A Unified Model for inelastic $e - N$ and $\nu - N$ cross sections at all $Q^2$

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Abstract. We present results using a new scaling variable, $\xi_w$ in modeling electron- and neutrino-nucleon scattering cross sections with effective leading order PDFs. Our model uses all inelastic charged lepton $F_2$ data (SLAC/BCDMS/NMC/HERA), and photoproduction data on hydrogen and deuterium. We find that our model describes all inelastic scattering charged lepton data, the average of JLAB resonance data, and neutrino data at all $Q^2$. This model is currently used by current neutrino oscillation experiments in the few GeV region.

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The field of neutrino oscillation physics has progressed from the discovery of neutrino oscillation [1] to the era of precision measurements of mass splitting and mixing angles. Currently, there are only poor measurements of differential cross sections for neutrino interactions in the few GeV region. This results in large systematic uncertainties in the extraction of mass splitting and mixing parameters (e.g. by the MINOS, NOνA, K2K and T2K experiments). Therefore, reliable modeling of neutrino cross sections at low energies is essential for precise (next generation) neutrino oscillations experiments. In the few GeV region, there are three types of neutrino interactions: quasi-elastic, resonance, and inelastic scattering. It is very challenging to disentangle each contribution separately, especially, resonance production versus deep inelastic scattering (DIS) contributions. There are large non-perturbative QCD corrections to the DIS contributions in this region.

Our approach is to relate neutrino interaction processes using a quark-parton model to precise charged-lepton scattering data. In a previous communication [2], we showed that our effective leading order model using an improved scaling variable $\xi_w$ describes all deep inelastic scattering charged lepton-nucleon scattering data including resonance data (SLAC/BCDMS/NMC/HERA/Jlab) [3, 4] from very high $Q^2$ to very low $Q^2$ (down to photo-production region), as well as high energy CCFR neutrino data [5].

The proposed scaling variable, $\xi_w$ is derived using energy momentum conservation, assuming massless initial state quarks bound in a proton of mass $M$.

$$\xi_w = \frac{2x(Q^2 + M_f^2 + B)}{Q^2[1 + \sqrt{1 + (2Mx)^2/Q^2}] + 2Ax},$$

here, $M_f$ is the final quark mass (zero except for charm-production in neutrino processes). The parameter $A$ accounts for the higher order (dynamic higher twist) QCD terms in the form of an enhanced target mass term (the effects of the proton target mass...
are already taken into account using the exact form in the denominator of $\xi_w$. The parameter $B$ accounts for the initial state quark transverse momentum and final state quark effective $\Delta M_f^2$ (originating from multi-gluon emission by quarks). This parameter also allows us to describe the data also in the photoproduction limit (all the way down to $Q^2=0$).

A brief summary of our effective leading order (LO) model is given as follows:

- The GRV98 LO PDFs $[6]$ are used to describe the $F_2$ data at high $Q^2$ region.
- The scaling variable $x$ is replaced with the improved scaling variable $\xi_w$ (Eq. 1).
- All PDFs are modified by $K$ factors to describe low $Q^2$ data in the photoproduction limit.

\[ K_{\text{sea}}(Q^2) = \frac{Q^2}{Q^2 + C_s}, \quad K_{\text{valence}}(Q^2) = [1 - G_D^2(Q^2)] \left( \frac{Q^2 + C_v}{Q^2 + C_{v1}} \right), \]  

(2)

where $G_D = 1/(1 + Q^2/0.71)^2$ is the proton elastic form factor. At low $Q^2$, $[1 - G_D^2(Q^2)]$ is approximately $Q^2/(Q^2 + 0.178)$. Different values of the $K$ factor are obtained for $u$ and $d$ quarks.

- The evolution of the GRV98 PDFs is frozen at a value of $Q^2 = 0.80$ which is the minimum $Q^2$ value of this PDFs. Below this $Q^2$, $F_2$ is given by:

\[ F_2(x, Q^2 < 0.8) = K(Q^2) \times F_2(\xi, Q^2 = 0.8) \]  

(3)

- Finally, using these effective GRV98 LO PDFs ($\xi_w$) we fit to all inelastic charged lepton scattering data (SLAC/BCDMS/NMC/H1) and photoproduction data on hydrogen and deuterium. Note that no resonance data is included in the fit. We obtain excellent fits with: $A=0.538$, $B=0.305$, $C_{v1}=0.202$, $C_{v1}=0.291$, $C_{v2}=0.255$, $C_{v2}=0.189$, $C_{v1}=0.621$, $C_{v1}=0.363$, and $\chi^2/DOF = 1874/1574$. Because of the $K$ factors to the PDFs, we find that the GRV98 PDFs need to be multiplied by a factor of 1.015.

