SPIN STRUCTURE AT THE PARTONIC LEVEL
I : DEEP INELASTIC LEPTON SCATTERING

E. Leader
Westfield College, London NW3 7ST

ABSTRACT

The fundamental internal structure of hadrons can only be probed fully using polarised beams and targets. We describe some of the essential features that can be studied in electromagnetic and weak charged current reactions and make some comments about Drell-Yan processes.

1. INTRODUCTION

It is totally meaningless to consider spin structure as a separate, isolated subject. The spin quantum number of quark or gluon is an intrinsic and fundamental attribute that plays a deep role in the properties of individual hadrons and in their behaviour when reacting with other hadrons. For this reason my report is written in two sections, the first dealing with leptonic scattering, where one learns, via the lepton probe, about the internal spin structure of the hadrons, and the second,* dealing with hadronic reactions, where one uses this information together with perturbative QCD to calculate the experimental observables. The latter, then, constitutes an effective tool for probing and testing the basic QCD elements. As to the former, ultimately of course one will test what one has learnt about the hadrons against the predictions of non-perturbative QCD, but it will probably be some years before that can be done reliably.

The questions that are answered in deep inelastic lepton scattering are: "What is the fundamental internal structure of a hadron? How is it built from quarks and gluons? What is their wave-function, i.e. how, in a hadron of definite helicity, are the momenta and spins of the constituents distributed?"

Unpolarised experiments have already provided an enormous amount of information on such quantities u(x,Q^2), the number density of "up" quarks with momentum fraction x, to be found in an unpolarised proton when probed by a lepton with q^2 = -Q^2. Experiments with polarised hadrons teach us about number densities such as

\[ u(x,Q^2), \quad \Lambda(x,Q^2) \]

which are the number densities of "up" quarks with spin parallel (\(\Lambda\)) or anti-parallel (\(\Lambda\)) to the helicity of the parent hadron. The usual number density is, of course, related to these:

\[ u(x,Q^2) = \Lambda(x,Q^2) + u(x,Q^2) \quad ... \quad (1) \]

*) See page 384.
It must not be forgotten that in order to study the partonic spin dependence one has to have both polarised hadrons and a polarised initial or final lepton beam. This is because the parity-conserving single spin asymmetries all vanish as a consequence of the simplicity of the dynamical mechanism, i.e. one-photon or one-weak boson exchange. For the latter the lepton polarisation is guaranteed since neutrinos are 100% polarised. (There are, of course, parity-violating single spin asymmetries, but they are best thought of as teaching us about the structure of electroweak theory. Some of these are dealt with in the contribution by Soffer).

In the following our main concern is to expose the basic physical ideas and not to look into the detailed dynamics. Thus we shall talk always in terms of the simplest parton model concepts and we shall leave questions as to QCD effects, $Q^2$ dependence etc. to the contribution by Craigie.

2. ELECTROMAGNETIC INTERACTIONS : $\mu^+ p \rightarrow \mu' X$

In general, with a polarised nuclear target, there are four independent structure functions\(^1\) $W_1$, $W_2$, $G_1$ and $G_2$. They are expected to scale in the following fashion:

\[
\begin{align*}
mW_1(v, Q^2) & \rightarrow F_1(x) \\
vW_2(v, Q^2) & \rightarrow F_2(x) \\
m^2G_1(v, Q^2) & \rightarrow g_1(x) \\
vG_2(v, Q^2) & \rightarrow g_2(x)
\end{align*}
\]

where $m$ is the nucleon mass and the scaling is in the Bjorken limit.

In the simple parton model we have, in addition,

\[
\begin{align*}
F_2(x) &= 2xF_1(x) \\
G_2(x) &= g_1(x)
\end{align*}
\]

Assuming the validity of these scaling forms one has for the observable quantities\(^2\):

a) Unpolarised

\[
\frac{d^2\sigma}{d\Omega dE'} + \frac{d^2\sigma}{d\Omega dE'} \propto \frac{y + 2(1-y)}{2y} F_2(x)
\]

where $\rightarrow$ and $\leftarrow$ imply nucleon and lepton helicities parallel or anti-parallel respectively.

b) Longitudinally polarised lepton beam and target:

\[
\frac{d^2\sigma}{d\Omega dE'} - \frac{d^2\sigma}{d\Omega dE'} \propto \frac{(1-y/2)}{[2xg_1(x)]}
\]

c) Longitudinally polarised leptons, transversally polarised nucleons:
\[
\begin{align*}
\frac{d^2\sigma^{\uparrow}}{d\omega dE'} &= \frac{d^2\sigma^{\downarrow}}{d\omega dE'} \cdot C \cdot \frac{\sin (1-y)(1+y/2)[2xg_1(x)]}{y} 
\end{align*}
\] ...

\[
\text{(7)}
\]

Measurements of these thus give us \( g_1(x) \) and also test the validity of (4) which, like (3), only holds strictly in the simple parton model.

