STUDY OF THE FRAGMENTATION $p K^- \Lambda^0$ AND $K^- p \Lambda^0$ AT 4.2 GeV/c

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1. INTRODUCTION

The inclusive production of $\Lambda^0$ by kaons on protons has been investigated in a number of experiments in the momentum range 4-32 GeV/c\(^{\text{(1)}}\).

In this paper we are mainly interested in the $\Lambda^0$ particles coming from target and beam fragmentation. The strangeness exchange and baryon exchange reactions respectively yield information on strangeness annihilation at the $K\bar{K}$ vertex and on $pp$ annihilation at the $\bar{p}p$ vertex. It has been suggested by T. Inami and H. Miettinen\(^{\text{(2)}}\) that the strangeness annihilation in the $K\bar{K}$ interaction may explain the experimentally observed rapid fall of the $\Lambda^0$-production in the beam fragmentation region.

In this spirit one should distinguish the reactions:

\[
\begin{align*}
K^- p &\rightarrow \Lambda + \text{pions} \quad \text{(1)} \\
\text{and} \quad \bar{K}^- p &\rightarrow \Lambda + K\bar{K} + \text{pions} \quad \text{(2)}
\end{align*}
\]

The cross section for these inclusive reactions is in the low missing mass region proportional to the forward Reggeon-particle amplitudes of the processes

\[
\begin{align*}
'K^+ K^- &\rightarrow \text{pions} \quad \text{(3)} \\
'K^+ K^- &\rightarrow K\bar{K} + \text{pions} \quad \text{(4)}
\end{align*}
\]

where $'K^+$ stands for a $K^+$ or $K^+(890)$ Regge trajectory.

Similarly in the beam fragmentation region the reaction

\[
K^- p \rightarrow \Lambda + \text{mesons} \quad \text{(5)}
\]

is related\(^{\text{(3)}}\) to the antinucleon-nucleon annihilation process

\[
'p'p \rightarrow \text{mesons} \quad \text{(6)}
\]

Definitions and the sample are presented in section 2, the missing mass distributions in the fragmentation regions in section 3. Fits to the distributions of crossing symmetric variables and interpretation in terms of trajectories are presented in section 4. Section 5 contains the multiplicity distributions of the residual pion system. Conclusions and discussion are presented in section 6.
2. DEFINITIONS AND SAMPLE

Variables and notations to be used in the study of the inclusive process
\[ K^- p \rightarrow \Lambda^0 X \]  \hspace{1cm} (7)
are illustrated in Fig. 1.

The \( t \) versus \( u \) scatter plot is a suitable representation for
discussing contributions from different processes.

In our experiment \( s = 9.1 \text{ GeV}^2 \) and \( (M_X)_{\text{max}} = 1.90 \text{ GeV/c}^2 \). The
variable \( t + u \) is a linear function of \( M_X^2 \), so lines of \( t + u = \text{constant} \)
in Fig. 1 mean \( M_X = \text{constant} \). The \( M_X^2 \) distribution is the projection of
the \( t - u \) plot onto the plane \( t - u = \text{constant} \). The variable \( t - u \) is
proportional to \( p_L \) or \( \cos \theta_{\text{CMS}} \). We define:
- target fragmentation region by \( t > -1.0 \text{ (GeV/c)}^2 \)
- beam fragmentation region by \( u > -1.0 \text{ (GeV/c)}^2 \)

There is no overlap between the regions. The selection is also illustrated
in the \( p_L - p_T \) plot, Fig. 2.

The sample consists of 30,000 events with seen \( \Lambda^0 \) in 4.2 GeV/c \( K^- p \)
interactions. The number of events in different channels are given in
Table 1.

