HADRON PHYSICS AT THE SPS: THEORETICAL REMARKS

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ABSTRACT

A few simple remarks are given about the relevance of the present experimental programme of hadron physics at the fixed-target SPS for quantitative tests of quantum chromodynamics.

The discussion is divided into three parts: the first deals with the tests of QCD in the "generalized scaling limit", where the simple scaling laws of the parton model are modified by usually mild logarithmic corrections. The second concerns a brief analysis of the effects which are suppressed by inverse powers of the large kinematical invariants characterizing a given process ("higher twists"); and the third reviews the relevance of exclusive processes (like the elastic scattering) which, for particular kinematical conditions, have a rather simple description in perturbative QCD.

Concerning the first part, the general pattern of predictions of perturbative QCD for hadron physics is in good qualitative agreement with the experimental results accumulated in the past few years. Can we now go further and establish an agreement on a more quantitative basis? There are a few obvious limitations to this: the first comes from the machine energy (SPS-fixed target mode) which puts a bound on the hardness reachable in a scattering process. The second is the still rather poor knowledge of some structure/fragmentation functions (the sea and the gluon distributions) in terms of which the normalization of different hard processes is settled; this is particularly crucial for those reactions, such as large $p_T$ one-particle inclusive spectra, which involve many subprocesses at the parton level. A third limitation is the theorist's energy: while a complete calculation of next-to-leading corrections to all relevant hard processes seems feasible (in many cases, it has already been done), the next to the next order appears to be a formidable task. Finally, the influence of phenomenological parameters like the "intrinsic $p_T$" introduced in order to get a good agreement with the data, can obscure the analyses of the effects which are genuinely predicted by perturbative QCD.

The knowledge of various hadron beam energies and intensities and of different parton structure functions allows us to deduce the parton beam facilities available at CERN: in Fig. 1 are reported the number of particles per pulse of valence up-quarks, sea up-quarks and gluons coming from a 400 GeV proton beam with $10^7$ ppp, the number of valence up-antiquarks from a 300 GeV $\pi^-$ beam with $10^7$ ppp and, for further comparison, the spectrum of the photon beam.
As one can see, the monochromaticity of the hadron beam is lost at the parton level; in particular, the way the hadron beam energy is transmitted to the partons depends on the type of hadrons: indeed, anti-up-quarks coming from $\bar{p}$ at 400 GeV are roughly equivalent to those coming from $\pi^-$ at 300 GeV. The worst-known beam is clearly the gluon one: therefore, a first criterion according to which various experiments can be graded is their sensitivity to the gluon distribution. Figure 2 contains a list of processes, a sketch of the lowest order diagrams and a "Michelin" type note for each of them. Obviously, the large $p_t$ jet/one-particle inclusive processes get the lowest note. For reactions where $g$-$q$ and $q$-$\bar{q}$ channels are both present, and an electromagnetic current is coupled to the quarks (direct $\gamma$ at large $p_t$ on- or off-shell, photoproduction), the subtraction of $\pi^- - \pi^+$ either in the incoming beam or in the outgoing particle can isolate the contribution of the qq annihilation (or $\gamma q$ for photoproduction) and subtract, at the same time, most of the purely hadronic background ($\pi^0$ for direct $\gamma$ at large $p_t$).

The picture in terms of lowest order diagrams is credible only if the process is "hard" enough. This statement can only be made quantitative by calculating the higher order corrections. It is precisely the test of these higher order corrections which can bring the agreement of the theory with the experiments from a qualitative level to a quantitative one.
Unfortunately, the actual normalization of next-to-leading corrections depends in general upon the renormalization scheme used: for, the electromagnetic corrections to low energy $\mu e$ scattering are very large if $\alpha_{\text{e.m.}} (M_{\text{Planck}})$, the fine structure constant renormalized at the Planck mass, is used.

Various optimization procedures have been proposed for fixing the proper renormalization scheme\(^1\); the best seems to be the one proposed by P. Stevenson\(^2\) (PMS) which tries to make the result of the optimization stable against mild readjustments of the scheme adopted. These gymnastics often result in a reabsorption of the largest part of the correction by lowering the scale entering the running coupling constant.

