MULTIPICLITY DISTRIBUTION OF $\bar{p}p$ INTERACTIONS
IN THE HIGH-ENERGY REGION

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(Presented by T. Kitagaki)

1. ANALYSIS OF 15 GeV/c $\bar{p}p$ INTERACTIONS

We have analysed 25K pictures of 15 GeV/c $\bar{p}p$ interactions. The pictures were taken by the NAL group in the Brookhaven National Laboratory 80-inch bubble chamber, and some of the pictures have been analysed by Dao et al. at NAL1). The present data were obtained independently from the analysis of independent samples of the pictures, and both data are in good agreement. Table 1 shows the multiplicity distribution of charged particles. The quantities quoted later are:

$$\langle n \rangle = \frac{\sum n \sigma_n}{\sum \sigma_n} = 4.15 \pm 0.07$$

$$D = \left[ \sum (n - \langle n \rangle)^2 \sigma_n / \sum \sigma_n \right]^{1/2} = 2.06 \pm 0.04$$

$$W = \langle n \rangle / D = 2.01 \pm 0.05$$

where $n$ is the number of charged particles and $\sigma_n$ is the inelastic cross-section of an $n$-prong event. The 0-prong is included in the values.

The analysis of $\gamma$ was made by measuring electron-positron pairs in liquid hydrogen. Approximately 3,800 $\gamma$ conversions associated with primary interactions were obtained out of 25K pictures.

The bias due to the loss of low-energy $\gamma$ was corrected, and the averaged efficiency of $\gamma$ conversions in our fiducial volume was 3.92%. The source of $\gamma$ other than $\pi^0$ produced in the primary interactions is negligibly small in this case, and the cross-sections of $\bar{p}p \rightarrow \pi^0 s + n$ charged particles are estimated on the assumption that the only source of $\gamma$ is $\pi^0 \rightarrow 2\gamma$. Table 2 shows the $\pi^0$ production cross-sections. The total $\pi^0$ production is $\sigma_{tot}(\pi^0) = 72 \pm 4$ mb. The estimated $\pi^0$ production is $\sigma_{non}(\pi^0) = 38$ mb for the non-annihilation and $\sigma_{ann}(\pi^0) = 34$ mb for the annihilation, as will be discussed later. The average number of $\pi^0$ in each $n$-prong event, $\langle n_\pi \rangle_n$, will be shown in Fig. 4a, and the linear approximation is

$$\langle n_\pi \rangle_n = 0.97 + 0.37(n/2).$$

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2. MULTIPLICITY DISTRIBUTION OF CHARGED PARTICLES

One of the recent topics in the high-energy region is the Koba, Nielsen and Olesen scaling on the multiplicity distribution\(^2\)). It has been pointed out by Slattery\(^3\)) that the distribution \(\langle n \rangle_{\text{inel}} / \sigma_{\text{n}}\) has an asymptotic form \(F(n/\langle n \rangle)\) as a consequence of the KNO scaling, and the experimental data seem to show excellent fit to the asymptotic curve at more than 50 GeV/c in various hadron-proton interactions. Also it has been pointed out by Dao et al.\(^4\)) that the inverse width \(W = \langle n \rangle / D\), has an asymptotic limit \(W_0\), which seems experimentally to be \(W_0 = 2.0\).

The asymptotic behaviour of the inverse width \(W\) is a natural consequence of the KNO scaling. However, the value of \(W\) is very sensitive to the definition of quantity except in the very high energy region. For instance, if we consider a linear transformation of \(n\) to \(n' = a + b \cdot n\), we obtain \(D' = b \cdot D\) and \(W' = W + a/D'\).

Recently, Froyland and Skontorp\(^5\)) showed that the value of \(W = \langle n \rangle / D\) is almost on a constant line, \(W_0 = 1.92\), in a wider interval of 10-300 GeV/c, by deriving the quantity from the number of produced pions instead of charged pions.

However, in the following we will use the \(W\) calculated by the total number of charge, since the experimental data are available only on the charged particles. The data on pp show that the value of \(W\) decreases smoothly with the increase of the incident momentum \(p_{lab}\), and it almost reaches the asymptotic value \(W_0\) at 100 GeV/c as shown in Fig. 1\(^6\)). The data on \(\pi^- p\) are close to those on pp.