We want to study the processes with only pions produced in addition
to the \( \Lambda^0 \). Other reactions which contribute to our sample are:
\[ K^- p \rightarrow \pi^0 + \text{anything}. \]  \hspace{1cm} (8)
and
\[ K^- p \rightarrow \Lambda + \bar{K} + \text{anything}. \]  \hspace{1cm} (9)

Only the fitted events of reaction (8) have been identified. We estimate that
25% of our no fit events are due to reaction (8) and correct for this in an
average sense by taking 75% of the cross section for the no fit channels. As
for reaction (9), which contributes \( \sim 10\% \) to the total sample, again only the
fitted events have been well identified. Our estimate is, that the target
fragmentation region contains \( \sim 5\% \) of reaction (9) and in the beam fragmenta-
tion region reaction (9) will be absent. In the missing mass distributions
reaction (9) will contribute to the mass region \( > 1.0 \text{ GeV/c}^2 \).
3. MISSING MASS DISTRIBUTION

The distributions of missing mass are shown in Fig. 3 for the target fragmentation region \((t > -1)\) and in Fig. 4 for the beam fragmentation region \((u > -1)\). A striking feature is immediately noticed: Below 1.1 GeV/c² in the \(t\)- and \(u\)-channels the missing mass distributions consist dominantly of single resonances. In the \(t\)-channel we find contributions from:

\[
\Lambda^0, \eta(550), \Lambda^0\rho(770), \Lambda^0\omega(785), \Lambda^0\eta'(960), \Lambda^0\phi(1020), \Lambda^0f(1270)
\]

and

\[
\Lambda^0A_2(1310)
\]

The total resonance production for \(M_x < 1.1\) GeV/c² is \(60\pm10\%\). In the \(u\)-channel we find contributions from:

\[
\Lambda^0, \eta(550), \Lambda^0\rho(770)
\]

and

\[
\Lambda^0\omega(785)
\]

and \(30\pm5\%\) resonance and \(\pi^0\) production for \(M_x < 1.1\) GeV/c².

The problem of reflection of hyperon resonances has been investigated. The hyperon resonances give in general rise to a smooth background or can be neglected in particular in the low mass region.

4. ANALYSIS OF THE INVARIANT CROSS-SECTION

The mass distributions vary as functions of \(t\) or \(u\). Low masses populate the region \(t \rightarrow 0\). As \(t \rightarrow -1\) the low masses are depleted and the distribution peaks towards the kinematical limit. A smooth power law \((s/s_0)^a(t)\) describes the data at high masses.

We assume that the inclusive production of \(\Lambda^0\) takes place by means of Regge trajectory exchange in the \(t\)- and \(u\)-channels. We assume furthermore that the processes can be parametrized by a triple Regge amplitude \((4)\).
The corresponding graphs are shown in Fig. 5a) and b). The cross section then is

\[ \frac{d^2 \sigma}{dt \, ds_x/s} = f(s, s_x/s, t) = s^{xM(0) - 1} (s_x/s)^{xM(0) - 2} (t) \]  

(10)

The conditions for the validity of the formula are

\[ s \to \infty, \quad s_x \gg 1, \quad s/s_x \gg 1 \quad \text{and fixed} \ t \ (u). \]

\[ a_{M}(0) \] stands for the intercept of the trajectory in the forward scattering amplitude for \('K^+ K^- \to K^+ K^-' or \('p''p \to 'p''p'; \ \alpha(t) \) is the trajectory in the t-channel or u-channel. Compared to the typical scale \( xO \sim 1 \). \((\text{GeV})^2, s = 9.1 \) is \( > > 1 \) and we choose in general \( s/s_x > 4 \) which is also \( > > 1 \). The condition least fulfilled is \( s_x \gg 1 \) as \( s_x \approx 2.3 \). However it has been shown \( (5) \) that the Regge pole amplitude also describes the low \( s_x \) resonance region in an average sense, and therefore it may not be too unreasonable to try to apply the cross-section parametrization (10) even at 4.2 GeV/c.