The measured structure functions data are corrected for the relative normalizations between the SLAC, BCDMS, NMC and H1 data. The deuterium data are corrected for nuclear binding effects $[7]$. We also add a separate charm pair production contribution to lepton-nucleon scattering using the photon-gluon fusion model. This component is not necessary at low energies, and is only needed to describe the highest $\nu$ HERA $F_2$ and photoproduction data (since the GRV98 LO PDFs do not include a charm sea).

Our effective LO model describes various DIS (SLAC, BCDMS, NMC, and HERA) and photo-production data down to the $Q^2 = 0$ limit. Fig. 1 shows some of comparisons. Furthermore, based on duality arguments $[8]$, it appears that this model also provides a reasonable description of the average value of $F_2$ for SLAC and Jlab data in the resonance region, as shown in Fig. 2 (left). Although no resonance data has been included in our fit, our model gives a good description of the most recent $2xF_1$ electron-proton Rosenbluth separated data in the resonance region from Jefferson Lab Hall C E94-110 Collaboration $[9]$, and also data from the first phase of the JUPITER program at Jlab. Our predictions for $2xF_1$ are obtained using our $F_2$ model and $R_{1998}^1$ $[10]$. For heavy targets, nuclear effects are important, especially at low $Q^2$. Recent results from Jlab indicate that
FIGURE 1. Comparisons of the predictions of our effective LO model for $F_2$ to charged lepton inelastic scattering data. [left] DIS $F_2$ proton data (SLAC, BCDMS, NMC), [right] H1 $F_2$ proton.

FIGURE 2. Comparisons of the predictions of our effective LO model to resonance electro-production data on protons (which was not included in our fit). Shown are $F_2$ proton [left], and $2xF_1$ proton data [right] (from the Jefferson Lab Hall C E94-110 Collaboration). The predictions for $2xF_1$ are obtained from our model for $F_2$ with $R_{1998}$. In the right plot, the solid line uses GRV98 PDFs, and the dashed line is our previous model using GRV94 LO PDFs.
the Fe/D ratio in the resonance region is the same as the Fe/D ratio from DIS data for the same value of $\xi$ (or $\xi_w$). Future Jlab experiments with deuterium and heavy nuclear targets (e.g. JUPITER) will provide a high statistics data in the resonance region which will be very important to improve our model at $Q^2$ region.

In neutrino scattering, in addition to the vector structure function, there is an axial vector structure function contribution. At the $Q^2 = 0$ limit, the vector structure function goes to zero, while the axial-vector part has a finite contribution. At high $Q^2$, these two structure functions are expected to be the same. Thus, it is important to understand the axial-vector contribution at low $Q^2$ by comparing to future low energy neutrino data (e.g. MINERvA [14]). As a preliminary step, we compare the CCFR and CDHSW [11] high energy neutrino data with our model, assuming that the vector contribution is the same as the axial vector contribution. We find that the CCFR/CDHSW neutrino data are well described by our model.

We are currently working on constraining the low $Q^2$ axial vector contribution using low energy CDHSW and CHORUS [12] data. The form of the fits we plan to use is motivated by the Adler sum rule [13] for the axial vector contribution as follows:

$$K_{sea-ax}(Q^2) = \frac{Q^2 + C_{2s-ax}}{Q^2 + C_{1s-ax}}, \quad K_{valence}(Q^2) = \left[1 - F_A^2(Q^2)\right] \left(\frac{Q^2 + C_{2v-ax}}{Q^2 + C_{1v-ax}}\right),$$

where $F_A(Q^2) = -1.267/(1 + Q^2/1.00)^2$. Nuclear effects for heavy target are also important and may be different for the vector and axial vector structure functions. Future measurements on the axial vector contribution from the MINERvA experiment [14] will be important in constraining this model.

REFERENCES