DEEP INELASTIC LEPTON-HADRON INTERACTIONS
AT HIGH ENERGIES

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PREFACE

The purpose of these lectures - to state the main ideas and elements of theoretical tools in studying the deep inelastic processes of high energy lepton-hadron interactions.

This field of elementary particle physics has achieved in the last years great successes both experimental and theoretical. About ten years ago M. Markov put forward an idea on point-like behaviour of total cross sections of inelastic neutrino reactions. The first SLAC-NIT experiments on deep inelastic scattering of electrons on proton were performed in 1968 and the foregoing DESY experiments supported this idea. A similar point-like picture of inelastic neutrino-nucleon interactions has been observed in the CERN experiments.

Moreover, the experiments on deep inelastic electroproduction reveal a remarkable regularity - the inelastic form factors of this process are determined asymptotically by functions of dimensionless ratios of kinematical variables and do not depend on any dimensional parameters. Such a phenomena - called usually the Bjorken scaling - has stimulated many theoretical investigations. There are now a number of different approaches and models - more or less sophisticated - which are trying to explain the main regularities of various processes of high energy lepton-hadron
interactions (Feynman's parton model, the current algebra and light-cone analysis, dual-resonance model and other).

It is the principle of automodality, proposed in 1969 by N. Mekhvedyan, A. Tavkhelidze and the author, which states the scale invariant behaviour to be a universal model-independent property of all the deep inelastic processes determining by the physical similarity laws and the dimensional analysis. A similar approach which uses the dimensional analysis was recently proposed by T.D. Lee.

As it was noticed by N. N. Bogolubov, the automodel or scale invariant behaviour of the inelastic form factors of deep inelastic processes is in an intimate analogy with the so-called automodel solutions of a problem of strong point explosion in hydrodynamics, well-known for a long time.

The successful applications of the automodality principle to the analysis of the muon pair production experiments performed in Brookhaven by the Columbia University group and the recent experiments on the inclusive hadron electroproduction show the fruitfulness of this hypothesis in studying deep inelastic processes.

The recent fundamental investigations by N. Bogolubov, A. Tavkhelidze and V. Vladimirov on automodel asymptotics in quantum field theory have provided a rigorous basis for the light-cone analysis, developed by R. Brandt, H. Leutwyler and others.

Notice, that the automodel or scale invariant behaviour was also observed in pure hadronic inclusive reactions, for the first time on the Serpukhov machine and then on the ISR and NAL. The theoretical studying of the inclusive processes was initiated by pioneer's works of A.A. Logunov and Collaborators.

An extension of the automodality hypothesis to relativistic nucleus collisions has led A. Baldin to a prediction of the so-called cumulative production effect, confirmed in 1970 in Dubna.

It will be extremely hard task to give a full history of development of physics of deep inelastic interactions.

This course of lectures is only an introductory review and does not pretend to represent all of the various aspects of the physics of deep inelastic lepton-hadron interactions at high energies. There are many excellent reviews on this field to which we may refer the readers.

1. INTRODUCTION

1.1. Deep inelastic lepton-hadron processes as a tool for studying the elementary particle structure.

In the recent years the main interest in studying high energy interactions of particles have shifted from the field of elastic and quasielastic interactions to that of essentially inelastic multiparticle processes. The reason is that in studying elastic processes one deals,
in essence, with \textit{global} properties of particles (effective radii, "transperance", etc.) whereas in studying the inelastic multiparticle interactions one gets a possibility to investigate the local structure of particles. In essentially inelastic processes when extremely large number of channels are opening, we deal, in some sense, not with properties of individual particles, but with those of "hadron matter" (an excitation of continuum occurs).

Processes of essentially inelastic lepton-hadron interactions at high energies and large momenta transferred from leptons to hadrons, accompanied by the secondaries production, are usually called the deep inelastic processes. Due to the local nature of weak and electromagnetic interactions, deep inelastic lepton-hadron scattering is an unique tool for studying the local properties of particles, i.e., a behaviour of "hadron matter" at small distances.

As experiment reveals one of the typical properties of all the deep inelastic lepton-hadron processes is their "point-like" character. The essence of this property consists in the following. In the limit of high energies and large momentum transfer, the total deep inelastic cross sections, having been summed over all open channels probability of each of them is considerably suppressed because of form factors of strongly interacting particles, are comparable by magnitude with those of point-like structureless particles.

The "point-like" nature of lepton-hadron interactions at high energies has led to an idea on automodel or scale-invariant behaviour of the deep inelastic differential cross sections.

The \textit{automodality principle} consists in an assumption that the asymptotic behaviour of deep inelastic processes is independent of any dimensional quantities, such as masses, "elementary lengths", etc.; therefore the structure functions or form factors of these processes are functions of the invariant kinematical variables, the general form of which is defined by the dimensional analysis.

A number of approaches to the theoretical interpretation of automodality or scale invariance for deep inelastic scattering do exist. In what follows we will discuss only some of them, which are based on the very attractive from the physical viewpoint idea on hadrons as composite extended objects with internal degrees of freedom. Among such model heuristic approaches there is the \textit{parton model} which is treating, in some sense, hadrons as a gas, of elementary constituents - partons. The parton model allows one to obtain relations between cross sections and form factors of various deep inelastic processes, starting from concrete assumptions about a type of partons (valent, quarks, quark-antiquark pairs, glyons, etc.) and on a degree of compositeness of hadrons. The other more or less
traditional approach of quantum field theory to studying
deep inelastic processes are developing such as vector
dominance, model, current algebra and analysis of singula-
rities of current commutators on the light cone ("physics
on the light cone") and some other methods which will be
mentioned in these lectures only in passing.

2. The kinematics of deep inelastic processes.

2.1. The classification of deep inelastic lepton-

hadron processes.

In respect with that whether the four-dimensional
momentum $q$ transferred from leptons to hadrons is space-
like ($q^2 < 0$) or time like ($q^2 > 0$), all the deep
inelastic processes of the lepton-hadron interactions can
be separated into two classes:

1. $q^2 < 0$ : "excitation" of hadrons by leptons;
2. $q^2 > 0$ ; annihilation and production lepton pairs
with participation of hadrons.

To the first class such processes as the inelastic electro-
production or neutrino/antineutrino production on nucleons
belong

1. $e^+ N \rightarrow e^+ \text{hadrons}$,
2. $\nu_e + N \rightarrow \ell^+ \text{hadrons}$,
$\bar{\nu}_e + N \rightarrow \bar{\ell}^+ \text{hadrons}$, $\ell = e, \mu$.

The studying of these processes put basis of the
theory of deep inelastic reactions.

To the second class one can assign the process of
electron-positron pairs annihilation into hadrons:

3. $e^+ e^- \rightarrow \text{hadrons}$,

and the process of the muon-pair production in proton-
nucleon collisions:

4. $p + N \rightarrow \mu^+ \mu^- + \text{hadrons}$

To the last process there is closely related the
process of the charged lepton pairs $\nu_\ell/\bar{\nu}_\ell \ell$ production
via, as one usually believe, the generation of the
intermediate charged $W^\pm$-mesons:

5. $p + N \rightarrow W^\pm + \text{hadrons}$

In the last years there arises a special logic in
studying pure hadronic inelastic reactions, namely, inclu-
sive experiments, i.e. the multiparticle production
processes in high energy hadron collisions when one or
more particles in the final state are identified while
the rest being undetected. The classic deep inelastic
processes of the types 1, and 2 are in fact, the inclu-
sive type processes when a secondary lepton is identified
in final states (so called "one arm" experiment).

An extension of inclusive description on hadron
system in the final states of deep inelastic lepton
hadron collisions leads to a special type of deep inelastic processes, for example, an inclusive electron
or neutrino/antineutrino/ - production on nucleons:

6. $\bar{e} + N \rightarrow e^- + A_i + \text{hadrons}$;

7. $\gamma_e + N \rightarrow \ell + A_i + \text{hadrons}; \quad \ell = e^-, \mu^-$;
   $\bar{\nu}_e + N \rightarrow \bar{\ell} + A_i + \text{hadrons}$;

or a process of inclusive annihilation of electron-
positron pairs into hadrons:

8. $e^+ + e^- \rightarrow A_i + \text{hadrons}$,

where $A_i$ - is some particular hadron.

For all the processes listed above the lowest approximations in electromagnetic and weak interactions are usually assumed to be hold. Besides, some deep inelastic processes of lepton-hadron interactions are discussed which require to appeal to more higher orders in the electromagnetic or the weak coupling constants.

2.2. Inelastic electroproduction on nucleon.

The matrix element of deep inelastic electroproduction on nucleon $e^- + N \rightarrow e^- + \ldots$ corresponding to the diagram, pictured on Fig. 1,

\begin{center}
\begin{tikzpicture}
  \node[proton] (p) at (0,0) {Proton};
  \node[electron] (e) at (1,0) {Electron};
  \node[ell] (ell) at (1,1) {Ell};
  \node[theta] (theta) at (1.5,1) {\theta};
  \node[Q] (Q) at (1,-1) {q};
  \node[final hadron] (fh) at (2,0) {Final Hadron State};
  \draw[->] (e) -- (ell);
  \draw[->] (ell) -- (theta);
  \draw[->] (ell) -- (Q);

diagram, pictured on Fig. 1.
\end{tikzpicture}
\end{center}

is determined by the expression

$$T = \frac{2\pi}{q^2} \langle N | j_\mu (e) | \rho \rangle$$

where

$$j_\mu = \bar{u}(\kappa') \gamma_\mu u(\kappa); \quad z = \kappa - \kappa'.$$

We are interested in the total differential cross section of this process summed over all the possible final hadron states, symbolically:

$$E_k \frac{d\sigma}{d\Omega} (N) \sim \sum_N | \begin{array}{c} N \end{array} |^2 \sim$$
where cross line denotes an integration over a phase space of real secondary particles.