The variable \( s_x \) in formula (10) is an approximation of the crossing symmetric variables

\[ v_t = s - t - m_k^2 \]

\[ v_u = s - u - m_p^2 \]  

(11)

The variable \( v \) is negative in some cases. We replace \( v_t/s \) by \( v_t/s + 0.03 \) and \( v_u/s \) by \( v_u/s + 0.10 \) which corrects this. The distributions in \( s_x/s \) and \( v/s \) have been fitted by a power law

\[ (s_x/s)^{a(t)} \quad \text{or} \quad (v/s)^{a(t)} \]

for 10 bins in the interval \(-0.9 < t < 0\), where \( a(t) \) stands for:

\[ a(t) = a_{M}(0) - 2 \ a(t) \]  

(12)

The narrow and abundant resonances produced (section 3) make it difficult to fit the distributions directly. Instead we fit the first moment integrals as has been suggested by Chan Hong-Mo, H.I. Miettinen and R.G. Roberts (6)

\[ I(X,t) = \int_X^X f(s) \left( \frac{v}{s}, t \right) d\left( \frac{v}{s} \right) \]

which gives

\[ I(X,t) = G(t) \frac{x a(t) + 2}{a(t) + 2} \]  

(13)

In order to interpret the results for \( a(t) \) in (12) we need information from an \( s \)-dependent analysis in order to estimate \( a_{M}(0) \). We use the results
in the experiment of A. Borg et al.\(^7\). The cross sections at 3.93 and 14.3 GeV/c are given in the reference. The average ratio of the cross sections for \(K^+ p \rightarrow \Lambda^0 + p\) ions and \(-0.9 < x < -0.5\) and \(0.5 < x < 0.9\) is \(3.04 \pm 0.31\) and \(2.81 \pm 0.20\) respectively, where \(x\) is the variable \(p_L/p_{MAX}\). The ranges selected correspond roughly to target and beam fragmentation. This implies for the intercepts that

\[
\alpha_M(0) = 0.06 \pm 0.08 \text{ in the t-channel}
\]

\[
\alpha_E(0) = 0.13 \pm 0.06 \text{ in the u-channel}
\]

The value for \(\alpha_M(0)\) in the t-channel is in agreement with the results of T. Inami and H. Miettinen\(^2\) \(\alpha_M(0) \approx 0\). The trajectory for antiproton-proton annihilation is expected to be exotic\(^3\). The result indicates that it is a low lying trajectory.

We can now obtain the values for the trajectories \(\alpha(t)\) and \(\alpha(u)\). The results are shown in Figs 6a, and 6b, for \(\alpha(t)\) and \(\alpha(u)\) respectively. Straight trajectories with slope 1 are drawn through the points \(\alpha(m^2_{K(494)}=0),\)

\(\alpha(m^2_{K(890)}=1)\) and \(\alpha(m^2_{D}=0.5)\) to guide the eye.

A fit to the points gives

\[
\alpha(t) = (-0.05 \pm 0.04) + (1.02 \pm 0.02)t
\]

and

\[
\alpha(u) = (-0.36 \pm 0.06) + (0.98 \pm 0.07)u
\]

The measured points in the t-channel are between the K and K(890) trajectories. This is in agreement with earlier published data\(^8\) from this experiment, which gave evidence for both K and K(890) exchange in this region. The trajectory in the u-channel is in excellent agreement with an anti-nucleon trajectory.

Our analysis may be compared to a similar experiment at 14.3 GeV/c\(^9\), where the reaction (7) has been studied. In view of the limited statistics at 14.3 GeV, the larger contribution from \(K\bar{K}\) states and the different choices of parameters the results are in fair agreement.

5. MULTICILITIES OF THE RESIDUAL SYSTEM

The number of \(\pi^0\) produced in the states with missing mass will be estimated. It can be done in two ways, either we use

\[
\langle n_{\pi^0} \rangle = \frac{<E_{MM}>}{<E_{\pi^0}>}
\]

where \(<E_{MM}>\) and \(<E_{\pi^0}>\) are the average total energies computed in the same
final state or take \( \langle E_\pi \rangle \) from events with known number of pions and vary \( \langle n_{\pi^0} \rangle \) until the relation is satisfied. The total energies are computed in the CMS.

