (C^\Gamma)^2 \tag{8} \\
C^\Gamma_r &= -\frac{3}{8} \tau^2 \mu + \frac{1}{\mu} \pi_{ab} \pi^{ab} - \mu R. \tag{9}
\end{align*}
\]
In the \( \Gamma \)-variables, \( C^\Gamma \) play the role of the diffeomorphism constraints. The PBs amongst \( C^\Gamma \) and \( C^R \) satisfy the Lie algebra for conformorphisms on the spatial hypersurface \[13\ \[14\ \[17\]. It follows that \( \{C^\Gamma, C^\Gamma_a, C^\Gamma_i\} \) form a set of independent first class constraints, by noting the preservation \( \{\mathcal{H}_\perp, \mathcal{H}_\perp(x')\}^G = \{\mathcal{H}_\perp(x) \mathcal{H}_\perp(x')\}^\Gamma \) and that all summands in \[6\] and \[8\] are function-proportional to these constraints.

**Triad formalism.** Let us take one step of the canonical analysis back to the G-variables. Introduce the triad \( e_i^a \) with inverse \( e^i_a \) and densitized triad \( E_i^a = \mu e_i^a \) with inverse \( E^a_i \), so that
\[
g_{ab} = \mu^2 E^a_i E^b_j \tag{10}
\]
with its inverse given by \( g^{ab} = \mu^{-2} E^a_i E^b_j \), using the “spin indices” \( i, j, \ldots = 1, 2, 3 \). We choose the orientation so that det \( e^a_i = \mu > 0 \). Introduce the spin-valued extrinsic curvature \( K^i_a \) so that the extrinsic curvature tensor is
\[
K_{ab} = \frac{\mu}{2} K^i_a E^i_b. \tag{11}
\]
The “\( K \)-variables” \( (K^i_a, E^i_a) \) are coordinates of an extended phase space of GR. The redundancy is due to the spin transformation and can be eliminated by means of the constraint:
\[
K_{ab} := \mu^2 K^i_a E^i_b. \tag{12}
\]
The \( K \)-variables are canonical since by regarding the \( G \)-variables as functions of them via \[11\] and \[10\] we have
\[
\begin{align*}
\{g_{ab}(x), p^{cd}(x')\}^K &= \delta^d_a \delta(x, x') \tag{13} \\
\{g_{ab}(x), g_{ab}(x')\}^K &= 0 \tag{14} \\
\{p^{ab}(x), p^{cd}(x')\}^K &= K^{abcd}(x) \delta(x, x'). \tag{15}
\end{align*}
\]
In terms of \( K^{abcd} := 1/8 (g^{ac}K^{bd} + g^{ad}K^{bc} + g^{bc}K^{ad} + g^{bd}K^{ac}) \). From \[13\ \[14\], we have in general
\[
\{A, B\}^K = \{A, B\}^G + \int \delta A \frac{\delta B}{\delta p^{ab}(x)} K^{abcd}(x) d^3x.
\]
However, by exploiting the property \( K^{abcd} = K^{(ab)(cd)} = -K^{(cd)(ab)} \) and algebraic dependence of \( \mathcal{H} \) on \( p^{ab} \) we get
\[
\{\mathcal{H}(x), \mathcal{H}(x')\}^K = \{\mathcal{H}(x), \mathcal{H}(x')\}^G \tag{16}
\]
which will prove useful in simplifying our canonical analysis tasks. Instead of \( K_{ab} \), it is advantageous to adopt
\[
C_i^K := \epsilon_{ijk} K^i_a E^a_k = \frac{1}{\mu^2} \epsilon_{ijk} K_{ab} E^a_j E^b_k \tag{17}
\]
serving as the canonical generator for rotation (spin). A natural connection associated with the spin indices is the LC spin connection \( \Gamma^a_i \), so that the associated spin covariant derivative of any spin-valued scalar \( S^a \) is given by
\[
\nabla_a S^i = \partial_a S^i + \epsilon_{ijk} \Gamma^j_a S^k. \tag{18}
\]
The spin connection itself is uniquely determined in terms of \( E^a_i \) by the torsion-free condition
\[
\nabla_i E^a_i = 0. \tag{19}
\]
By varying \[14\] we see a relation of the form
\[
\delta E^a_i = E^a_{bj} \delta \Gamma^b_j \tag{20}
\]
where \( E^a_{bj} \) is algebraic in \( E^a_i \) and so \( \nabla_c E^a_{bj} = 0 \). This form has two remarkable consequences: First, for any scalar \( \varphi \) satisfying \( d \varphi = 0 \), if \( \delta E^a_i = E^a_i \delta \varphi \) then \( \delta \varphi = 0 \). Hence, like the LC connection \( \Gamma^a_{bc} \), the LC spin connection is invariant under constant conformal transformations. Secondly, the form in \[20\] implies that \( E^a_i \delta \Gamma^b_j \) is a total divergence, given specifically by
\[
E^a_i \delta \Gamma^b_j = \frac{1}{2} (\epsilon_{ijk} E^a_i E^b_j \delta E^a_k)_b. \tag{21}