The theoretical interpretation of the above quantities in terms of the quark number densities \( q_i^p(x), q_i^n(x) \) is very simple \(^3\):

\[
\begin{align*}
F_2(x) &= x\Sigma_i^2 q_i(x) \quad \ldots \quad (8) \\
2xg_1(x) &= x\Sigma_i^2 [q_i^p(x) - q_i^n(x)] \quad \ldots \quad (9)
\end{align*}
\]

Thus the experiments directly yield information on the fundamental quark distributions.

3. CHARGED CURRENT WEAK INTERACTIONS : \( \mu^+ \rightarrow \nu^e \)

These reactions are experimentally more difficult than the electromagnetic ones and there is no hope of having a large enough polarised target to do \( \mu^+ \rightarrow \nu^e \). But to compensate, their theoretical interpretation is simpler. The charged weak current involves only left-handed quarks and right-handed anti-quarks; moreover either a "u" or a "d" quark participates, not both. Thus, in the approximation that the Cabibbo angle is zero, and at \( x \)-values where anti-quarks can be neglected:

\[
\begin{align*}
\mu^- \text{ on } p^- &\rightarrow u^e(x) \\
\mu^- \text{ on } p^+ &\rightarrow u^p(x) \\
\mu^+ \text{ on } p^- &\rightarrow d^e(x) \\
\mu^+ \text{ on } p^+ &\rightarrow d^p(x)
\end{align*}
\] ...

\[
\text{(10)}
\]

where \( \leftarrow \) means proton helicity \( +\frac{1}{2} \) in the lepton-hadron C.M. We thus see that C.C. reactions with a polarised target measure the spin-dependent quark distributions directly.

4. Theoretical Models for the Spin-dependent Quark Densities

In principle, if one could solve the non-perturbative dynamical problem of the binding of three quarks to form a nucleon, one would have available the wave function

\[
\Psi_\Lambda(x_1, \tau_1, \lambda_1; x_2, \tau_2, \lambda_2; x_3, \tau_3, \lambda_3)
\]

for a proton of helicity \( \Lambda \) made up of quarks with momentum fractions \( x_i \), third components of isospin \( \tau_i \), and helicities \( \lambda_i \), from which all the quark
distributions could be calculated directly. We are a long way from the goal, but it is not inconceivable that lattice calculations will begin to yield this sort of information within the next few years.

In the absence of detailed dynamical calculations one resorts to "educated guesses" about the structure of the wave-function, guided by what is known from low energy quark models of the hadron spectrum.

The simplest assumption is that the effective Hamiltonian conserves both spin and isospin, so that the wave-function can have SU(2)_{Spin} \times SU(3)_{Isospin} structure. Assuming that the colour wave-function is anti-symmetric, and that we require a symmetric ground state space wave-function, we are forced to seek a symmetric spin \times isospin wave-function in order to guarantee overall adherence to the Pauli principle. However the only spin or isospin wave-functions with \( S = \frac{1}{2}, I = \frac{1}{2} \) are of mixed symmetry. For example for a proton, spin up, one could have, for the spin part, in an obvious notation\(^4\),

\[
\chi_{MS} = \frac{1}{\sqrt{6}} \left\{ (\uparrow\uparrow + \downarrow\downarrow) \uparrow - 2 \downarrow\uparrow \right\} 
\]

or

\[
\chi_{MA} = \frac{1}{\sqrt{2}} \left\{ (\uparrow\downarrow - \downarrow\uparrow) \uparrow \right\} 
\]

When \( M \) stands for mixed, and the \( S, A \) refer to the symmetry or anti-symmetry under interchange of quarks 1 and 2.

The isospin wave functions \( \phi_{MS}, \phi_{MA} \) look identical, with \( \uparrow \) replaced by "u", \( \downarrow \) by "d".

The only completely symmetric spin \times isospin wave-function turns out to be \( \chi_{MS} \phi_{MS} + \chi_{MA} \phi_{MA} \) so one has, for the proton wave-function, the form

\[
\psi_{\lambda = \frac{1}{2}} = \int (x_1, x_2, x_3) \left\{ \chi_{MS} \phi_{MS} + \chi_{MA} \phi_{MA} \right\} 
\]

which is, even though we did not demand it, an SU(6) wave-function. One then finds for the valence quark distributions in a proton:

\[
u(x) = \frac{29}{18} f(x), \quad \bar{u}(x) = \frac{7}{18} f(x), \\
d(x) = d(x) = \frac{1}{2} f(x) 
\]

where \( f(x) \) is normalised so \( \int_0^1 dx f(x) = 1 \).