Averaging over the polarization of the initial nucleon and electron we get, in the approximation $m_e = 0$, for the differential cross section of deep inelastic electro-production

$$E_{k'} \frac{d\sigma}{d^2 k'} = \int \frac{d\sigma}{8\pi^2} = \frac{1}{S - M^2} \left( \frac{4\pi\alpha}{\xi^2} \right)^2 m_{\mu \nu} \omega_{\mu \nu} ;$$

where $S = (p+k)^2$ and

$$m_{\mu \nu} = \frac{1}{2} \left[ \mu \nu (k^2 + m_e^2) + \lambda_{\nu \mu} (k^2 + m_e^2) \lambda_{\nu} \right]_{m_e = 0} \quad = 2 \left( k^\mu k'^\nu + k^\nu k'^\mu - g_{\mu \nu} k k' \right) -$$

is the result of averaging over the initial electron polarization and summing over the final electron polarization;

$$W_{\mu \nu} (p, t) = \frac{1}{8\pi} \sum_{\text{Spin}} \int d\xi \epsilon^* \langle p | J_{\mu} (x) J_{\nu} (0) | p \rangle \left< N | J_{\mu} (x) / p \right> \left< N | J_{\nu} (0) / p \right> .$$

The current product in this expression can be interchanged at $\xi_0 > 0$ by the current commutator.

Due to the hermitian property of the hadron electromagnetic current $J_{\mu}^* = J_{\mu}$ and the continuity equation $\frac{2}{\xi \mu} J_{\mu} (x) = 0$, the quantity $W_{\mu \nu}$ should satisfy the following conditions:

$$q_{\mu} W_{\mu \nu} (p, t) = 0 \quad W_{\mu \nu}^* (p, t) = W_{\mu \nu} (p, t) .$$

Starting from these conditions one can expand the tensor $W_{\mu \nu}$ on the gauge-invariant structures:
The form factors $W_1$, $W_2$ which are usually called the structure function of deep inelastic electroproduction, depend on the invariant variables $q^2$ and $y = p_1^-$. Notice, that the quantity $f(p, k, k')$ which determines a spectrum of the final electron momenta, is a function of three independent invariant variables, e.g.: $p k$, $p k'$, $k k'$. The invariant variables can be chosen in a number of ways, for example:

1. $S = (p + k)^2$, $q^2 = (k - k')^2$, $1 = p q = p (k - k')$;

2. $E, E', \theta$ - energies of an initial and a final electrons, and scattering angle in the laboratory system ($\hat{p} = 0$), respectively.

In the laboratory system it is convenient some times to use instead of the energy of a final electron $E'$ the inelasticity parameter $\eta = E'/E$. In accordance with a choice of invariant variables, an element of the phase volume of the final electron takes the form:

$$\frac{d^2 k'}{E_{k'}} = \frac{2\pi}{s - M^2} d q^2 d y$$

$$\frac{d^2 k'}{E_{k'}} = E' d E' d \Omega = E^2 d \eta d \Omega$$

The region of the deep inelastic scattering is determined by

$$S, y, |q^2| \gg M^2, \quad \ell = -\frac{q^2}{2y} \text{ fixed}$$

The character of this asymptotic limit in terms of various invariant variables is discussed in Appendix I.

Now we find the form of the function $f$ in the limit of high energies.

Using (2.1) and (2.2) we obtain

$$m_{\mu \nu} W_{\mu \nu} = -2q^2 W_1 + \left[ (s - \eta^{-2})^2 + q^2 (M^2 - \nu^2) \right] W_2 \Rightarrow$$

$$\Rightarrow -2q^2 W_1 + s (s - 2\nu) \cdot W_2$$

$$s \gg M^2$$
Thus, the cross section of inelastic electroproduction in the limit of high energies \( S \gg M^2 \) is determined by the expression:

\[
\frac{d^2 \sigma}{dE'd\Omega} = \frac{2\pi \alpha^2}{\gamma^4} \left[ \frac{(1 - \frac{2\nu}{S})}{S} W_2 - \frac{2\nu^2}{S^2} W_1 \right], \tag{2.3}
\]

which by using the variables \( \xi \) and \( \eta = 1 - \frac{2\nu}{S} \) is transformed to the form:

\[
\frac{d^2 \sigma}{dE'd\xi} = \frac{2\pi \alpha^2}{\xi^4} \left[ \eta \xi (\nu W_2) + (1 - \eta)^2 W_1 \right]. \tag{3.4}
\]

The expressions (2.3) and (3.4) have the evident relativistic invariant forms and can be applied in an arbitrary reference system. In analysis of the inelastic electroproduction in the laboratory system \( \vec{p} = 0 \) the variables \( E, E' \) and \( \theta \) are the most convenient. In terms of these variables we have

\[
\sigma_{\text{tot}} \sim \frac{1}{\text{"flux"}} \left[ \frac{1}{\text{"flux"}} \right]^{2} = \frac{4\pi \alpha}{\text{"flux"}} \epsilon_{\mu} \epsilon'^{\nu} W_{\mu\nu}(p, \epsilon)
\]

that gives for the differential cross section the result:

\[
\frac{d^2 \sigma}{dE'd\Omega} = \frac{4\pi \alpha^2}{Mq^4} E'^2 \left\{ \cos \frac{2\nu}{2} M^2 W_2 + 2 \sin \frac{2\nu}{2} W_1 \right\}.
\]

As it follows from this formula for the differential cross section, the studying of the angle distribution of the secondary electron momenta allows to separate the invariant form factors \( W_1 \) and \( W_2 \), or, what is the same, to separate the contributions of the transverse and the longitudinal polarizations of the virtual photon.

**Photoabsorption cross sections:**

A relation between the form factors \( W_1 \) and \( W_2 \) can be expresssed in terms of the total cross sections of the photoabsorption on a nucleon:

\[
\gamma^* + N \rightarrow \text{hadrons}
\]

corresponding to the various directions of the virtual photon polarization:

\[
\sigma_{\text{tot}} \sim \frac{1}{\text{"flux"}} \sum \left[ \left. \frac{1}{\text{"flux"}} \right]^{2} \right] = \frac{4\pi \alpha}{\text{"flux"}} \epsilon_{\mu} \epsilon'^{\nu} W_{\mu\nu}(p, \epsilon)
\]
Choose, following the tradition, the flux factor, which coincides with \( \text{flux} \) or "equivalent" real photons producing the same invariant mass of final hadrons \( W \):

\[
W^2 = M^2, \xi^2 + 2\nu = (p - t^\prime)^2 \left/ \frac{\xi}{\xi} = 0 \right. = M^2 + 2 M \xi^0
\]

Then define the normalized polarization vectors of virtual photon \( \xi^\mu \), which satisfy the gauge condition \( \xi^\mu \xi_\mu = 0 \):

\[
\xi^\mu = \frac{i}{\sqrt{2}} (0, 1, \pm i, 0)
\]

- transverse polarization;

\[
\xi^\mu = \frac{1}{1 - \xi^2} (\xi, 0, 0, \xi)\]

- longitudinal polarization.

where \( \xi = (\xi, 0, 0, \xi) \) - is the moment of the virtual photon in the laboratory system: \( M \xi^0 = \nu \), \( M \xi_\perp \equiv \sqrt{\nu^2 - M^2} \).

Thus we obtain the result:

\[
\sigma_T = \sigma_T + \sigma_T^\perp = \frac{\frac{4\pi^2}{\nu + \xi^2}}{\nu + \xi^2} \cdot W_1
\]

\[
\sigma_\perp = \frac{\frac{4\pi^2}{\nu + \xi^2}}{\nu + \xi^2} \left[ (\nu W_2) \left/ \frac{\nu}{\nu + \xi^2} \right. - W_1 \right]
\]

Notice, that due to the positivity of the cross sections \( \sigma_T, \sigma_\perp \) the form factors \( W_{1,2} \) should satisfy the inequality:

\[
(\nu W_2) \left/ \frac{\nu}{\nu + \xi^2} \right. \geq W_1 \geq 0
\]

Let us introduce the parameter \( R \) which determines the ratio of the photoabsorption cross sections with the longitudinal and the transverse polarizations:

\[
R = \frac{\sigma_\perp}{\sigma_T} = (1 - \frac{\nu^2}{M^2}) \frac{W^2}{M^2} - 1
\]

or

\[
2\nu W_1 = \frac{\frac{1}{\nu}}{1 + \frac{\nu^2}{M^2}} (\nu W_2) (1 + \frac{\nu^2}{M^2})
\]

For the case of point-like particles in connection with a spin, the parameter \( R \) takes the values:

\[
\begin{align*}
S = 0 : & \quad \sigma_T = 0 ; \quad R = \infty ; \\
S = \frac{1}{2} : & \quad \sigma_\perp = 0 ; \quad R = 0 ; \\
S \geq 1 : & \quad \sigma_\perp / \sigma_T \not= 0 ; \quad R = \text{const} \not= 0 .
\end{align*}
\]
In terms of the total photoabsorption cross sections \( \sigma_T \) the form factors \( W_1 \) and \( W_2 \) are expressed as follows:

\[
W_1 = \frac{\gamma + \frac{1}{2}}{4\pi^2} \cdot \sigma_T ;
\]

\[
W_2 = \frac{\gamma + \frac{1}{2}}{4\pi^2} \cdot \frac{2\mu^2}{\varepsilon \mu^2} \cdot \frac{1}{\nu^2} (\sigma_T + \sigma_L).
\]

Under conditions

\[
\sigma_L \to 0 ; \sigma_T \to \sigma_{TN} \neq 0 \text{ at } \varepsilon^2 \to 0,
\]

where \( \sigma_{TN} \) -total cross section of the real photon absorption on unpolarized nucleon, it follows that

\[
\gamma W_2 \sim C \varepsilon^2 \quad ; \quad C = \frac{M^2}{4\pi^2} \sigma_{TN}.
\]

\[
\varepsilon^2 \sim 0
\]

Obviously, that under the same conditions \( \sigma_T \to 0 \), as \( \varepsilon^2 \to 0 \).

