Spontaneous emergence of contrarian-like behaviour in an opinion spreading model

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PACS. 83.23.Ge – Dynamics of social systems.
PACS. 89.70.+q – Lattice theory and statistics.
PACS. 05.10.Gg – Stochastic analysis methods.

Abstract. – We introduce stochastic driving in the Sznajd model of opinion spreading. This stochastic effect is meant to mimic a social temperature, so that agents can take random decisions with a varying probability. We show that a stochastic driving has a tremendous impact on the system dynamics as a whole by inducing an order-disorder nonequilibrium phase transition. Interestingly, under certain conditions, this stochastic dynamics can spontaneously lead to agents in the system who are analogous to Galam’s contrarians.

Introduction. – The study of complex systems, in particular the application of statistical physics methods to social phenomena, has recently attracted the attention of theoretical physicists [1–5]. From a statistical physics perspective, social systems are modeled as a collection of agents interacting through simple rules. In particular, the building (or the lack) of consensus in social systems has attracted much attention in recent years. A number of models have been considered in order to mimic the dynamics of consensus in opinion formation, cultural dynamics, etc [6]. Among those models, Sznajd dynamics of opinion spreading has been subject of a great deal of work in recent years. Sznajd model is duly based on the trade union lemma: “united we stand, divided we fall”, and has been studied on different network topologies and for (slight) variations of the dynamics [6–11].

An important aspect of social and economic systems, recently discussed within opinion formation models, has been the presence of some agents called contrarians – namely, people who are in a “nonconformist opposition”. That is, people who always adopt the opposite opinion to the majority [12–14]. In stock markets for instance, contrarians are those investors who buy shares of stock when most others are selling and sell when others are buying. The existence of a high proportion of contrarians in a society may play an important rôle in social dynamics (think for instance of referendums or stock market dynamics) [12].

In an attempt to include the contrarian effect in existing social models, a number of previous studies have considered contrarian agents as a initial condition, i.e. a given density...
of contrarians is introduced in the model by hand for instance [12–14]. This is somewhat artificial and one would expect that simple models of opinion spreading should spontaneously lead to the existence of a fraction of contrarians among the population as some sort of emergent property.

In this Letter we show that a contrarian effect can spontaneously emerge when stochastic driving is included in the model. As a typical model example we introduce stochastic dynamics on the Sznajd model. This randomness in the update of an agent opinion is meant to be a highly simplified description of the interplay between fashion/propaganda and a collective climate parameter, which is usually referred to as social temperature of the system [15–17]. We show here that social temperature in Snajd-type models leads to the spontaneous appearance of contrarian agents in the system.

Mean-field approach. – Following the mean-field approach in [8], we have considered the Sznajd model (the so-called “two over one” case in [8]), where two agents are chosen randomly and, if they are in consensus, then another randomly chosen agent is convinced by them. The Fokker-Planck equation (FPE) for the probability \( P(m,t) \) of having a “magnetization” \( m \), at time \( t \) given a certain initial condition at time \( t_0 < t \) is given by

\[
\frac{\partial}{\partial t} P(m,t) = -\frac{1}{2N} \frac{\partial}{\partial m} \left( m(1 - m^2)P(m,t) \right) + \frac{1}{2N} \frac{\partial^2}{\partial m^2} \left( (1 - m^2)P(m,t) \right) + O(N^{-3}),
\]

where \( N \) is the total number of agents. The magnetization density \( m = (N^+ - N^-)/N \) measures the opinion state of the system and \( N^+ \), \( N^- \) are the number of agents supporting the + or the - opinion, respectively (with \( N^+ + N^- = N \)).