For the unpolarised distributions we then have
\[ u(x) = 2 \, d(x) = 2f(x) \quad \ldots \quad (15) \]

But these distributions, particularly \( u(x) = 2d(x) \), are manifestly unacceptable, as can be seen from the unpolarised data for the ratio of electromagnetic ep to en cross-sections.

Various attempts, none very deep, have been made to improve the theoretical picture. In the "broken SU(6)" model one takes

\[ y_{A=\frac{1}{2}} = f_0(x_1) x_{MA} \phi_{MA} + f_1(x_1) x_{MS} \phi_{MS} \quad \ldots \quad (16) \]

which leads to ep and en electromagnetic structure functions of the form:

\[ F_2^p(x) = \frac{4}{9} f_0(x) + \frac{2}{9} f_1(x) + \text{sea} \]

\[ F_2^n(x) = \frac{1}{9} f_0(x) + \frac{1}{3} f_1(x) + \text{sea} \quad \ldots \quad (17) \]

and

\[ 2xg_1^p(x) = \frac{2}{27} \left[ 6f_0(x) - f_1(x) \right] \quad \ldots \quad (18) \]

\[ 2xg_1^n(x) = \frac{1}{9} \left[ f_0(x) - f_1(x) \right] \]

(Note that the SU(6) limit, \( f_0 = f_1 \), implies \( g_1^n(x) \equiv 0 \).)

The unpolarised ep and en data give us \( f_0(x) \) and \( f_1(x) \), and one finds\(^5\)

\[ f_1(x) \approx 1.3 \left( 1 - x \right)f_0(x) \quad \ldots \quad (19) \]

The polarised proton data of the SLAC-YALE group\(^6\) is shown in Fig (1) and compared with various theoretical models. The parameter displayed, \( A_1 \), is obtained from the experimental asymmetry with longitudinal polarisations [Eqn (6) divided by Eqn (5)], and in the simple parton model is given by

\[ A_1 = \frac{2xg_1(x)}{F_2(x)} \quad \ldots \quad (20) \]
The SU(6) result (14) gives $A_1$ independent of $x$. The broken SU(6) model, Eqns (17) (18) and (19) gives a reasonable fit for medium to large $x$, but cannot be correct at small $x$ where the sea is important, and presumably being unpolarised, will dilute the asymmetry. To account for the latter Carlitz and Kaur multiply the R.H.S. of Eqn (18) by an $x$-dependent dilution factor designed to approach 1 as $x \to 1$ and to approach 0 as $x \to 0$ like $\sqrt{x}$, the latter suggested by Regge theory. The prediction is shown as Curve 4 in Fig.(1), and is clearly an acceptable fit at the present level of accuracy. However the prediction involves one free function of $x$ so might not be considered as very significant.

On the other hand there exists a rigorous sum rule due to Bjorken, which, in the parton model becomes

$$\frac{1}{2} \int_0^1 dx [g_1^P(x) - g_1^n(x)] = \frac{1}{3} \left| \frac{g_A}{g_V} \right| = 0.418 \pm 0.002 \quad \ldots \quad (21)$$

The dilution factor in ref,(7) was taken as a simple function of $x$ with only one free parameter, which was fixed by satisfying the above sum-rule. So there is some significance in the agreement with the data of Fig.(1).
Much more interestingly, the asymmetries in electromagnetic en and in C.C. reactions can be predicted without invoking any further functions (detailed results are given in Kaur), so a future comparison is important. We show below for example a comparison of $2\text{g}_{1}(x)$ for protons and neutrons as given in ref.(5), and the en asymmetry which is predicted to be small, except at $x=1$ where it tends to unity.

None of the above models is particularly convincing, but a knowledge of the detailed internal structure of hadrons must ultimately be of the greatest importance. It is up to the experimentalists to provide this knowledge.

CONCLUSION:

We have explained briefly, how experiments on deep inelastic lepton scattering (electromagnetic or C.C.) with polarised hadron targets provide vital, detailed information about quark distributions in the hadrons. All this was presented at the level of the simple parton model, but eventually questions of the $Q^2$ dependence too will be worthy of examination.

We have not had space to discuss polarised Drell-Yan reactions. It should be noted that detailed predictions exist for all sorts of two-spin asymmetries i.e. involving both polarised beam and target. More important, and a crucial test of the whole elegant dynamical picture, all single spin asymmetries should vanish. The latter must be tested!

References
1) J.D. Bjorken, Phys. Rev. 148, 1467(1966)
2) A.J.G. Hey and J.E. Mandula, Phys. Rev. D5, 2610(1971). The proportionality factor in (5), (6) and (7) is : $8\alpha^4 E^4 / Q^4$
3) Sec. e.g. : F.E. Close : An Introduction to Quarks and Partons. Academic Press 1979, Chapter 13.
4) See Chapter 4 of ref.(3).