Both ways agree very well. The results are:

\[
\begin{align*}
\Lambda^0_{\pi^+} & \quad \langle n_{\pi^0} \rangle = 2.9 \pm 0.1 \\
\Lambda^0_{\pi^+ \pi^-} & \quad \langle n_{\pi^0} \rangle = 2.2 \pm 0.1 \\
\Lambda^0_{2\pi^+ 2\pi^-} & \quad \langle n_{\pi^0} \rangle = 2.0 \pm 0.1
\end{align*}
\]

The multiplicities might be slightly underestimated as the events with missing mass contain \( \sim 25\% \Sigma^0_{\pi^+ \pi^-} \ldots \). The total energy of the photon is not, however, very different from that of the pion.

The average charged and total pion multiplicities are plotted as functions of \( s \) in Figs. 7 and 8. Fits give

\[
\begin{align*}
\langle n_{\pi^+} \rangle &= (1.29 \pm 0.04) + (0.42 \pm 0.02)s_x \\
\langle n_\pi \rangle &= (2.44 \pm 0.03) + (0.53 \pm 0.02)s_x
\end{align*}
\]

in the t-channel and

\[
\begin{align*}
\langle n_{\pi^+} \rangle &= (1.34 \pm 0.02) + (0.47 \pm 0.01)s_x \\
\langle n_\pi \rangle &= (2.35 \pm 0.04) + (0.62 \pm 0.02)s_x
\end{align*}
\]

in the u-channel.

Interpolating \( \langle n \rangle \) in the u channel one obtains

\[
\begin{align*}
\langle n_{\pi^+} \rangle &= 3.00 \pm 0.02 \\
\langle n_\pi \rangle &= 4.54 \pm 0.05
\end{align*}
\]

at \( s = (2m)^2 \), in excellent agreement with the results from \( \bar{p}p \) annihilation at rest (10):

\[
\begin{align*}
\langle n_{\pi^+} \rangle &= 3.07 \pm 0.06 \\
\langle n_\pi \rangle &= 4.9 \pm 0.10
\end{align*}
\]

Several properties of the off-mass-shell multiplicity distributions emerge. The standard deviation \( D \) is plotted as function of \( \langle n_{\pi^+} \rangle \) in Fig.9. The Mueller moment \( f_2 = D^2 - \langle n \rangle \) is large and negative as can be inferred from the plot.

6. CONCLUSIONS AND DISCUSSION.

We have studied target and beam fragmentation processes in \( K^- p \rightarrow \Lambda + \) pions reactions at 4.2 GeV/c. The total sample consisted of 70,000 events
with measured $\Lambda^0$. 20664 events had $t > -1$ and 9068 events had $u > -1$.

The main results are:

i) The target fragmentation is dominated by resonance and particle formation for $M_X < 1.1$ GeV/c$^2$. States with $\Lambda^0$ and $\pi^0$, $\eta(550)$, $\rho(770)$, $\omega(785)$, $\eta'(960)$, $f(1020)$, $f(1270)$ and $\Lambda_2(1310)$ make up 60\pm10\% of the cross section.

ii) In the beam fragmentation region $\Lambda^0$ and $\pi^0$, $\eta(550)$, $\rho(770)$ and $\omega(785)$ contribute 30\pm5\% to the cross section for $M_X < 1.1$ GeV/c$^2$.

iii) In a Regge particle exchange model a $K^+$ ($K^0(890)$) trajectory can be exchanged in the $t$-channel and an antinucleon trajectory in the $u$-channel. The result (i) is a confirmation of the prediction that the annihilation $'K^+$, $K^-$ pions should proceed via intermediate resonances only.

iv) The $\nu_t/s$ and $\nu_u/s$ distributions have been fitted in order to obtain the effective trajectories in a triple Regge pole model. The resulting trajectory in the $t$-channel has slope and intercept between the expected $K^+$ and $K^0(890)$ trajectories. The trajectory in the $u$-channel agrees with the nucleon trajectory.

v) The average charged and total pion multiplicities as a function of missing mass squared can be extrapolated to $s_X = (2m_p)^2$ and agree perfectly with the results from $\bar{p}p$ annihilations at rest. It lends further support to the interpretation of the exchange particle as an antinucleon trajectory.

vi) The average multiplicities for $'K^+$, $K^-$ pions show the same $s_X$ dependence as the $'\bar{p}'p$ annihilations.