The contributions of the various virtual photon polarizations to the differential cross section of inelastic electroproduction are described by the Hand's formula:

\[
\frac{s(s-2\nu)}{4\pi M^2} \frac{d^2 \sigma}{d\epsilon^2 d\nu} = \frac{d^2 \sigma}{dE d\Omega} = \Gamma_T \cdot (\sigma_T + \epsilon \sigma_L)
\]

where

\[
\Gamma_T = \frac{d^2}{2\pi^2 M^2} \frac{\gamma + \frac{1}{2}}{\varepsilon^2} \frac{E'}{E} \frac{1}{1 - \epsilon};
\]

\[
\varepsilon = \left[ 1 + (1 - \frac{\gamma^2}{M^2}) \frac{2\gamma^2 \theta}{2} \right]^{-1}
\]

We cite here the expressions for the form factors and cross sections for electron scattering off point-like particles with spins: \( S = 0 \) and \( \frac{1}{2} \).

**Point-like spinless particle, \( S = 0 \):**

Using the expression for the electromagnetic current of point-like spinless particle with an electric charge \( Q \):

\[
\langle p' j_\mu (\omega) / p \rangle = Q \cdot (p' + p) \nu
\]

we find

\[
W_{\mu \nu} = \frac{Q^2}{4\pi} \int \frac{dP'}{2P_0'} \delta^{(4)}(p + \xi - p') \delta^{(4)}(p + \xi)' \mu (p + \xi)' \nu
\]
\[ = 2Q^2 \left( p + \frac{\xi}{2} \right)_\mu \left( p + \frac{\xi}{2} \right)_\nu \delta^2 \left[(p+\xi)^2 - M^2 \right] = \]

\[ = 2Q^2 \left( p - \frac{\xi}{2} q \right)_\mu \left( p - \frac{\xi}{2} q \right)_\nu \cdot \delta^2(q^2 + 2\nu), \]

from where it follows

\[ W_1 = 0 ; \quad \nu W_2 = Q^2 \delta(1 - \xi). \]

The double differential cross section has formally the scale-invariant (=automodel) form:

\[ \frac{d^2\sigma}{d\xi^2 d\nu} = \sigma_{\text{Mott}} \cdot Q^2 \delta(1 - \xi) ; \quad S \gg M^2, \]

\[ \sigma_{\text{Mott}} = \frac{d\sigma(s=0)}{d\xi^2} = \frac{4\pi\alpha^2}{\xi^4} \eta_0 ; \quad \eta_0 = \left(1 + \frac{\xi^2}{2}\right). \]

Point-like particle with spin \( S = 1/2 \).

Using the expression for the electromagnetic current of point-like Dirac particle with charge \( Q \):

\[ \langle p' | J_\mu | p \rangle = Q \bar{\psi}(p') \gamma_\mu \psi(p), \]

we find

\[ W_{\mu\nu} = \frac{Q^2 \int d^4p' (2\pi) \delta(p + p' - p) \delta(p + M) \chi_\mu(p') \gamma_\nu(p' + M) \gamma_\nu = \]

\[ = Q^2 \delta(q^2 + 2\nu) \cdot \left[ p_\mu p'_\nu + p_\nu p'_\mu + g_{\mu\nu} (M^2 - p p') \right] p' + \xi, \]

Thus, at \(|\xi|^2 \gg M^2\) the form factors are determined by

\[ 2W_1 = \nu W_2 = Q^2 \delta(1 - \xi) \]

For the differential cross sections we get the result

\[ \frac{d^2\sigma(s=1/2)}{d\xi^2 d\nu} = \sigma_0 \cdot Q^2 \delta(1 - \xi) ; \quad S \gg M^2, \]

\[ \sigma_0 = \frac{d\sigma(s=1/2)}{d\xi^2} = \frac{d\sigma(\psi \rightarrow \psi \nu \bar{\nu})}{d\xi^2} = \frac{4\pi\alpha^2}{\xi^4} \left[ \eta_0 + \frac{1}{2} (1 - \eta_0)^2 \right] = \]

\[ = \frac{4\pi\alpha^2}{\xi^4} \left[ 1 + \frac{\xi^2}{2} + \frac{1}{2} \left( \frac{\xi^2}{5} \right)^2 \right] ; \quad \eta_0 = 1 + \frac{\xi^2}{5}. \]

In conclusion we write down the contribution of an elastic electron scattering on nucleon to the total cross section of electroproduction.
Single-nucleon term:

The electromagnetic current of nucleon is determined by

\[ \langle p'|J_{\mu}(p)\rangle_p = \bar{u}(p')\left[F_1(q^2)v_\mu + \frac{q^2}{2M}\sigma_{\mu\nu}\varepsilon^\nu\right]u(p), \]

where \( \sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) \), \( F_1 \) and \( F_2 \) are the electric and the anomalous magnetic Pauli form factors of nucleon. Using this formula we find:

\[ W_1 = M^2 \frac{G_E^2(q^2)}{1 + \frac{q^2}{4M^2}} \delta(1 - \epsilon) \]

\[ W_2 = \frac{G_M^2(q^2)}{1 + \frac{q^2}{4M^2}} \delta(1 - \epsilon) \]

where \( \epsilon = -\frac{q^2}{4M^2} \) and

\[ G_E(q^2) = F_1(q^2) - \frac{\epsilon}{M} F_2(q^2) \]

\[ G_M(q^2) = F_1(q^2) + \frac{\epsilon}{M} F_2(q^2) \]

are the total electric and magnetic Sachs form factors of nucleon, normalized at \( q^2 = 0 \) by

\[ G_E(0) = \text{charge;} \]

\[ G_M(0) = \text{magnetic moment (in units of } \frac{e^2}{2M} \text{)} \]

Under condition, that at \( -q^2 \gg M^2 \), \( \epsilon \ll 1 \), we find approximately

\[ W_1 \approx W_2 \]

as for the point-like spin 1/2 particle.

2.3. Deep inelastic neutrino reactions.

The weak interactions of leptons with hadrons are described by an effective Lagrangian of the form:

\[ \mathcal{L}_{\text{weak}} = \frac{G}{\sqrt{2}} \left( J_\mu^+ \gamma_\mu^- + \text{h.c.} \right) \]

where

\[ \gamma_\mu^- = \overline{\nu}_\mu \gamma_\mu (1 + \gamma_5) \nu_\mu \]

\( \gamma_- = e^-, \mu^- \)

and \( J_\mu^\pm \) - are the weak charged currents of hadrons:

\[ G_M^2 = 10^{-5} \times 1.02 \]

The matrix elements of the processes of deep inelastic neutrino and antineutrino scattering on nucleon

\[ \nu_\ell + N \rightarrow \ell + \ldots \]

\[ \overline{\nu}_\ell + N \rightarrow \ell + \ldots \]

in accordance with the diagram (Fig. 2):
are determined by the expressions:

\[ T^{\nu_\mu} = \frac{G}{\lambda} \langle N | J^\pm_{\mu} (0) | p \rangle \phi \Phi, \]

where \( J^\pm_{\mu} \) - the corresponding charged lepton currents:

\[ \nu_e \rightarrow \ell : \quad J^+_{\mu} = \bar{\nu}(k') \gamma_{\mu} (1 + \gamma_5) \nu(k); \]

\[ \bar{\nu}_e \rightarrow \bar{\ell} : \quad J^-_{\mu} = \bar{\nu}(k') \gamma_{\mu} (1 - \gamma_5) \nu(k). \]

The total cross sections of interactions for unpolarized nucleons summed over all the possible final states of hadrons are described by

\[ E_k \frac{d\sigma}{dR^{\nu \bar{\nu}}} \sim \sum_N \left| \frac{\nu_{\mu}}{\nu_{\nu}} \right|^2 \sim m_{\nu \bar{\nu}} \cdot W_{\nu \bar{\nu}} (p, \xi) \]

where

\[ m_{\nu \bar{\nu}} = \sum (\nu_{\mu})^2 = \sum \bar{\nu}_{\mu} (1 + \gamma_5) (k' \pm m_e) \nu_{\nu} (1 - \gamma_5) = \]

\[ = \bar{\delta}[\bar{\nu}_{\mu} \nu_{\nu} + \nu_{\mu} \nu_{\nu}' - g_{\mu \nu} \nu_{\nu} ' + i e_{\mu \nu \lambda \rho} \nu_{\lambda} \nu_{\rho} ] \]

and

\[ W_{\mu \nu} (p, \xi) = \frac{1}{8 \pi} \sum_{\text{spin}} \int d\xi \cdot e^{i \xi \cdot x} \left< \rho | [J^\mp_{\mu}(x), J^\pm_{\nu}(0)] | \rho \right> = \]

\[ = (-g_{\mu \nu} + \frac{\xi_\mu \xi_\nu}{\xi^2}) W_{3,4} \nu_{\nu} \nu_{\nu} + (p - \frac{\nu}{\xi^2}) \mu \cdot (p - \frac{\nu}{\xi^2}) \cdot W_{2,6} \nu_{\nu} \nu_{\nu} - \]

\[ - \frac{i}{2} e_{\mu \nu \lambda \rho} \rho_{\lambda \rho} W_{2,6} \nu_{\nu} \nu_{\nu} + e_{\mu \nu \lambda \rho} \nu_{\lambda} \nu_{\rho} W_{2,6} \nu_{\nu} \nu_{\nu} + \]

\[ + i (p_{\mu} \nu_{\nu} - p_{\nu} \nu_{\mu}) \cdot W_{2,6} \nu_{\nu} \nu_{\nu} \]

The quantity \( W_{\mu \nu} (p, \xi) \) represents the result of summing over all the final hadronic states

\[ W_{\nu \bar{\nu}} (p, \xi) \sim \]

and contains all the information on the dynamics of neutrino/antineutrino interactions with unpolarized nucleons.

Starting from the hermitian conjugation property of the weak currents of hadrons \( J^+_{\mu} \) and \( J^-_{\nu} \), the invariance under the time revision (\( \mathcal{T} \)-invariance),
and locality (CPT-invariance) one can find the following symmetry properties of the tensor $W_{\mu \nu}^{\nu, \bar{\nu}}(p, q)$:

1. $[W_{\mu \nu}^{\nu, \bar{\nu}}(p, q)]^* = W_{\mu \nu}^{\nu, \bar{\nu}}(p, q)$;
   i.e. $W_{\mu}^{\nu, \bar{\nu}}$ are all real; $i = 1, 2, \ldots, 6$ .