As already pointed out\(^4\)), the behaviour of the \(W\)-value in \(\bar{p}p\) interactions is quite different from that in other hadron interactions: a) The value of \(W\) seems to be already close to the asymptotic value in the 10 GeV/c region, whereas the values of other hadron interactions differ by more than 30% from the asymptotic value. Our value of 15 GeV/c \(\bar{p}p\), \(W = 2.01 \pm 0.05\) is very close to the values of pp interactions above 100 GeV/c. b) The sign of the slope of \(W\) in \(\bar{p}p\) interactions is opposite to that of other hadron interactions. It approaches from the lower side to the asymptotic value (see Fig. 1).

We found that such a feature of \(\bar{p}p\) interactions is attributed to the two components of the non-annihilation and annihilation processes. The inelastic \(\bar{p}p\) interaction is the sum of the non-annihilation process and the annihilation process; that is:

\[
\sigma_{\text{n}}^{\text{inel}}(\bar{p}p) = \sigma_{\text{n}}^{\text{non}}(\bar{p}p) + \sigma_{\text{n}}^{\text{ann}}(\bar{p}p)
\]

\(42 \text{ mb} = 30 \text{ mb} + 12 \text{ mb}\) at 15 GeV/c.
42 mb is the present total inelastic cross-section and 30 mb is estimated from
the world-wide data on \( \bar{p}p \) cross-sections. In the momentum region of 1.5-10 GeV/c,
the data show \( \sigma^{\text{ann}} \approx \sigma(\bar{p}p) - \sigma(pp) \). Therefore it may be a reasonable approximation
to substitute the \( pp \) interaction for the non-annihilation part of \( \bar{p}p \) interaction
at the same energy.

In Fig. 2a the charged multiplicity distribution in 15 GeV/c \( \bar{p}p \) interaction
is shown as crosses (x). Non-annihilation and annihilation data are given in
the figure as the dots and the open circles, respectively. The dash-dots repre-
sent pp data interpolated at 15 GeV/c. The data for the non-annihilation were
obtained from events with identified slow protons. The data were normalized to
30 mb, with the normalization factor 2.15, assuming that the whole non-annihil-
ation part has the same distribution as those events with the identified proton.
The agreement of the non-annihilation data with pp data is good within errors.

The multiplicity distribution for the annihilation was obtained as the difference
between the observed total multiplicity and the non-annihilation multiplicity.
The dashed curve was calculated for the annihilation by the Goldberg formula\(^7\),
where the values of the parameters given in Ref. 7 are used, and the total
annihilation cross-section is normalized to 12 mb. The Goldberg formula repre-
sents qualitatively the feature of the experimental data on the annihilation
multiplicity. We have found in Fig. 2a that both non-annihilation and annihil-
ation terms in \( \bar{p}p \) interaction at 15 GeV/c have \( W^{\text{non}} = 2.06 \) and \( W^{\text{ann}} = 3.07 \),
respectively, and peaks of the distributions in both processes were apart from
each other, while the sum of the two components has a smaller value,
\( W = 2.01 \pm 0.05 \). The similarity between the multiplicity distribution in
15 GeV/c \( \bar{p}p \) collisions and that in pp collisions above 100 GeV/c is quite appar-
ent. It should be emphasized that the important thing is the separation of the
two component peaks and not the details of the shape of each curve.

Figures 3a, b, and c show the change of the two components from 15 GeV/c
to 3.7 GeV/c. Here, the annihilation component is given by two models: solid
line by the multi-Regge model of Goldberg and dashed line by the statistical
model of Orfanidis and Rittenberg\(^8\)). Figure 3 shows that the separation of the
two component peaks becomes larger as the momentum is smaller. This is because
of the different shifts of Q-values in both component channels. The change of
the separation results in the unique feature of the slope of \( W \) in the \( \bar{p}p \) inter-
action. The estimated trajectories of \( W \) by the combination of two components,
non-annihilation (pp data) and annihilation (solid line by Goldberg and dashed
one by Orfanidis and Rittenberg) are shown in Fig. 1. Both lines predict the
positive slope of \( W \) qualitatively. But the quantitative fit is much better in
the statistical model at p_{lab} < 7 GeV/c, where the Orfanidis formula is in good
agreement with the annihilation multiplicity data. The feature that is mainly
due to the separation of two components predicts that the W-value rises slowly with energy and crosses the limiting value $W_0$ ($\approx 2$) around 15 GeV/c; it will then slightly exceed $W_0$ and approach the pp curve. The recent data of the 32 GeV/c $\bar{p}p$ interaction$^9$ seem to fit it well.