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### TABLE 1: Sample of events in the fragmentation regions a)

<table>
<thead>
<tr>
<th>Final state</th>
<th>$t &gt; -1.0$</th>
<th>$u &gt; -1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda^0$</td>
<td>1212</td>
<td>204</td>
</tr>
<tr>
<td>$\Lambda^+$</td>
<td>3143</td>
<td>979</td>
</tr>
<tr>
<td>$\Lambda^+ \pi^-$</td>
<td>2584</td>
<td>814</td>
</tr>
<tr>
<td>$\Lambda^0 \pi^+$</td>
<td>6200</td>
<td>2978</td>
</tr>
<tr>
<td>$\Lambda^+ \pi^- \pi^0$</td>
<td>4816</td>
<td>2262</td>
</tr>
<tr>
<td>$A_2\pi^+ 2\pi^-$</td>
<td>822</td>
<td>682</td>
</tr>
<tr>
<td>$A_2\pi^+ 2\pi^- \pi^0$</td>
<td>1385</td>
<td>915</td>
</tr>
<tr>
<td>$A_2\pi^+ 2\pi^- \pi^0$</td>
<td>502</td>
<td>234</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>20664</strong></td>
<td><strong>9068</strong></td>
</tr>
</tbody>
</table>

a) Corrected for decay probabilities and ambiguous hypotheses. 4-c events are required to have $P > 1\%$, 1-c events $P > 5\%$ in the selection. Correction factor for events with missing mass is 0.70 to account for $\Sigma^0, \Lambda\Xi$ etc.
FIGURE CAPTIONS

(1) a) Notations for the inclusive reaction $K^-p \rightarrow \Lambda^0X$

b) Boundaries in the t-u plane and definition of the target and beam fragmentation regions.

(2) Target and beam fragmentation boundaries in the $p_T$-$p_L$ plane.

(3) Mass distribution $M_X$ in the t-channel.

(4) Mass distribution $M_X$ in the u-channel.

(5) Tripple Regge pole graph for

a) target fragmentation

b) beam fragmentation

(6) Trajectories

a) $\alpha(t)$ in the t-channel

b) $\alpha(u)$ in the u-channel

$\alpha_X(0) = 0.06$ and $\alpha_Y(0) = 0.13$ have been obtained from ref. (7).

(7) Average number of pions as a function of $s_X$ in the t-channel.

(8) Average number of pions as a function of $s_X$ in the u-channel.

(9) Dispersion $D = \langle n^2 \rangle - \langle n \rangle^2$ versus $\langle n \rangle$. The full lines are smooth curves through the points.
4 GeV/c $K^- p \rightarrow \Lambda^0 X$

- $s = (p_a + p_b)^2$
- $t = (p_a - p_c)^2$
- $u = (p_b - p_c)^2$
- $s_x = (p_a + p_b - p_c)^2$
- $s + t + u = m_a^2 + m_b^2 + m_c^2 + s_x$

**Fig. 1**
Fig. 2

Longitudinal momentum CMS $p_L$ of $\Lambda^0$ GeV/c

Fig. 3

No of events / 0.025

$M_x$ GeV/c²
Fig. 4

a) target fragmentation

Fig. 5

b) beam fragmentation
Fig. 6

Fig. 7
- Miettinen:

One month ago we predicted [Takeo-Inami and H. Miettinen, Phys. Letters B49, 67 (1974)] that below the \( \bar{p}p \) threshold the resonances would be present. Moreover, we predicted no background under the low mass enhancements, in agreement with your experimental result. Your beautiful analysis shows a modern way of doing \( \bar{p}p \) physics.