Sometimes, the size of the correction cannot be hidden by a change of scheme: one then gets "observable" large corrections, which it is useful to test. A popular quantity which is quoted in these cases is the $K$ factor defined as

$$K = \frac{d_0 (1+2)}{d_0 (1)}$$

i.e., as the ratio of the prediction including the next-to-leading correction over that with the leading term only. In Fig. 3a and 3b are reported the "$K$" factors for the total Drell-Yan cross-section in $pp$ scattering\(^3\) and the one differential in the transverse momentum in $(\pi^- - \pi^+)p$ scattering\(^4\).

In the case of the total cross-section, the parton distribution functions used have been extracted from deep inelastic electroproduction (DIS). In the case of $p_t$ distribution, one can alternatively use parton densities normalized in DIS or fit the rapidity distributions of the Drell-Yan reaction itself: the resulting $K$ factor is quite different in the two cases.

<table>
<thead>
<tr>
<th>DY</th>
<th>![Diagram]</th>
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<tr>
<td>Direct $\gamma p_t$</td>
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<tr>
<td>$D_{p_t}$</td>
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<td>$\gamma N' + \text{jet} + X$</td>
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<td>$\gamma N' + \gamma + X$</td>
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<td>$hh + \text{jet}$</td>
<td>![Diagram]</td>
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- Fig. 2 -
The second order calculation for the deep Compton process has recently been completed\textsuperscript{5): no major corrections have been found to the $y$ distribution except for a dip around $y = 1 \pm 2$ at $p_\perp = 2$ GeV and $E_y = 200$ GeV (see Fig. 4). However, this comes from the gluon contribution to the higher order corrections and is therefore subjected to some amount of undetermination. As a counter-example, see the dashed line in Fig. 3a giving the gluon contribution in that case. The prompt photons at large $p_\perp$ have been studied beyond the leading order only in the soft gluon approximation\textsuperscript{6): the corrections found are of the order of 20 - 30%, at $p_\perp$ of the order of 8 GeV. To summarize, next-to-leading corrections are nice if large enough to be observable, if they can be resummed to be credible and if they are not constant over the explorable range of kinematics, so as to be unmistakable.

An interesting sector of large perturbative corrections are those arising from terms which behave as

$$\left[ \alpha_s(Q^2) \ln^2 \frac{Q^2}{Q_0^2} \right]^n$$  \hspace{1cm} (2)

where $n$ is the order of the perturbative expansion and $Q^2/Q_0^2$ are two scales sufficiently different to make the expansion parameter $\alpha_s \ln^2 Q_0^2$ of order one. The origin of those double logs is the emission of soft gluons and the imperfect balance of the terms left from the cancellation of infra-red singularities between real and virtual diagrams. The techniques aiming to resum
the terms like in Eq. (2) have been applied to several processes, such as
to the muon pair production at $p_t$ relatively lower than their invariant
mass$^7$) or the $E_T$ distributions in hadron-hadron scattering$^8$). Notice that
the techniques are applicable only if two different mass scales can be iden-
tified, both characterizing the hardness of the process: in particular, the
medium $p_t$ prompt photon cross-section cannot be handled in this way.

The impossibility of extending the predictive power of perturbation
theory down to low $p_t$ makes many processes very vulnerable to the contami-
nation of the "intrinsic $p_t$" effects. In Fig. 5 one can see as a rough in-
dication the effect of having on or off an intrinsic $p_{t1}$ of 1 GeV through
the ratio $R$ of the cross-section with the $p_{t1}$ on over the one with the $p_{t1}$
off. In this case, the single particle inclusive gets an inverse star.
In the case of Drell-Yan, the resummation which we already mentioned dra-
tically reduces these effects: the data can be fitted down to $p_t = 0$ with
only a $p_{t1} \approx 0.4$ GeV and the value of $R$ at $p_t = 3$ GeV is 1.3 only.