2. $W_{\mu \nu}^{\nu, \bar{\nu}}(p, q) = \pm [W_{\mu \nu}^{\nu, \bar{\nu}}(p, \bar{q})]^*$,
   where signs $+$ for either $\mu = \nu = 0$ or $\mu, \nu \neq 0$; $-$ for either $\mu = 0, \nu \neq 0$ or $\nu = 0, \mu \neq 0$;
   and hence $W_6^{\nu, \bar{\nu}} = 0$;

3. $W_{\mu \nu}^{\nu, \bar{\nu}}(p, q) = - W_{\mu \nu}^{\nu, \bar{\nu}}(p, -q)$;

from where it follows the cross symmetry relations between the form factors:

$N_i^{\mu, \nu}(p, q) = - N_i^{\mu, \nu}(p, -q)$; $i = 1, 2, \ldots, 5$.

Under neglecting the lepton masses the form factors $W_{4, 5}^{\nu, \bar{\nu}}$ do not contribute to the cross sections, so as

$$\frac{\partial^2 \sigma}{\partial \phi \partial \theta} = 0, \quad \frac{\partial \sigma}{\partial E} = 0$$

For the differential cross sections describing the momentum spectra of the final leptons one gets

$$E_{\nu'} \frac{\partial \sigma}{\partial q^2} = \int \frac{\sigma^{\nu, \bar{\nu}}(p, q)}{s \gg H^2} \frac{C}{4\pi^2} \left[ -2 \frac{i}{2} W_1^{\nu, \bar{\nu}} + 5(s-2\nu) W_2^{\nu, \bar{\nu}} + \frac{1}{2} (s-\nu) W_3^{\nu, \bar{\nu}} \right]$$

Using the variables $C = - \frac{\pi}{2\nu}$ and $\eta = (1 - \frac{2\nu}{s})$

we find

$$\frac{\partial^2 \sigma^{\nu, \bar{\nu}}}{\partial \phi \partial \theta} = \frac{C}{E} \left[ \eta^2 (W_1)^2 + (1-\eta)^2 W_2^{\nu, \bar{\nu}} + \frac{1}{2} (1-\eta)(\nu W_3^{\nu, \bar{\nu}}) \right]$$

In terms of the variables $E, E'$ and $\theta$, defined in the laboratory system ($\mathbf{p} = 0$), the expressions for the differential cross sections take the form:

$$\frac{\partial^2 \sigma^{\nu, \bar{\nu}}}{\partial E \partial \phi} = \frac{M E' \frac{\partial \sigma^{\nu, \bar{\nu}}}{\partial E}}{\frac{2\pi}{4}} = \frac{C}{E} \left[ \cos^2 \theta \sin^2 \frac{\theta}{2} M^2 W_2^{\nu, \bar{\nu}} + 2 \sin^2 \frac{\theta}{2} W_1^{\nu, \bar{\nu}} + M (E+E') \sin^2 \frac{\theta}{2} W_3^{\nu, \bar{\nu}} \right]$$
Introduce by an analogy with the electroproduction processes the normalized polarization vectors, which satisfy the gauge condition \( \xi \cdot \epsilon = 0 \):

\[
\begin{align*}
\xi_+ = \frac{1}{\sqrt{2}} (0, 1, i, 0) & \quad \text{right} \\
\xi_- = \frac{1}{\sqrt{2}} (0, i, 1, 0) & \quad \text{left} \\
\xi_s = \frac{1}{\sqrt{-\xi_s^2}} (\xi_s, 0, 0, \xi_s) & \quad \text{scalar or longitudinal polarization}
\end{align*}
\]

where \( \xi = (\xi_s, 0, 0, \xi_s) \).

The absorption cross sections which correspond to these polarizations of the currents are expressed through the invariant form factors by the following way:

\[
\begin{align*}
\sigma_+ &= \sigma_0 \left[ \frac{1}{\sqrt{2}} (1 - \frac{\xi_s^2}{\xi_s^2}) \right] \nu W_3 ; & \nu \rightarrow \bar{\nu} \\
\sigma_- &= \sigma_0 \left[ \frac{1}{\sqrt{2}} (1 - \frac{\xi_s^2}{\xi_s^2}) \right] \nu W_3 ; & \bar{\nu} \rightarrow W_3 \\
\sigma_s &= 2 \sigma_0 \left[ -W_1 + (1 - \frac{\xi_s^2}{\xi_s^2}) \right] \nu W_2 
\end{align*}
\]

where for the pure conditional case of an absorption of a massless particle with the quantum numbers of the \( \nu^+ \) -mesons, coupled to the weak hadron currents with a coupling constant \( g \), is equal:

\[
\sigma_0 = \frac{\beta_0}{\gamma + \xi_s^2/2}
\]

From the positivity condition of the cross sections \( \sigma_+ \geq 0, \sigma_- \geq 0 \), the inequalities on the form factors:

\[
0 \leq \frac{1}{2} \left( 1 - \frac{M^2}{\nu^2} \right)^{1/2} W_3 \left/ \nu W_3 \right. \leq \sigma_+ \leq \left( 1 - \frac{\nu^2}{M^2} \right) M^2 W_2
\]

We write down here as an example the form factors and cross sections for the neutrino and antineutrino interactions with a point like Dirac particle, say, an electron:

**Point-like Dirac particle:**

\[
\begin{align*}
\sigma_+ &= \delta (1 - \xi_s) ; \\
\sigma_- &= 2 \delta (1 - \xi_s) ; \\
\sigma_s &= \pm 2 \delta (1 - \xi_s), \quad \text{for } \nu \text{ and } \bar{\nu}, \text{ respectively} ; \\
\frac{d\sigma}{d\xi_s^2} &= \frac{C_2}{\pi} \quad \text{for } \nu e^- \text{ - scattering} ; \\
&= \frac{C_2}{\pi} \left( 1 + \frac{\xi_s^2}{2} \right)^2 \quad \text{for } \bar{\nu} e^- \text{ - scattering}.
\end{align*}
\]
Integrating over all transferred momenta in the physical region \( 0 \leq |\xi| \leq S \), we find the total cross sections of \( \nu e \) - and \( \bar{\nu} e \) - interactions:

\[
\sigma_{\nu e} = \frac{G^2}{\pi}; \\
\sigma_{\bar{\nu} e} = \frac{1}{3} \frac{G^2}{\pi}; \quad S \gg m_e^2.
\]

#### 2.5. Annihilation of lepton pairs into hadrons:

The annihilation of electron-positron pairs into hadrons is a simplest example of a deep inelastic process with the time like transferred momenta.

In the one-photon approximation (Fig. 3) the matrix elements of this process is given by:

\[
\mathcal{M}(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi\alpha}{\xi^2} \epsilon_{\mu} \langle N | J_{\mu}(\xi) | 0 \rangle, 
\]

where

\[
\epsilon_{\mu} = \bar{\nu}(\xi^+) \gamma_{\mu} \nu(\xi^-); \quad \xi = \xi^+ + \xi^-.
\]

For the total cross section of annihilation process, summed over all the final hadronic states, we get:

\[
\sigma_{e^+e^- \rightarrow \text{hadrons}}(\xi^2) \sim \sum_N \left| \mathcal{M}(e^+e^- \rightarrow N) \right|^2 = \frac{1}{216(2\xi^2-4m_e^2)} \left( \frac{4\pi\alpha}{\xi^2} \right)^2 \epsilon_{\mu} \epsilon^{\nu} \cdot \rho_{\mu\nu}(\xi);
\]

where

\[
\epsilon_{\mu} \epsilon^{\nu} = \frac{1}{4} \delta_{\mu\nu} (\xi^+ + m_e) \eta(\xi^+ - m_e) = (\xi^+ k^- + k^+ \xi^- - \frac{1}{2} g_{\mu\nu}(\xi^+ + \xi^-)^2).
\]

Fig. 3.

represents the result of averaging over the polarizations of electron and positron, and
$$\rho_{\mu} (\xi) = \sum_{N} (2\pi)^4 \delta (\xi - p_N) \langle 0 | J_\mu (x) | N \rangle \langle N | J_\nu (0) | 0 \rangle =$$

$$= \int dx e^{i \xi x} \langle 0 | J_\mu (x) J_\nu (0) | 0 \rangle =$$

$$= (-g_{\mu \nu} x^2 + q \cdot x) \rho (x^2)$$

is a gauge-invariant quantity, which contains all dynamical information on transferring of a virtual photon into hadrons.

Thus, in the approximation $m_e = 0$, we have

$$\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-} (\xi) = \frac{\delta x^2}{\xi^2} \rho (\xi^2).$$

Notice, that at $q_0 > 0$ the quantity $\rho_{\mu \nu}$ coincides with the Fourier transform of the vacuum expectation value of the commutator of the electromagnetic currents of hadrons, i.e.

$$\rho_{\mu \nu} (\xi) = \int dx e^{i \xi x} \langle 0 | [J_\mu (x), J_\nu (0)] | 0 \rangle / q_0 > 0$$

Write down here, as an example, the form of the spectral function $\rho (\xi^2)$ for the case of the muon pair production in the final state:

$$\rho_{\mu^+ \mu^-} (\xi^2) = \frac{1}{4 \pi} \frac{1}{(1 - \frac{\xi^2}{4m_e^2} \sqrt{\frac{\xi^2}{4m_e^2}})} = \frac{1}{6 \pi}, \quad m_e = 0$$

and the cross section

$$\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-} = \frac{4 \pi \alpha^2}{\xi^2}.$$


Consider the process of an inelastic collision of two hadrons $a$ and $b$, with production of a lepton pair, say $\mu^+ \mu^-$, and some system of hadrons $N$ with the total momentum $P_N$:

$$a (p) + b (p') \rightarrow \mu^- (x) + \mu^+ (x') + N (p_N).$$

In the lowest order in the electromagnetic interaction the amplitude of the process, which corresponds to the diagram
is determined by

\[ T_{ab \rightarrow \mu^+ \mu^- \ldots} = \frac{4\pi \alpha}{4\pi} \mathcal{E}_\mu \left< N_{\text{out}} \left| J_\mu^{(a)} \right| p, p'; \text{in} \right> \]

where \( \mathcal{E}_\mu = \bar{U}(k) U(k') \) is the electromagnetic current of the lepton pair \(^1\).