\]
One therefore has \( \Gamma^a_i(x) = \delta F/\delta E^a_i(x) \) in terms of the “generating function” \( F := \int \Gamma^a_i E^a_i d^3x, \) together with the following integrability identity \[14\]:
\[
\frac{\delta \Gamma^a_i(x)}{\delta E^a_j(x')} = \frac{\delta \Gamma^a_j(x')}{\delta E^a_i(x)}.
\tag{22}
\]
Spin gauge formalism. By virtue of (22), a one-parameter family of phase spaces of GR can be constructed as:
\[
A_i^a := \Gamma_{ia} + \beta K_i^a, \quad P_i^a := \frac{E_i^a}{\beta} \tag{23}
\]
parametrized by a nonzero complex constant \(\beta\). We refer to the \((A_i^a, P_i^a)\) as the “A-variables” and denote the corresponding PB by \(\{\cdot, \cdot\}^A\). The transformation from the \(K\)- to \(A\)-variables is canonical since the following equations
\[
\begin{align*}
\{K_i^a(x), E_j^b(x')\}^A &= \delta_i^j \delta_a^b \delta(x, x') \tag{24} \\
\{K_i^a(x), K_j^b(x')\}^A &= 0 = \{E_i^a(x), E_j^b(x')\}^A \tag{25}
\end{align*}
\]
hold for any \(\beta\). In validating the first equation in (24), the identity (22) has been evoked. It follows that all PB relations in the \(K\)-variables are strongly preserved in the \(A\)-variables. By design of relations in (24), a spin connection with \(\tilde{\beta}\) has been evoked. It follows that all PB relations in the \(K\)-variables are strongly preserved in the \(A\)-variables. By design of relations in complete analogy with (18)–(22) hold with the substitutions
\[
\begin{align*}
\Gamma_{ia} &\rightarrow \Gamma_{ia} + \beta K_i^a, \\
P_i^a &\rightarrow P_i^a + \epsilon_{ijk} A_i^a S^k, \\
\{E_i^a(x), E_j^b(x')\}^A &\rightarrow \{E_i^a(x), E_j^b(x')\}^A\tag{26}
\end{align*}
\]
where we have introduced \(K_i^a := A_i^a - \Gamma_{ia} = \beta K_i^a\) as the torsion contribution. The associated covariant differentiation and curvature 2-form are denoted by \(D\) and \(F_{iab}\) respectively, so that any spin-valued scalar \(S^i\) have
\[
\begin{align*}
D_i S^i &= \partial_i S^i + \epsilon_{ijk} A_i^a S^k \tag{27} \\
\{D_i, D_k\} S^i &= \epsilon_{ijk} F_{iab} S^k.
\end{align*}
\]
By using (26) and (24) we see that \(C_i^K = \epsilon_{ijk} K_i^a E_j^b = P_{iab}^a + \epsilon_{ijk} A_i^a P_b^k\) which allows one to express the spin constraint in the form
\[
C_i^A := D_i P_i^a \tag{28}
\]
alogous to the “Gauss constraint” in the Maxwell and Yang-Mills gauge theories. Furthermore, the momentum constraint becomes
\[
\begin{align*}
\mathcal{H}_a &= C_a^A + \Gamma_{a}^k C_k^A - \frac{1}{2} \epsilon_{ijk} E_i^a E_j^b C_k^{c} \tag{29} \\
C_a^A &:= F_{iab} P_b^k - A_i^a C_k^A. \tag{30}
\end{align*}
\]
The constraints \(C_k^A\) and \(C_a^A\) respectively generates rotations and diffeomorphisms through their PBs with the \(A\)-variables. The Hamiltonian constraint then becomes:
\[
\begin{align*}
\mathcal{H}_\perp &= C_\perp^A + \frac{2\beta^2}{\mu} P_{k}^c C_{k;c} + \frac{1}{8\mu} C_k^C C_k^C \tag{31} \\
C_\perp^A &:= \frac{1}{\mu} \left[ \epsilon_{ijk} \beta^2 F_{iab} - \frac{3\beta^2}{2} K_i^a \tilde{K}_j^b \right] P_i^a P_j^b \tag{32}
\end{align*}
\]
Note that terms like \(P_{k}^c C_{k;c}\) in (31) are regarded as proportional to the constraints involved, as the LC covariant differentiation of the constraint there will be swapped over to their coefficients once “smeared” over the spatial hypersurface. The constraints \(C_k^A, C_a^A\) and \(C_\perp^A\) hence form a set of independent first class constraints. Ashtekar’s original gauge formalism of GR corresponds to the choice \(\beta = \pm i/2\) so that the non-polynomial term in (32) vanishes. In Barbero’s modified approach, \(\beta\) is considered as a real and positive parameter in order to resolve the reality problem on quantization.