We now include a social temperature effect in the model by allowing the possibility that at every time step an agent follows the rules of the Sznajd model with probability \( p \) \( (p \leq 1) \), while there is a probability \( 1 - p \) that those rules are not fulfilled (an agent adopts the opposite option than the one dictated by the rules). Then, for a given probability \( p \), we arrive at a FPE for \( P_p(m,t) \) that reads

\[
\frac{\partial}{\partial t} P_p(m,t) = -\frac{1}{2N} \frac{\partial}{\partial m} \left( \left[ (6p - 5)m - (2p - 1)m^3 \right] P_p(m,t) \right) + \frac{1}{2N} \frac{\partial^2}{\partial m^2} \left( \left[ 3 - 2p - (2p - 1)m^2 \right] P_p(m,t) \right).
\]

The stationary solution \( P_p^{stat}(m) \) results to be

\[
P_p^{stat}(m) \approx \exp \left\{ N \int_{-1}^{1} \frac{(6p - 5)u(2p - 1)u^3}{3 - 2p - (2p - 1)u^2} du - \ln \left[ N(3 - 2p - (2p - 1)m^2) \right] \right\}.
\]

The analysis of this stationary solution for varying \( p \) shows that there is a threshold value, \( p = p_c \), such that for \( p > p_c \) the system is bistable with a probability density \( P_p^{stat}(m) \) having two maxima at \( m_\pm = \pm \sqrt{(6p - 5)/(2p - 1)} \). In this case the system gets ordered by spontaneously selecting one of the stable solutions \( m_\pm \). On the contrary, for \( p < p_c \) the system becomes monostable and disordered with a magnetization density peaked at \( m = 0 \) in which no dominant opinion survives. The threshold \( p_c \) can be calculated in this mean-field approximation equating \( m_+ = m_- \), at which all three extreme coalesce into a single minimum at \( m = 0 \), so that we find \( p_c = 5/6 \). This behavior is shown in Fig. 1.
The picture emerging from the mean-field approach is clear. The effect of including thermal fluctuations in Sznajd type models immediately leads to a contrarian-like effect. Some agents randomly take decisions that oppose the rules of the model, indicating some undecidedness in a fraction of the population. If such a fraction overcomes the critical threshold ($1 - p_c = 1/6 \approx 0.1667$) the system will reach a stalemate situation, analogous to the contrarians effect discussed in [12–14].

Monte Carlo simulations. – In what follows we report on Monte Carlo simulations, in order to test the above discussed mean-field results. We have studied the model on regular lattices and small-world networks (which in the limit of high rewiring probability should reproduce the mean-field results). To make such an analysis, and in order to avoid the spurious antiferromagnetic solution of the Sznajd original model, we have studied a convenient variation proposed in [18]:

- **rule 1’**: Chose an agent at random, say $i$, and if $s_i \times s_{i+1} = 1$, then $s_{i-1}$ and $s_{i+2}$ adopt the direction of the selected pair $[i, i + 1]$,

- **rule 2’**: if $s_i \times s_{i+1} = -1$, then $s_i$ adopts the direction of $s_{i-1}$ and $s_{i+1}$ the direction of $s_{i+2}$.

Following [18], in case of disagreement of the pair $(s_i, s_{i+1})$, the rule 2’ makes that the agent $i$ "feels more comfortable" since it ends up with at least one neighbor having his same opinion. This variation of Sznajd model does not affect the basic behaviour and indeed has been shown to exhibit the same type of scaling features as the original model.

We now introduce a stochastic mechanism in the dynamics as follows. At each Monte Carlo step we assume that, with a probability $p$, the rules are fully applied as indicated above,
while the opposite option to the one dictated by the rules happens with a probability $1 - p$. The probability $p$, in analogy with Weidlich [1, 2] and Babinec [16], is defined according to

$$p = \Lambda \exp\left[\frac{\alpha}{\theta}\right],$$

where $\alpha$ is some fixed parameter related to the strength of nearest neighbour interactions (we assume $\alpha > 0$), and $\theta$ is the collective climate parameter and plays the role of a (social) temperature. The normalization constant is $\Lambda^{-1} = \exp(\alpha/\theta) + \exp(-\alpha/\theta)$.

The asymptotic behaviour of $p$ is:

- if $\theta \to 0$, we have $p \to 1$, indicating that without thermal fluctuations we recover Sánchez (and Sznajd) dynamics;
- if $\theta \to \infty$, we have $p \to 0.5$, the probability of fulfilling the model rules or its opposite are the same. The model has a complete random behavior.