The summary of the notes given to the different reactions according to:
i) the complication in terms of elementary processes, ii) the visibility of
higher order corrections and iii) the sensitivity to phenomenological
smearing procedures is given in Fig. 6. For the large $p_t$ jet cross-section,
a complete second order calculation is still lacking: there are, however,
indications that they would suggest an "effective" scale for the process
much lower than the ones commonly used. Incidentally, this might have
important effects on the normalization of this cross-section at the collider.

<table>
<thead>
<tr>
<th>$\gamma p \to hX$</th>
<th>$p_t = 3$ GeV</th>
<th>$\approx 1.4$</th>
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<tr>
<td>$E = 100$ GeV</td>
<td>$p_t = 3$ GeV</td>
<td>$\approx 13$</td>
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<tr>
<td>E</td>
<td>$p_t = 6$ GeV</td>
<td>$\approx 4$</td>
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<th>$\approx 4.2$</th>
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<td>$E = 200$ GeV</td>
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<td>$\approx 3.5$</td>
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<th>$\approx 2$</th>
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<tbody>
<tr>
<td>$E = 400$ GeV</td>
<td>$p_t = 6$ GeV</td>
<td>$\approx 1.3$</td>
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- Fig. 5 -
<table>
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<tr>
<th>Subprocesses</th>
<th>K factors</th>
<th>intrinsic $p_\perp$</th>
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<tr>
<td>DY</td>
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<td>DY$<em>{p</em>\perp}$</td>
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<td>Direct $\gamma$</td>
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<td>Photoproduction</td>
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<tr>
<td>Deep Compton</td>
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<tr>
<td>$h-h+\text{jet}/\pi_0$</td>
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- Fig. 6 -

What I have been discussing so far might be considerably shaken if the factorization theorem of perturbative QCD should be proved to fail in hadron-hadron reactions. The situation at present is still controversial\textsuperscript{9}), but there are some encouraging results indicating the absence of the problem\textsuperscript{10}).

Another problem which might severely affect the discussion is the presence of nuclear effects of the type observed by the EMC collaboration at CERN\textsuperscript{11}): the Drell-Yan experiments seem to be the best placed to verify those effects\textsuperscript{*}). In particular, if the effect is due to "sea" (pion) quarks\textsuperscript{12}), the ratio of $\pi^-$ over $\pi^+$ cross-sections as a function of $p_\perp$ should approach 1/4 in a different way on deuterium or platinum targets.

We turn now to the discussion of the effects which die as inverse powers of the large invariants: the higher twist effects. I will make a distinction between inclusive higher twists and exclusive ones. The first are those arising from the participation of more than one parton per hadron to a hard reaction. In general, they involve multiparton reactions like the one depicted in Fig. 7. The picture in terms of multiparton processes is not unique: through the equations of motion, the off-shellness can be traded for more partons incoming\textsuperscript{13}). This makes the parametrization of higher twist effects simple in some "languages" (operator basis) and complicated in others\textsuperscript{14}). A difficulty which cannot be avoided anyway is the inclusion of many more input functions with respect to the leading twist case. Models

\textsuperscript{*}The observed A dependence for the total cross-section might come from a compensation of the EMC effect over the $x_1-x_2$ range of values explored.
are therefore required to provide good ansätze for those new functions and, at present, it is hard to believe that they can be fully determined on an experimental basis. This makes the analysis of inclusive higher twists rather problematic for the moment.

![Diagram](image)

**Fig. 7**

Exclusive higher twists originate from processes where, at the price of a form factor decreasing as an inverse power of some invariant, hadrons are directly produced (not through a standard fragmentation process) at small distances. In Fig. 8a there is an example of a diagram describing, in the lowest order, the direct production of a meson at large $p_1$ in photo-production\(^{15}\). For a pion, this contribution is suppressed by $\frac{\omega_{\pi}^2}{p_1^2}$ with respect to the leading twist large $p_1$ cross-section. These effects, which, however, are generally rather small, have the advantage of having a rather clear signature ("prompt" hadrons in the final state) and of requiring a modest $p_1$ range to be detected. In Fig. 8b there is another example of these reactions, where a pion is completely "eaten" (at least its valence quarks are) during a high $p_1$ scattering\(^{16}\). The signature of this process is the reduction of pion fragments in the forward direction; if this should become an important contribution in some kinematical configurations (forward angles and large $p_1$), a similar mechanism, with a $\rho$ meson incoming (Fig. 8b) could be a worrying background to three-jet events in photoproduction, which are taken as signatures of a direct coupling of the photon (Fig. 8c). If the significance of testing the exclusive higher twists is well established, the actual estimate of their size is still at a rather qualitative level; one should look more at the clearness of the signature than at the absolute normalization of these effects.