3. AVERAGE NUMBER OF $\pi^0$

Figure 2b shows $\pi^0$ production in n charged prongs. The dots and open circles represent the contributions from non-annihilation and annihilation, respectively. An estimate of the non-annihilation cross-section for $\pi^0$ was made with the same procedure and the assumption previously mentioned for the multiplicity distribution of charged particles. Besides, an assumption was made that the fraction of events with the proton identified in the total non-annihilation $\pi^0$ cross-section is the same as in the total non-annihilation cross-section. That is, the normalization factor of 2.15 was used to get the total non-annihilation cross-section for $\pi^0$. The total non-annihilation cross-section for $\pi^0$ thus obtained $\sigma_n^{\pi^0}$ is 37 mb excluding 0-prong; this agrees with the cross-section interpolated from the data at 12 GeV/c$^{10}$ and 19 GeV/c$^{11}$ pp interactions. The remaining part of $\pi^0$ production is the annihilation, $\sigma_{ann}^{\pi^0} \approx 34$ mb. The dotted curve in the figure shows the distribution for annihilation calculated by the Goldberg formula. Similarly to the case of the charged events, the Goldberg formula is consistent with the data except for the small difference between the positions of the peaks of the data curve and those of the theoretical one.

Figure 4a shows the present data of the average number of $\pi^0$ for n-prong events in the 15 GeV/c $\bar{p}p$ interactions. The linear approximation of the distribution is given by $\langle n_0 \rangle_n = 0.97 + 0.37$ (n/2). The strong positive correlation $\langle n_0 \rangle_n = a + n/2$ is expected at the high-energy limit on the basis of isospin independence, but the distributions are flat in pp data at 12 GeV/c$^{10}$ and 19 GeV/c$^{11}$. The above gradient of 0.37 in the 15 GeV/c $\bar{p}p$ data is close to that of the 69 GeV/c pp data$^{12})$. That is, 15 GeV/c $\bar{p}p$ interactions seem to show the high-energy behaviour again in the distribution of $\langle n_0 \rangle_n$. In Fig. 4b the average number of $\pi^0$ in n-prong events, $\langle n_0 \rangle_n'$, are shown for the non-annihilation part. The data are estimated from the events with slow protons identified. The open circles and crosses are the data of pp interactions at 19 GeV/c$^{11}$ and 12 GeV/c$^{10}$, respectively. This figure shows that the pp non-annihilation data are in good agreement with the pp data. Figure 4c shows a contribution from annihilation to $\langle n_0 \rangle_n'$, which is obtained as a difference between the total and non-annihilation data. The curve in Fig. 4c was calculated for pp annihilation on the basis of the Goldberg formula. The prediction is consistent with the experimental data. The Goldberg formula gives a weak negative correlation for
the annihilation part, since it is based on the multi-Regge model without any consideration of the positive correlation between \( \pi^0 \) and \( \pi^\pm \) productions. It is important to notice that although the production of \( \pi^0 \) does not have any correlation with the production of charged pions for each of annihilation and non-annihilation processes, the over-all production of \( \pi^0 \) has strong positive correlation with the production of charged pions, as seen in Fig. 4a. The phenomenon is understood as follows:

\[
\langle n_\pi \rangle_n = \frac{\langle n_\pi \rangle_{\text{non}} + \langle n_\pi \rangle_{\text{ann}}}{\sigma_{\text{non}} + \sigma_{\text{ann}}},
\]

where \( \langle n_\pi \rangle_n \), \( \langle n_\pi \rangle_{\text{non}} \), and \( \langle n_\pi \rangle_{\text{ann}} \) represent the average number of \( \pi^0 \) in an \( n \)-prong event for over-all, non-annihilation, and annihilation processes, respectively. The topological cross-sections for non-annihilation and annihilation are represented by \( \sigma_{\text{non}} \) and \( \sigma_{\text{ann}} \), respectively. Firstly, \( \langle n_\pi \rangle_{\text{ann}} \) is about twice as large as \( \langle n_\pi \rangle_{\text{non}} \). Secondly, \( \sigma_{\text{non}} \) has a large value for the relatively small \( n \), on the other hand \( \sigma_{\text{ann}} \) has a large value for the relatively large \( n \). These two points induce the positive correlation for \( \langle n_\pi \rangle_n \).