\(^1\) In this expression for the amplitude we have used the "in" and "out" representations of hadron states because, in contrary to one-particle states, for many-particle states the "in" and "out" representations are unequivocal.

An element of the differential cross section for collision of unpolarized hadrons, summed over all the final hadron states and polarizations of the lepton pair:

\[ d\sigma \sim \sum_N \frac{1}{4\sqrt{(pp')^2 - m^2 m' \xi^2}} \left( \frac{4\pi \alpha}{\xi^2} \right)^2 (\xi^2)(e_{\mu \nu} e \cdot e ' + e_{\nu \nu} e \cdot e') \rho_{\mu \nu} \frac{d^4 e}{(2\pi)^4} \]

where, as early, the cross line denotes an integration over a phase space of real hadrons; \( m \) and \( m' \) - masses of the colliding hadrons \( a \) and \( b \), respectively. Here we use the notations

\[ \rho_{\mu \nu}(p, p'; \xi) \sim \]

\[ = \sum_N \delta(p_p' \xi - p_N) \left< p, p'; \text{in} \left| J_\mu^{(a)} \right| N \right> \left< N \left| J_\nu^{(b)} \right| p, p'; \text{in} \right> \]

where the averaging over hadron polarizations has to be done;
\[ \Pi(q^2) = -g_{\mu\nu}q^2 + \varepsilon_{\mu}\varepsilon_{\nu} = \frac{1}{\xi} \sum_{\text{spins}} \int \frac{d^2k'}{2\pi^2} \frac{1}{2k_0'2k_0}(2\pi)^3 \delta(q-k-k') \varepsilon_{\mu} \varepsilon_{\nu}' = \]

\[ = \left( \frac{\varepsilon^2 - 4m^2}{\xi^2} \right)^{1/2} \int \frac{d\Omega}{8\pi^2} \left[ k_\mu k_\nu + k_\nu k_\mu - \frac{i}{2} g_{\mu\nu} (k+k')^2 \right] \]

where \( d\Omega \) is an element of solid angle, related to the directions of a momentum of one of the muons in the centre of mass system of the muon pair \( (q^2 = 0, k_0 = k_0') \), \( m_\mu \) - mass of muon.

Performing the integration over the angles in the centre of mass system of muon pair, where

\[ k' = \frac{1}{\xi} \left( \sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta \right), \]

\[ k_0 = k_0' = \frac{1}{\xi} \sqrt{q^2 + m_\mu^2} = \frac{1}{\xi} \sqrt{\xi^2} , \]

we find

\[ \Pi(q^2) = \frac{1}{6\pi} \left( 1 + \frac{2m^2}{\xi^2} \right) \frac{\xi^2 - 4m^2}{\xi^2} = \frac{1}{6\pi} \]

We discuss now the properties of the tensor \( \rho_{\mu\nu} \), which contains all the dynamical information on the lepton pair production in collisions of unpolarized hadrons.

Due to the electromagnetic current conservation this tensor should satisfy the gauge invariance condition

\[ \xi_{\mu} \rho_{\mu\nu} = \xi_{\nu} \rho_{\mu\nu} = 0 \]

and from the hermiticity \( \rho_{\mu\nu}^* = \rho_{\nu\mu} \) it follows that the real part of this tensor must be symmetrical and its imaginary part antisymmetrical under replacing the indices \( \mu \leftrightarrow \nu \).

In accordance with the current conservation the tensor \( \rho_{\mu\nu}(p, p', \xi) \) can be decomposed into the five independent gauge-invariant structures:

\[ \rho_{\mu\nu}(p, p', \xi) = \rho_1 (g_{\mu
u} + \frac{\xi_{\mu} \xi_{\nu}}{\xi^2}) + \rho_2 \cdot p_{\mu} p_{\nu} + \rho_3 \cdot p'_{\mu} p'_{\nu} + \]

\[ + \rho_4 \left( \delta_{\mu\nu} p'_{\mu} + p_{\mu} p_{\nu} \right) + i \rho_5 \left( \delta_{\mu\nu} (p'_{\mu} - p_{\mu} p'_{\nu}) \right) ; \]

where

\[ \delta_{\mu\nu} = p_{\mu} - \frac{p_{\mu} \xi_{\nu}}{\xi^2} ; \quad \delta_{\mu\nu}' = p'_{\mu} - \frac{p'_{\mu} \xi_{\nu}}{\xi^2} . \]

All the scalar form factors \( \rho_i \) are real functions which depend on four independent invariant variables constructed from the vectors \( p, p' \) and \( \xi \). For
example, one can choose the square of the virtual photon momentum \( q^2 \) (\( q^2 \) - muon invariant mass squared), and the ordinary Mandelstam's variables \( S, t \) and \( u \):

\[
S = (p + p')^2 = m^2 + m'^2 + 2mE;
\]

\[
t = (p' - q)^2 = \Delta^2;
\]

\[
u = (p - q)^2 = m^2 + q^2 - 2\tau;
\]

where the invariant \( \tau = p_\perp^2 \) in the laboratory frame system (\( p_\perp = 0 \)) is proportional to the energy of the virtual photon, \( E = \frac{1}{m} (p \cdot p') \) - is an energy of projectile hadron in the laboratory frame system.

Introducing one more invariant variables \( m_N^2 = p_N^2 \) the invariant final hadron mass squared, we get a linear relation between five invariant variables:

\[
S + t + u = m^2 + m'^2 + q^2 + m_N^2.
\]

It is convenient to decompose the tensor \( \rho_{\mu\nu} \) into the structures corresponding to definite virtual photon polarization vectors: \( \tilde{e}^2(\tau_1), \tilde{e}^2(\tau_2) \) and \( \tilde{e}^2(\tau) \) in the rest frame system of a virtual photon, \( E_\perp = 0 \), i.e. in the centre of mass system of the muon pair (see Fig. 4).

The c.m.s. of the muon pair. The axis \( z \) is directed along the momentum \( p_\perp \), and the momentum \( p_\perp' \) lies in the production plane \( xz \). The normal to the production plane is directed along the \( y \) - axis.

\[\text{Fig. 5}\]

Then the corresponding four-dimensional polarization vectors can be represented in a covariant form:

\[
es_{\mu}(\tau_1) = \frac{1}{\sqrt{-\overline{p}^2 + (p \cdot p')^2}} \left( p' - \frac{p' \cdot p}{p_2} p_\mu \right);
\]

\[
es_{\mu}(\tau_2) = \frac{1}{\sqrt{-\overline{p}^2 + (p \cdot p')^2}} \epsilon_{\mu \nu \lambda \rho} p_\nu p_\lambda e_\rho;
\]

\[
es_{\mu}(\tau) = \frac{1}{\sqrt{-\overline{p}^2}} p_\mu.
\]

It is easy to see that the polarization vectors are orthogonal to the virtual photon momentum \( q \) and
one to another, their norm is \(-1\), i.e.
\[
\varepsilon_\mu \varepsilon_\mu (i) = 0 ;
\]
\[
\varepsilon_\mu(\mu) \varepsilon_\mu (\kappa) = -\delta^{i\kappa} ;
\]
\(i, \kappa = T_1, T_2, \xi ;\)

and the completeness condition
\[
\sum_i \varepsilon_\mu (i) \varepsilon_\nu (i) = (-\delta_{\mu\nu} + \frac{\varepsilon_\mu \varepsilon_\nu}{\varepsilon^2})
\]
\(i = T_1, T_2, \xi \)

holds.

Using these polarization vectors, we decompose the tensor \(\rho_{\mu\nu}\) as follows:
\[
\rho_{\mu\nu} = \rho_{T_1} \varepsilon_\mu (\eta) \varepsilon_\nu (\eta) + \rho_{T_2} \varepsilon_\mu (\tau_1) \varepsilon_\nu (\tau_2) + \rho_{\xi} \varepsilon_\mu (\xi) \varepsilon_\nu (\xi) + \rho_{T_2}^{(3)} \varepsilon_\mu (\eta) \varepsilon_\nu (\omega) + \rho_{T_2}^{(3)} \varepsilon_\mu (\tau_1) \varepsilon_\nu (\tau_2) + \rho_{\xi}^{(3)} \varepsilon_\mu (\xi) \varepsilon_\nu (\omega).
\]

We note that in the system \(\omega = 0\) there is a simple relation between the space components of the tensor \(\rho_{ij}\). \((\rho_{00} = \rho_{0i} = \rho_{i0} = 0\) \) and of the form factors with the definite polarizations.

\[
\begin{pmatrix}
\rho_{xx} & 0 & \rho_{x\xi} \\
0 & \rho_{yy} & 0 \\
\rho_{\xi x} & 0 & \rho_{\xi\xi}
\end{pmatrix}
= \begin{pmatrix}
\rho_{T_1} & \rho_{T_2}^{(3)} + i\rho_{T_2}^{(1)} \\
0 & \rho_{T_2} & 0 \\
\rho_{T_2}^{(3)} - i\rho_{T_2}^{(1)} & 0 & \rho_{\xi}
\end{pmatrix}
\]

Up to a normalization this is the density matrix of the virtual photon given in a linear basis.

Now, the differential cross section of the process can be represented in an invariant form using the formula:
\[
d^3q = \frac{1}{4\sqrt{(p\rho')^2 - m^2\omega^2}} \cdot d\omega \cdot d^3p \cdot d\phi;
\]
where \(\phi\) is an azimuth angle.

The physical region, which determines by the total momentum conservation, is given in the Appendix I.