![Diagrams](images)

**Fig. 8a**  
**Fig. 8b**  
**Fig. 8c**
The last part of this discussion concerns the exclusive processes like the elastic pion-pion scattering. Two different mechanisms can contribute to the regime $s \geq t > \Lambda_{QCD}^2$: they are reported in Figs. 9a and 9b. The first, originally due to Brodsky and Farrar (BF)\(^{17}\), is very close to the mechanism acting in the e.m. form factor and, the second, due to Landshoff\(^{18}\) (L), imagines that the scattering takes place for each constituent independently. The two interpretations lead to rather different predictions: (BF) predict

$$\sigma_{\text{elastic}} \propto \frac{1}{s^\frac{1}{8}} f(\theta)$$

and (L) predicts

$$\sigma_{\text{elastic}} \propto \frac{1}{t^\frac{1}{8}} \tilde{f}(\theta)$$

The experiments at PS energies and large angles seem to favour the BF mechanism, while at SPS-ISR energies and $3 \leq -t \leq 14 \text{ GeV}^2$ one observes a very clear $t^{-\frac{1}{8}}$ behaviour. The SPS energy seems the one where there is the cross-over between the two regimes, and it would certainly be interesting to have more experimental information.

![Diagram of pion-pion scattering](image)

- Fig. 9a -  
- Fig. 9b -

In the framework of the BF picture, Mueller has proposed an interesting test, where a pion hits a nucleus producing a pion, a proton and an unbroken nucleus\(^{19}\):

$$\pi \cdot N(A) + p \cdot N'(A-1)$$

(3)

The $N'$ remains unbroken because in the BF mechanism the pion wave function before the scattering shrinks down to dimensions of the order of $1/|t|$, becoming transparent then to the nuclear matter. In the (L) type of approach, the pion maintains its normal shape during the scattering and it would certainly re-interact with the nuclear matter: the reaction in Eq. (3) may then serve as a useful way of separating the two mechanisms. In the field of exclusive processes, there are still some unsolved problems, mainly for what concerns the proton: for example, its electromagnetic form factor, which is predicted to behave as $\propto q^2 / (q^2)^2$, does not show experimentally
any signal of the sizeable dependence on $\alpha_s$. Also, there is still some
debate as to whether the (L) type of contribution would not be partially
depressed by a Sudakov form factor.

Let me stress a few points in the conclusion. The "generalized scaling"
regime of QCD is the only one which can be quantitatively tested at present.
On this I would remark that:

A) the reaction of hadron-hadron scattering into a jet or a single particle
at large $p_t$ seems a rather hopeless domain, given that:
   i) the "effective" scale is probably much lower then the natural one;
   ii) the sensitivity of the predictions to the smearing of the
"intrinsic $p_t$" is very large (the contamination only becomes
reasonable for $p_t \gtrsim 8$ GeV);
   iii) there are too many subprocesses contributing to the reaction

B) For a quantitative test of large $p_t$ Drell-Yan pairs it is important
to have a precision of the order of 20% at $p_t \sim 5$ and both $\pi^-$ and $\pi^+$
beams.

C) The high $p_t$ obtainable with prompt $\gamma$ hadroproduction can compensate
for the rather large sensitivity of this process to intrinsic $p_t$ smearing
effects. In this respect, large $p_t$ hadrons in photoproduction are
rather insensitive to this contamination and may become an important
testing ground of QCD.

Exclusive higher twists should be looked for: this type of search fits
very nicely with the energy range and the operation mode (fixed target) of
the SPS.

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11) See, for example, the review by 

12) See the talk by 
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    For a recent review, see: 


19) A.H. Mueller, Columbia preprint CU-TM.232, talk given at the XVII Rencontre de Moriond, 
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