Firstly we report on our results on the one-dimensional lattice. Each lattice site is occupied by one agent with opinion (spin) $s_i \in \{+1, -1\}$. Starting from a random initial condition we let the system evolve towards its stationary state. For $\theta = 0$ a consensus state arises ($m_{\pm} = \pm 1$). However, as $\theta$ is increased we observe transition towards the stalemate state as predicted by mean-field theory, nonetheless the transition is discontinuous (first order) as shown in Fig. 2.

In order to compare with the mean-field results, we have studied our model in a fully connected network, which is expected to behave as a mean-field system. Indeed, in a fully connected network we observe that the transition between order and stalemate state becomes continuous. The qualitative agreement with mean-field results is apparent in Fig. 3.
Fig. 3 – Stationary distribution of $m$ for a fully-connected lattice of $N = 512$ agents. Data were averaged over 100 independent runs. From left to right and top to bottom $\theta = 0.30, 2.30, 2.60,$ and $2.90$. A second order transition towards a “stalemate” state is apparent: the most probably values of the order parameter $m$ changes continuously from $m = \pm 1$ (bistable) to $m = 0$ (monostable).

We have also studied the intermediate cases between one-dimensional and mean-field (infinite dimensional) limits by analyzing the model on small-world networks. Starting from a regular a one-dimensional lattice with periodic boundary conditions the links between neighbours are rewired with a certain probability $r$ to a random site. Even for small values of $r$ we observe a continuous phase transition from order to disorder as predicted by mean-field theory. We also observed that for increasing values of the rewiring probability $r$, the critical temperature, $\theta^*$, also increases, as shown in Fig.4.

The critical density of contrarians that it is required to reach the threshold ($\rho_c \sim 1 - p(\theta^*)$) is relatively large (the mean-field result $\rho_c \sim 0.33$ in the case 2 against 2). However, such values are reasonable baring in mind that such a density of contrarians corresponds to a statistical average, dynamically generated by a large value of the social temperature (that is, each agent could sustain, convince or change its opinion dynamically). This mechanism is notably different from setting a fixed number or density $\rho_c$ of agents in a “nonconformist opposition” that will never follow the rules, as was done in previous studies [12–14]. In physical terms this difference is similar to the distinction between annealed and quenched disorder.

**Conclusions.** – We have proposed a dynamical mechanism that leads to a contrarian-like effect analogous to the one described in [12]. However, in contrast to [12], we found that contrarians may spontaneously emerge from the dynamics when social temperature effects are taken into account. For low temperatures the system gets to a consensus where a majority opinion emerges just like in Sznajd type models. However, when temperature is above a critical threshold the density of contrarians is (on average) then high enough to make impossible for the system to reach a consensus and the opinion is equally divided between both options.

Here, we have considered different forms and the most convenient prescriptions of the Sznajd model for the analytical and the numerical analysis. However, we have checked that the phenomenon is robust and does not depend on the particular form of the model. Moreover, since the Sznajd model (as well as many other two state opinion formation models) is similar to Ising type model [19] up to a certain extend, we can regard our results as a sophisticated manifestation in social systems of the ferromagnetic transition in spin systems.
Fig. 4 – The critical temperature $\theta^*$ vs. the rewiring probability $r$. All simulations shown correspond to $N = 512$ agents and averages over 100 independent realizations.

To conclude, the possibility of some external stochastic and/or deterministic influence on the agents of an opinion formation model, particularly regarding the possibility of some form of stochastic resonance [20], was recently analyzed by several authors [16, 17]. It would be of great interest to study such an stochastic resonance effect (particularly its dependence on the size of the system [17]) in our model. This can actually be done by including a fashion external field (for instance a periodic signal) combined with the noise effect coming from the social temperature. This is the subject of a forthcoming work.

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We acknowledge financial support from Ministerio de Educación y Ciencia (Spain) through Grant No. BFM2003-07749-C05-03 (Spain). MSL is supported by a FPU fellowship (Spain). HSW thanks the European Commission for the award of a Marie Curie Chair at Universidad de Cantabria (Spain).

REFERENCES