The triple differential cross section of the process, when in the final state only one muon pair with definite \(q^2\), \(\Delta^2\) and \(\delta = \frac{1}{m} p\rho')\) is detected and a summation is performed over all the possible hadron states, reads:
\[
d^3\sigma = \frac{a^2}{8\pi^2} \frac{1}{2^3} \left[ 1 - \frac{2\Delta^2 - 4m^2}{\Delta^2} \right] \sqrt{\frac{4\omega - 4m^2}{\Delta^2}} \frac{m p(s, \xi') s \Delta^2}{2 \sqrt{s - (m + m)^2}} \sqrt{s - (m - m)^2}.
\]
where

\[ \rho(s, t, \lambda, \delta) = (g_{\mu \nu} + \frac{t_{\mu \nu}}{t^2}) \rho_{\mu \nu} = \rho_{T_1} + \rho_{T_2} + \rho_L. \]

The distribution over the squared effective mass of the muon pair is obtained by integrating over \( d^2 \Delta^2 \) and \( d^2 \delta \) inside the physical region. Neglecting the muon mass we get the following formula for the muon mass spectrum:

\[ \frac{d\sigma}{d\xi^2} = \frac{\alpha^2}{12\pi^4 (\sqrt{s} - m^2) \sqrt{\xi^2 - (m^2 - m'^2)^2}} \int_{\Delta_{\text{min}}}^{\Delta_{\text{max}}} \int_{\delta_{\text{min}}}^{\delta_{\text{max}}} \left| \frac{d^2 \Delta^2}{d\Delta^2 d\delta} \rho(s, t, \lambda, \delta) \right|^2 \]

For the purpose of applying the vector dominance hypothesis it is convenient to represent the mass spectrum formula in the form

\[ \frac{d\sigma}{d\xi^2} (ab \rightarrow \mu^+ \mu^- + ...) = \frac{\alpha^2}{12\pi} \left( \frac{2m^2 - 4m^2}{\xi^2 - 4m^2} \right) \frac{d\sigma_{\gamma^*}}{d\xi^2} \]

where

\[ \sigma_{\gamma^*}(s, t^2) = \sigma_{T_1} + \sigma_{T_2} + \sigma_L. \]

is the total cross section of production of a virtual \( \gamma^* \) photon with the mass \((\xi^2)^{1/2}\) in the process:

\[ \alpha + b \rightarrow \gamma^* + \text{hadrons}. \]

In the limit of high energies, when \( \xi \gg m, m' \), we get

\[ \sigma(\xi) = \frac{\alpha^2}{8\pi} \frac{4m^2}{\xi^2 - 4m^2} \int \int \frac{d^2 \Delta^2}{d\Delta^2 d\delta} \rho(s, t, \lambda, \delta) \]

where \( \xi^* = \xi - \frac{\Delta^2}{\xi^2 - 4m^2} \), and \( \delta^* = \Delta_\delta \) - is a transferred energy in the laboratory system.

Notice, that the total cross section of the muon production process determines only the sum of the invariant form factors with definite polarizations

\[ \rho_{T_1} + \rho_{T_2} + \rho_L, \]

and does not depend on \( \rho_{TE} \).

Measuring of all the form factors separately (except \( \rho_{TE} \)) can be one in studying the angular distributions in directions of a momentum of one of the muons in the centre of mass system, where \( \xi = 0 \):
\[ W(\theta, \phi) = \frac{1}{4\pi \rho} \frac{1}{1 - \frac{1}{3} \beta^2} \left[ \rho_T(1 - \beta^2 \sin^2 \theta \cos^2 \phi) + \rho_L(1 - \beta^2 \sin^2 \phi) + \rho_L^+(1 - \beta^2 \cos^2 \theta) - \rho_L^-(\beta^2 \sin 2\theta \cos \phi) \right]. \]

where \( \rho \) is given by \( \rho_T + \rho_L + \rho_L^- \), \( \beta = \frac{E}{E_0} = \sqrt{1 - \frac{4m^2}{E^2}} \)

is the muon velocity in the c.m.s.

The form factor \( \rho_T^-(\gamma) \) can be found studying a polarization of one of the muons along the normal to the production plane (along \( \gamma \)-axis on Fig. 5).

\[ \langle S_\gamma \rangle \sim m_\mu E_0 \rho_T^- \gamma. \]

Integrating over \( d\phi \) or \( d\cos \theta \) we find the distribution in only \( \theta \) or \( \phi \), respectively:

\[ W(\theta) = \frac{1}{2\pi} \frac{1}{1 - \frac{1}{3} \beta^2} \left[ (\rho_T + \rho_L)(1 - \beta^2 \sin^2 \theta) + \rho_L(1 - \beta^2 \cos^2 \theta) \right]. \]

\[ W(\phi) = \frac{1}{2\pi \rho} \frac{1}{1 - \frac{1}{3} \beta^2} \left[ \rho_T(1 - \frac{2}{3} \beta^2 \cos^2 \phi) + \rho_L(1 - \frac{2}{3} \beta^2 \sin^2 \phi) + \rho_L^+(1 - \frac{2}{3} \beta^2 \cos^2 \phi) - \rho_L^-(\beta^2 \sin 2\theta \cos \phi) \right]. \]

We notice in conclusion, that the form factors \( \rho_T, \rho_L, \rho_L^+ \)

have kinematical singularities. Its relations to the form factors \( \rho_T, \rho_L, \rho_L^+ \) introduced above, which are free (under some conditions) of kinematical singularities are given by:

\[ \rho_T^+ = \rho_T - \frac{(\vec{p}_F)^2}{p^2} \rho_T^1; \]

\[ \rho_L^+ = \rho_L^+; \]

\[ \rho_L^- = \rho_L^-; \]

\[ \rho_L^+ = \frac{1}{p^2} \left[ \frac{1}{p^2} (\vec{p}_T^2 \cdot (\vec{p}_F)^2)^1 \right] \rho_3 + \]

where \[ + \left[ \frac{1}{p^2} \vec{p}_T^2 \cdot (\vec{p}_F)^2 \right] \rho_4 \pm i \rho_5; \]

\[ \vec{p}^2 = \frac{1}{2\mu} \left( m^2 + \gamma \right); \]

\[ \vec{p}_F^2 = \frac{1}{2\mu} \left( m^2 + (\vec{p}_T)^2 \right); \]

\[ (\vec{p} \vec{F}) = \frac{1}{2\mu} \left( \vec{m} \vec{e}^2 - \nu (\vec{p}_T)^2 \right). \]
3. Dimensional analysis and the automodelity principle in physics of deep inelastic interactions

3.1. The concept of similarity and the dimensional analysis

As experimental data indicate, one of the most general tendencies observed in superhigh energy behaviour of particle interactions is exposing of definite properties of similarity which connect a behaviour of amplitudes and cross sections at different values of energy and momentum transfer.

Formulation and interpretation of these similarity properties is one of the most important problems of the contemporary theory of elementary particles.

Various approaches to solving this problem do exist. In this section we will present the main ideas of the method called the automodelity principle, which provides an unique description of all the processes of deep inelastic lepton-hadron interactions.

The automodelity principle is based on the concept of similarity and on the dimensional analysis.

Notice, that the dimensional theory has emerged as a result of application to physical phenomena of concepts of the geometrical similarity, ratio and proportion already known by Greeks. For the first time this apparently has been done by Galilej in determining the beam stability of a given material as depending on its linear size. Many great scientists starting from Mariotte, Newton, Fourier, etc. have contributed to developing of methods of the dimensional theory and to their basic concept of physical similarity.

The dimensional theory is based on the fundamental property of invariance of the physical laws and their mathematical formulations with respect to the concrete choice of units for physical quantities.

Transformations of similarity related to changes of scales, i.e. units of the main m dimensional physical quantities, are called the dimension transformations or scale transformations. The concept of dimensionality of physical quantities is tightly related to the dimension or scale transformations.

**Dimensionalities:**

Let among the given variable and constant quantities characterizing some class of phenomena we can pick out a minimum number of basic quantities $E_1, E_2, \ldots, E_K$, for which independent units or scales denoted as $e_i = [E_i]$ $i = 1, 2, \ldots, K$ are determined.

We will say that a physical quantity $\rho$ has a dimensionality $[\rho] = e_1^{\alpha_1} e_2^{\alpha_2} \ldots e_K^{\alpha_K}$, where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_K)$
is a set of some numbers, if under the scale transformations of the basic units \( E_i \rightarrow \lambda_i E_i \); \( i = 1, 2, \ldots, k \); the quantity \( \rho \) transforms as a homogeneous function

\[
E_1^{d_1} E_2^{d_2} \cdots E_k^{d_k} ,
\]

i.e.,

\[
\rho \rightarrow \lambda_1^{d_1} \lambda_2^{d_2} \cdots \lambda_k^{d_k} \cdot \rho.
\]

The main laws and functional relations characterizing the given range of physical phenomena do not depend on a choice of basic units and consequently do not change their form under the scale transformations of dimensional quantities. This property of mathematical forms of physical laws results in one of the main theorems of the dimensional theory - the so-called \( \Pi \) - theorem.

\( \Pi \) - theorem.

Let the given class of phenomena be characterized by a set of \( n \) dimensional quantities among which \( k \) quantities have independent dimensionalities and can be chosen as "primary" fundamental quantities \( E_i \), defining the basic units:

\[
e_i = [E_i] ; \quad i = 1, 2, \ldots, k ; \quad k < n .
\]

Then a set of others, "secondary" quantities \( P_i = 1, 2, \ldots, n-k \) will have dimensionalities determined via basic ones in the following way:

\[
[P_i] = E_1^{d_1 (i)} E_2^{d_2 (i)} \cdots E_k^{d_k (i)} ; \quad i = 1, 2, \ldots, n-k .
\]

Let us form dimensionless combinations of the quantities:

\[
\pi_i = \frac{\rho_i}{E_i^{d_1 (i)} E_2^{d_2 (i)} \cdots E_k^{d_k (i)}} ; \quad i = 1, 2, \ldots, n-k ;
\]

which stay invariant under the scale transformations of the dimensional quantities.

Then any functional relation of the theory, of general form \( f(E_1, \ldots, E_k ; P_1, \ldots, P_{n-k}) = 0 \) independent of a choice of basic units, can be reduced to a functional relation between smaller number of the dimensionless variables:

\[
\Phi (\pi_1, \pi_2, \ldots, \pi_{n-k}) = 0 .
\]

The main difference between numbers of all dimensional quantities and basic quantities with independent dimensionalities, the larger simplicity is acquired by the functional relations of theory as a result of application of the \( \Pi \) - theorem. In particular case, when \( n-k = 1 \), the functional relation becomes of the extremely simple form and can be reduced to the form

\[
\rho = c \cdot E_1^{d_1} E_2^{d_2} \cdots E_k^{d_k} ,
\]

with one arbitrary dimensionless factor.