Conformal triad formalism. We shall develop a new set of real gauge variables for GR that contains no free parameters. The ambiguity due to \(\beta\) described above arises from an arbitrary scaling factor in defining the \(A\)-variables. If an alternative set of gauge variables for GR can be found that possesses a conformal symmetry, then such an arbitrariness may be absorbed. Therefore, the search for such variables naturally involves an extension of the GR phase space by incorporating this symmetry. By analogy with the passage from the triad to spin gauge formalism of GR discussed above, we shall first introduce a set of “conformal triad variables” as a precursor of the ultimate spin gauge variables of GR with conformal symmetry.

To this end, we introduce the conformal triad \(\tilde{e}_i^a\) with inverse \(e^a_i\) so that \(\gamma_{ab} = \tilde{e}_a^i \tilde{e}_b^i\) and \(\gamma^{ab} = \tilde{e}_a^i \tilde{e}_b^i\). Further, we introduce the densitized triad \(E_i^a = \mu \tilde{e}_a^i\) with inverse \(E_i^a = \mu^{-1} \tilde{e}_i^a\). The LC spin connection of \(\gamma_{ab}\) will be denoted by \(\nabla\) and the associated covariant differentiation also denoted by a subscript “\(^\perp\)”. Relations in complete analogue with (18)–(22) hold with the substitutions
\[
\begin{align*}
ge_{ab} &\rightarrow g_{ab}, \quad \Gamma_i^a \rightarrow \tilde{\Gamma}_i^a, \quad \nabla \rightarrow \nabla, \quad E_i^a \rightarrow E_i^a. \tag{33}
\end{align*}
\]
A trace-split of the extrinsic curvature \(K_i^a\) is then performed in a conformally invariant manner. These considerations lead to the spin version of (18) as
\[
\begin{align*}
E_i^a &= \phi^4 E_i^a, \quad K_i^a = \phi^{-4} K_i^a + \frac{1}{2} \phi^2 \tilde{e}_i^a E_i^a \tau \tag{34}
\end{align*}
\]
where we have introduced \(K_i^a\) to function as the “conformal extrinsic curvature”. We have thus arrived at a set of conformal triad description of GR using \((K_i^a, E_i^a) := (K_i^a, E_i^a; \tau, \mu)\), called the “K-variables”. Using the corresponding PB denoted by \(\{\cdot, \cdot\}^K\), we can show that these variables are indeed canonical, since (31) implies
\[
\begin{align*}
\{K_i^a(x), E_j^b(x')\}^K &= \delta_i^j \delta_a^b \delta(x, x') \tag{35} \\
\{K_i^a(x), K_j^b(x')\}^K &= 0 = \{E_i^a(x), E_j^b(x')\}^K. \tag{36}
\end{align*}
\]
From (34) one might see the factor \(\phi^4\) as a generalization of \(\beta\). However, instead of being a free parameter, it is important to note that this factor depends on the canonical variables which in turn have a conformal symmetry.

Conformal spin gauge formalism. Based on the preceding discussions, we shall now formulate our final phase space of GR by incorporating conformal symmetry while retaining a spin gauge structure. The basis of the canonical transformation from the \(K\)- to \(A\)-variables can be traced to the term \(E_i^a \tilde{K}_i^a\) being a total divergence due to (34). As such, up to an arbitrary constant coefficient, say \(1/\beta\), it can be added to the time-derivative terms in the canonical action for GR in the \(K\)-variables as follows:
\[
\begin{align*}
E_i^a \tilde{K}_i^a + \frac{1}{\beta} E_i^a \tilde{K}_i^a &\rightarrow P_i^a \tilde{A}_i^a. \tag{37}
\end{align*}
\]
The result is the time-derivative terms in the \( A \)-variables. In a similar fashion, the analogy of (22) with substitution (33) enables us to add the total divergence (1/\( \alpha \))\( E^a_i \Gamma^i_a \) to the time-derivative terms in the K-variables:

\[
\mu \dot{x} + E^a_i \dot{K}^i_a + \frac{1}{\alpha} E^a_i \dot{\Gamma}^i_a = \mu \dot{x} + \frac{E^a_i}{\alpha} (\Gamma^i_a + \alpha K^i_a) \quad (38)
\]

for any constant 1/\( \alpha \). However, owing to the “built-in” conformal symmetry of the K-variables, this constant can always be absorbed into the variables themselves using

\[
E^a_i \to \alpha E^a_i, \quad K^i_a \to \frac{K^i_a}{\alpha}, \quad \Gamma^i_a \to \Gamma^i_a
\]

so long as \( \alpha \) is real and positive. In this case, (38) yields the terms \( \mu \dot{x} + \Pi^a_i \dot{A}^i_a \) in a new set of variables:

\[
A^i_a := \Gamma^i_a + K^i_a, \quad \Pi^i_a := E^a_i \quad (40)
\]

We call \( (A^C, \Pi_C) := (A^i_a, \Pi^a_i; \tau, \mu) \) the “A-variables” and denote the associated PB by \( \{\cdot, \cdot\}^A \). The canonical nature of these variables can be verified by the PB relations:

\[
\{K^C(x), E_D(x')\}^A = \delta^C_D \delta(x, x') \quad (41) \\
\{K^C(x), K^D(x')\}^A = 0 = \{E_C(x), E_D(x')\}^A \quad (42)
\]

In deriving the first equation in (42), we have used the analogue of (22) with (33). The spin covariant derivative associated with \( A^i_a \) and its curvature 2-form are denoted by \( \bar{D} \) and \( \bar{F} \) respectively. Henceforth, \( K^i_a \) is understood in terms of the A-variables as \( A^i_a - \Gamma^i_a \). It follows that

\[
C^\Gamma = \frac{1}{2} K^i_a \Pi^a_i =: C^A \quad (43) \\
C^i_a = \epsilon_{ijk} K^j_b \Pi^a_b =: \epsilon^a_i \quad (44) \\
H_a = 2 C^a_i \Pi^i_a =: C^A_a \quad (45)
\]

The constraints \( C^\Gamma, C^i_a \) and \( C^A_a \) may be called the conformal, spin and diffeomorphism constraints in the A-variables respectively, as they generate the corresponding transformations using \( \{\cdot, \cdot\}^A \). Finally, the Hamiltonian constraint in the form of (8) with (9) becomes

\[
\mathcal{H}_\perp = C^A_a + \frac{1}{\mu} C^A k^A_k + \frac{2 \phi^8}{\mu} \Pi^i_c C^A_{ik} + \frac{\tau}{2} C^A + \frac{1}{2 \mu} (C^A)^2 \\
\mathcal{C}_\perp := -\frac{3}{8} \tau^2 \mu + 8 \mu \phi^{-5} \Delta \phi
\]

where \( \Delta := \gamma^{ab} \nabla_a \nabla_b \) is the Laplacian associated with the conformal metric \( \gamma_{ab} \). In (17), the third term is analogous to (22) with \( \beta \to \phi^4 \). There, the “additional” first term is due to the York time \( \tau \) being separated from the conformal part of kinematics whereas the second term counts for the conformal factor \( \phi \) being a local function of the A-variables.

By virtue of (14) and the preservation of the PB of \( \mathcal{H}_\perp(x) \) and \( \mathcal{H}_\perp(x') \) throughout all canonical transformations considered in this work, we conclude that \( C^A \) and the canonical generators \( C^A_a, C^A_i, C^A_a \) form a set of first class constraints for the above conformal spin gauge formulation of GR using the A-variables. There are no free parameters in the description. The main tradeoff seems to be the two extra leading terms in the effective Hamiltonian constraint in (17). While the first term looks quite simple, the second term may post new challenges in the regularization procedure on quantization. Nevertheless, the absence of any parameter in the presented formulation of GR opens up an interesting possibility for “conformal loop quantum gravity” that is at least free from the Barbero-Immirzi ambiguity. Furthermore, a unitary evolution of loop quantum gravity may also be addressed as per the discussion towards the end of (14). Discussions on the related quantum issues as well as the full detail of the present work are deferred to a future publication.

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