We would remind that, for example, in mechanics the CGS system with three independent dimensionalities is usually used

\( k = 3 \): length \( L \, (cm) \), mass \( M \, (gr) \); time \( T \, (sec) \).
In elementary particle physics, due to the presence of fundamental physical constants $\hbar$ and $c$ determining the nature of micro-world, there does exist only one independent dimensionality ($k = 1$):

length $L$ or mass $M$.

There is however an interesting possibility of increasing of a number of basic dimensionalities when using the so-called vector units of length $\vec{L} = (L_x, L_y, L_z)$. This possibility corresponds introducing of different scales for measurement of lengths along various space directions. Such a possibility, considered apparently firstly by Williams in 19... proved to be extremely fruitful in applying the methods of the dimensional theory to an analysis of multiparticle productions in hadron collisions, within the framework of the automodelity principle.\(^\star\)

\(^\star\) An application of the generalized dimensional analysis with vector units of length for description of the multiparticle process behaviour in hadron collisions is discussed in the lectures by Prof. R. Muradyan.

Not let us formulate the concept of physical similarity.

Two physical phenomena or processes are similar, if they can be connected through the scale transformation of basic dimensional quantities characterizing these phenomena or processes.

The necessary and sufficient condition for similarity of two phenomena is the universality of numerical values of essential dimensionless parameters, defining a character of the given processes.

Notice, that, for example, in hydrodynamics such dimensionless parameters, giving a criterion of similarity of processes of equilibrium flow of liquid are the Reynolds number ($Re = \frac{\nu L \rho}{\mu}$), Mach number ($M = \frac{V}{c}$), etc.

Then the given set of phenomena does not depend on some dimensional quantities or parameters, one can say about self-similarity of the phenomena with respect to the scale transformations which do not touch these quantities or parameters.

In problems of gas dynamics and hydrodynamics in the corresponding cases, when processes are independent of any dimensional parameters, the corresponding solutions are said to be automodel (self-similar). A classical example of such a situation is the problem of intensive "point" explosion, the solution of which in the limit of infinitely small size of a charge has the automodel character.

According to the $\Pi$-theorem of the dimensional
theory, an extra decrease of the number of essential dimensionless variables or parameters describing the process in question as compared with their maximal total number \((n-k)\) is peculiar to the automodel solutions. Consequently, the automodel solutions can be considered as partial solutions of the task which depend on smaller number of variables and parameters as compared with their total number (degeneration with respect to the parameters).

According to that said above, we will use the concepts of automodelity and automodel behaviour as an expression of self-similarity which is a result of dropping of definite dimensional quantities or parameters out of the physical picture of a phenomenon.

Thus, the theory of similarity and the dimensional analysis employ the following notions and definitions:

\[
\begin{align*}
\text{Dimensionality} & \quad = \quad \text{a degree of homogeneity of a physical quantity with respect to the given set of basic units;} \\
\text{Scale transformations} & \quad = \quad \text{transformations connected with a change of scales of basic units;} \\
\text{Similarity} & \quad = \quad \text{an invariance under scale transformations of the general form;} \\
\text{Self-similarity} & \quad = \quad \text{the invariance under scale transformations not touching a definite set of dimensional quantities;}
\end{align*}
\]

\[
\text{Automodelity} = \quad \text{the self-similarity, connected with degeneracy with respect to some set of dimensional variables or parameters.}
\]

3.2. The automodelity principle for deep inelastic lepton-hadron interactions.

In studying the processes of deep inelastic lepton-hadron interactions at high energies and large momentum transfers, there emerges the so-called "point-like" picture of behaviour which is characterized by disappearance of the dimensional parameters, defining the particle effective radii, in the differential cross sections of these processes.

Physically it means, that in the processes of weak and electromagnetic interactions leptons play the role of test particles with zero sizes, i.e. they behave as point-like particles.

The effective sizes of hadron targets disappear, in summing over all the open channels, what results in the above mentioned "point-like" behaviour of deep inelastic lepton-hadron interactions. The asymptotic behaviour of form factors of these processes are determined by functions depending on the dimensionless ratios of invariant variables only. Such an asymptotic behaviour of form factors of inelastic weak and electromagnetic processes has a character similar to the so-called automodel solu-
tions of a number of classical problems of hydrodynamics, for example, as was mentioned above, the task of strong point explosion. One can expect that at high energies and large momentum transfers, when the particle masses can be neglected, the situation is very simplified and, in some sense, becomes "hydrodynamical" one. Starting from this analogy, we formulate here the automodelity principle for the processes of weak and electromagnetic interactions at high energies and large momentum transfers, which provides a unique approach to the description of all deep inelastic lepton-hadron interactions by using the methods of similarity and dimensional analysis.

The automodelity principle.

Let us suppose that the asymptotic behaviour of deep inelastic lepton-hadron interactions does not depend on any dimensional parameters fixing a scale of lengths or momenta, such as masses, dimensional coupling constants, "elementary lengths", etc.; and, consequently, the structure functions of these processes would depend only on the kinematically invariant variables. Then under the scale transformations, connected with changing of the measurement scale of momenta, all physical quantities such as the cross sections, or form factors transform as homogeneous functions of the corresponding physical dimensionalities.

So, the physical quantity $F(q, p_i)$ characterizing a deep-inelastic process with momentum $q$ transferred from leptons to hadrons and with momenta of interacting hadrons $p_i$, having a dimensionality $n$, i.e.

$$[F(q, p_i)] = \xi^n \text{ or } m^{-n},$$

under the scale transformations

$$\xi \to \lambda \xi; \quad p_i \to \lambda p_i$$

behaves as a homogeneous function

$$F(\lambda q, \lambda p_i) = \lambda^{-n} F(q, p_i).$$

An applicability range of the automodelity principle and following from this properties of scale invariance is given by the conditions:

$$|q^2|, \quad s_i = p_i p_i; \quad s_{ij} = p_i p_j \geq M^2; \quad \frac{q^2}{s_i}, \quad \frac{q^2}{s_{ij}} \text{ fixed.}$$

Below we shall demonstrate the applications of the automodelity principle to description of various processes of deep inelastic lepton-hadron interactions. The following agreement concerning dimensionalities of basic quantities will be employed:

We take the system of units where $\hbar = c = 1$ and, consequently, the velocity and action are dimensionless. As a basic dimensional quantity we take some mass $m$. It is evident that an element of cross section will have dimensionality $[d\sigma] = m^{-2}$, while all basic kinematical invariants of the processes have dimen-
sionality:

\[ [t^2] = [v_i] = [s_{ij}] = m^2. \]

When defining dimensionalities for various quantities of the theory, such as current matrix elements, structure functions and form factors, we should take into account that the dimensionality of currents equals:

\[ [J_\mu] = [\bar{v} \gamma_\mu \nu] = [\bar{v} \gamma_\mu \tau_5 \nu] = m^3, \]

and the dimensionality of the one-particle state vectors \(|p\rangle\) which are normalized in the relativistically invariant way:

\[ \langle p' |p\rangle = 2 \rho \cdot (2\pi)^3 \delta(\vec{p}' - \vec{p}) \]

can be taken as follows:

\[ [|p\rangle] = m^{-1}. \]

**Annihilation of lepton pairs:**

In accordance with the automodality principle the total cross section of electron-positron pair annihilation into hadrons: \(e^+e^- \rightarrow \text{hadrons} \) at \(q^2 \gg M^2\), where \(M\) is some mass parameter of the order of the nucleon mass, is a function of the variable \(q^2\) only and does not depend on any dimensional fixed parameters.

Then, from the dimensional analysis it follows

\[ \sigma_{e^+e^- \rightarrow \text{hadrons}} (q^2) = \frac{C}{q^2} \]  \hspace{1cm} (3.1)

where \(C\) is some dimensionless constant.

In the lowest approximation in the electromagnetic interaction, where

\[ \sigma_{e^+e^- \rightarrow \text{hadrons}} (q^2) = \frac{8 \pi \alpha^2}{q^2} \rho (q^2), \]

the dimensionless constant \(C\) is determined by the limit:

\[ C = 8 \pi \alpha^2 \lim_{q^2 \to \infty} \rho (q^2). \]

Applying the dimensional analysis directly to the quantity \(\rho (q^2)\) defined by the expression

\[ \rho (q^2) (\vec{q} \delta_{\mu\nu} + q_\mu q_{\nu}) = \int dx e^{i \vec{q} \cdot x} \langle 0 | J_\mu (x) J_\nu (0) | 0 \rangle \]

we find, that the quantity \(\rho (q^2)\) is dimensionless and consequently under the scale transformations \(\xi \rightarrow \lambda \xi\) transforms as a homogeneous function of zero dimension, i.e.

\[ \rho (q^2) \rightarrow \rho (\lambda q^2) = \rho (q^2), \]

what is in accordance with the formula (3.1).

For the pure conditional example, when the final state of annihilation process coincides with a pair of point-like Dirac particles, say \(\mu^+\mu^-\) -pair, we have
\[ \rho \to \mu \mu^* = \frac{1}{6\pi} ; \quad \xi^2 \gg m^2. \]

Notice here an interesting circumstance, that the automodel asymptotics of the total cross section of the \( e^+ e^- \) pair annihilation into hadrons corresponds, with respect to the character of asymptotic behaviour, the upper bound on the total cross section of annihilation in the lowest \( e^2 \) approximation, which follows from the unitarity condition (see Appendix):

\[ \sigma_{e^+ e^- \to \text{hadrons}} \leq \frac{12\pi}{\xi^2}. \]

Thus, the automodel asymptotics for the total cross section of annihilation of \( e^+ e^- \) pairs into hadrons does not contradict, in the framework of \( e^2 \) approximation, the unitarity condition only when the following inequality holds:

\[ \lim_{\xi^2 \to \infty} \rho(\xi^2) \leq \frac{3}{2\pi m^{-1}}. \]

It is not difficult to generalize these results for the case of annihilation of charged lepton pairs into hadrons:

\[ \nu_e \bar{e} \to \text{hadrons} ; \quad \bar{\nu}_e \nu \to \text{hadrons} ; \quad \xi = e, \mu^- ; \]

the cross sections of which in the lowest approximation in weak interaction coupling constant \( G \) are determined by the expression

\[ \sigma_{\nu \bar{e} \to \text{hadrons}} = \sigma_{\bar{\nu} e \to \text{hadrons}} = G^2 \rho_{\text{weak}}(\xi^2) ; \quad \xi^2 \gg m^2; \]

where we use the definition:

\[ \int dx e^{-i\xi x} J_{\mu}^e(x) J_{\nu}^{\mu^*}(0) = \rho_{\text{weak}}(\xi^2) (\xi^2 \nu^2 + \xi \nu \bar{\nu} + \xi \bar{\nu} \nu) \xi^2. \]

Taking into account that the constant of weak interaction \( G \) is dimensional quantity

\[ [G] = m^{-2}, \]

we find that the quantity \( \rho_{\text{weak}} \) is dimensionless and in accordance with the automodelity principle must behave at large transferred momenta \( \xi^2 \) as constant, i.e.

\[ \sigma_{\nu \bar{e} \to \text{hadrons}} = \sigma_{\bar{\nu} e \to \text{hadrons}} \to \text{const.} \xi^2 G^2 ; \quad \xi^2 \gg M^2. \]

As an example, we notice that for pure leptonic processes of the type

\[ \nu_e + e \to \nu_e + \mu^+ ; \quad \bar{\nu}_e + e \to \bar{\nu}_e + \mu^- ; \]

the cross sections at large momenta are

\[ \sigma_{\nu \bar{e} \to \text{leptons}} = \sigma_{\bar{\nu} e \to \text{leptons}} = \frac{G^2}{2\pi} \xi^2 ; \quad \xi^2 \gg m^2. \]

As early, the unitarity condition and the optical theorem lead, in the framework of the lowest approximation in weak coupling constant, to an upper bound on the total cross section of charged lepton pair annihilation into hadrons:
\[ \sigma^{\gamma^*}_{ee} \rightarrow \text{hadrons} \leq \frac{8\pi}{\ell^2}. \]

Due to this bound there arises a limitation on the allowed region of energies for which the consideration has a sense, namely

\[ M^2 \ll \ell^2 < \frac{\text{const}}{G}. \]

Deep inelastic scattering of electron and neutrino

The differential cross section of deep inelastic electroproduction \( \frac{d^2\sigma}{d\xi d\ell} \), where \( \xi = -\frac{Q^2}{2M} \) is dimensional variable, can be represented in a general form as

\[ \frac{d^2\sigma}{d\xi d\ell} = \frac{1}{\xi^4} F(s, \xi^2, \ell) \]

where the function \( F(s, \xi^2, \ell) \) - dimensionless quantity.

Using the automodelity principle, we find that under the scale transformations of the virtual photon and nucleon moments

\[ \xi \rightarrow \lambda \xi ; \quad \rho \rightarrow \lambda \rho \]

the quantity \( F \) transforms as a homogeneous function with dimensionality equal to zero, i.e.

\[ F(s, \xi^2, \ell) \rightarrow F(\lambda s, \lambda^2 \xi^2, \ell) = F(s, \xi^2, \ell). \]

Thus, in the region

\[ |\xi^2|, \quad \ell = \rho \xi, \quad s \gg M^2 / \ell, \quad \xi^2 \text{ fixed}, \]

the function \( F(s, \xi^2, \ell) \) depends on the dimensionless kinematical variables \( \xi^2 \) and

\[ \eta = \frac{(1 - 2\nu)}{s} = \frac{(1 + \xi^2)}{s \xi^2} \]

only

\[ F(s, \xi^2, \ell) = F(\xi, \eta). \]

In the one-photon approximation a structure of the quantity \( F \) becomes much more simpler:

\[ F(\xi, \eta) = \frac{4\pi e^2}{\xi^3} \left[ (1 - \eta)^2 F_1(\xi) + \frac{\xi}{\eta} F_2(\xi) \right], \]

where \( F_1 \) and \( F_2 \) are related to the contributions of the transversal and longitudinal polarizations of the virtual photon.

To illustrate this point, apply the automodelity principle immediately to the tensor \( \gamma_{\mu\nu}^{(p,\xi)} \) which corresponds the Fourier transform of the electromagnetic current commutator:

\[ \gamma_{\mu\nu}^{(p,\xi)} = \left( g_{\mu\nu} + \frac{p \xi}{\xi^2} \right) \gamma_1 + \left( \frac{p - \xi}{\xi^2} \right) \gamma_2 \gamma_3. \]

Using the agreement on the dimensionality of currents and state vectors, we find
\[
\left[ W_1 (q^2, \nu) \right] = 1 ; \left[ W_2 (q^2, \nu) \right] = m^2.
\]

Under scale transformations the quantities \( W_1, W_2 \) undergo the transformations

\[
W_1 (q^2, \nu) \rightarrow W_1 (\lambda q^2, \lambda \nu) = W_1 (q^2, \nu);
\]

\[
W_2 (q^2, \nu) \rightarrow W_2 (\lambda q^2, \lambda \nu) = \frac{-1}{\lambda} W_2 (q^2, \nu).
\]

For this reason, at sufficiently large energies and transferred momenta the form factors \( W_1 \) behave as follows

\[
W_1 (q^2, \nu) = F_1 (\lambda);
\]

\[
W_2 (q^2, \nu) = \frac{1}{\nu} F_2 (\lambda);
\]

where \( F_1, F_2 \) — dimensionless functions of the variable \( \lambda = -q^2/2\nu \).

For the scattering off the point-like particle we have

\[
F_2 (\lambda) = \delta (1 - \lambda);
\]

\[
F_1 (\lambda) = 0 \quad \text{for spin } S = 0;
\]

\[
= \frac{1}{\nu} \delta (1 - \lambda) \quad \text{for spin } S = 1/2.
\]

In general case the scale-invariant (automodel) functions which correspond the transverse and longitudinal polarizations of a virtual photon, are the functions

\[
F_T = 2 F_1 \quad - \text{transverse polarization};
\]

\[
F_L = \frac{1}{\nu} F_2 - 2 F_1 \quad - \text{longitudinal polarization},
\]

in terms of which the differential cross section of deep inelastic scattering reads:

\[
\frac{d^2\sigma}{dq^2 d\lambda} = \frac{4\pi\alpha^2}{q^2} \left[ \eta F_L (\lambda) + \frac{1}{2} (1 + \eta^2) F_T (\lambda) \right].
\]

A generalization to the case of deep inelastic neutrino scattering requires only to consider in addition the form factor \( W_3 \), which determines in the matrix element of the commutator of weak charged hadron currents an interference of the vector and the axial parts of interaction, i.e.

\[
\frac{1}{2} \epsilon_{\mu\nu\lambda\rho} \epsilon^\mu \epsilon^\nu \cdot W_3 (q^2, \nu).
\]

As the form factor \( W_3 \) has a dimensionality \( [W_3] = m^2 \), the automodelity principle leads to the behaviour of the type:

\[
W_3 (q^2, \nu) = \frac{1}{\nu} F_3 (\lambda).
\]

Introducing the combinations of the automodel functions corresponding to the "right", "left" and the scalar polarizations of the current of lepton pair:
\[ F_R = F_1 + \frac{1}{2} F_3 \]  
\[ F_L = F_1 - \frac{1}{2} F_3 \]  
\[ F_\sigma = -2F_1 + \frac{1}{2} F_2 \]  
\[ = \text{right} ; \]  
\[ = \text{left} ; \]  
\[ = \text{scalar or longitudinal} \]

One represents the differential cross section of deep inelastic neutrino scattering in the form:

\[
\frac{d^2 \sigma}{d^2 \ell^2} = \frac{g^2}{2\pi} \left[ \gamma \cdot F_\sigma + \frac{1}{2} F_1 \right]_R \to \ell_R \to \gamma \]  
The distribution in the invariant transferred momentum squared is determined by

\[
\frac{d\sigma}{d\ell^2} = \frac{g^2}{\pi} f(\ell^2/\ell_s^2) \]

where the dimensionless function \( f(\ell^2/\ell_s^2) \) is a polynomial of second order degree in respect to the variable \( \ell^2/\ell_s^2 \).

For example, for pure leptonic processes one gets:

\[ \nu \ell \to \nu \ell : f(\ell^2/\ell_s^2) = 1 ; \]  
\[ \bar{\nu} \ell \to \bar{\nu} \ell : f(\ell^2/\ell_s^2) = (\ell^2/\ell_s^2) \]  

Integrating the distribution \( d\sigma/d\ell^2 \) over all the momentum transfers in the region \( 0 \leq \ell^2/\ell_s^2 \leq S \)

we find the total cross sections of neutrino processes:

\[ \sigma_N/\ell_s = \text{const.} \frac{g^2}{\pi} ; \quad S \gg M^2. \]

Thus, the cross section of a neutrino reaction must grow and at sufficiently large energies

\[ S \sim \frac{1}{G^2} \approx (300 \text{ GeV})^2 \]

reaches values forbidden by the unitarity in the framework of the lowest approximation in weak interactions.

In conclusion we would like to stress that

1. the automodelling principle provides a general method applicable to all the processes of deep inelastic lepton-hadron collisions.

2. the automodelling principle is not based on any specific dynamic mechanism of deep inelastic processes, it supposes only a degeneration of physical quantities with respect to dimension characteristics of particles, such as masses, effective sizes, etc. The predictions of the automodelling principle are based on the general concepts of similarity and dimensionality and, consequently, are model-independent.

3. deviations from predictions of the automodelling principle if exist would have a fundamental character and must point apparently to the existence of an "elementary" length or "superhigh" mass, which will require the cardinal reconsideration of our ideas on elementary particles.

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