4. CONCLUSION

It is found that the shape parameter \( W = \langle n \rangle / D \) of charged multiplicity is equal to about 2, and that \( \pi^0 \) production has positive correlation with \( \pi^\pm \) production in the 15 GeV/c \( \bar{p}p \) interactions. These two features are seen in the high-energy \( pp \) interactions above 100 GeV/c. The \( s \)-dependence of \( W \) for \( \bar{p}p \) interactions from a few GeV/c to a few tens of GeV/c has a positive slope in contrast to the negative slope of \( pp \) interaction in the same energy region. It was shown, however, that neither the annihilation process nor the non-annihilation process has high-energy behaviour.

It was noticed that the three features mentioned above in \( \bar{p}p \) interaction can be explained using the fact that \( \bar{p}p \) interaction consists of two components of annihilation and non-annihilation together with the characteristics of the multiplicity of each component.
REFERENCES

1) F.T. Dao, J. Lach and J. Whitmore, Int. Conf. on Elementary Particles, Aix-en-Provence, 1973; NAL-Pub-73/80-EXP (1974); NAL-Pub-73/81-EXP (1974). We were given the opportunity to analyse independent samples of 25K pictures, by courtesy of Dr. J. Lach of NAL.


Table 1
Topological cross-sections

<table>
<thead>
<tr>
<th>No. of prongs</th>
<th>No. of events observed</th>
<th>No. of events corrected</th>
<th>Fraction</th>
<th>Cross-section (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>148</td>
<td>166</td>
<td>0.022</td>
<td>1.13 ± 0.28</td>
</tr>
<tr>
<td>2</td>
<td>2603</td>
<td>3233</td>
<td>0.424</td>
<td>(el.) 9.93 ± 0.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(inel.) 12.05 ± 1.03</td>
</tr>
<tr>
<td>4</td>
<td>2239</td>
<td>2311</td>
<td>0.303</td>
<td>15.72 ± 0.61</td>
</tr>
<tr>
<td>6</td>
<td>1339</td>
<td>1360</td>
<td>0.179</td>
<td>9.25 ± 0.37</td>
</tr>
<tr>
<td>8</td>
<td>435</td>
<td>452</td>
<td>0.059</td>
<td>3.07 ± 0.21</td>
</tr>
<tr>
<td>10</td>
<td>89</td>
<td>83</td>
<td>0.011</td>
<td>0.57 ± 0.08</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>10</td>
<td>0.0013</td>
<td>0.07 ± 0.03</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>3</td>
<td>0.0004</td>
<td>0.02 ± 0.01</td>
</tr>
<tr>
<td>Total</td>
<td>6868</td>
<td>7618</td>
<td>1.000</td>
<td>(tot.) 51.8 ± 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(inel.) 41.9 ± 1.3</td>
</tr>
</tbody>
</table>

Table 2
Cross-sections for \( \bar{p}p \rightarrow \pi^0 + \text{anything} \) (\( \chi^2 \leq 200 \))

<table>
<thead>
<tr>
<th>No. of prongs</th>
<th>No. of ( \gamma ) observed</th>
<th>No. of ( \gamma ) weighted</th>
<th>Fraction</th>
<th>( \sigma_n(\pi^0) ) (mb)</th>
<th>( \sigma_n(\pi^0)/\sigma_n^{\text{inel}} ) (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>65</td>
<td>1610</td>
<td>0.020</td>
<td>1.44 ± 0.17</td>
<td>1.27 ± 0.36</td>
</tr>
<tr>
<td>2</td>
<td>710</td>
<td>17703</td>
<td>0.218</td>
<td>15.72 ± 0.55</td>
<td>1.30 ± 0.14</td>
</tr>
<tr>
<td>4</td>
<td>1198</td>
<td>29927</td>
<td>0.369</td>
<td>26.58 ± 0.73</td>
<td>1.69 ± 0.13</td>
</tr>
<tr>
<td>6</td>
<td>833</td>
<td>20766</td>
<td>0.256</td>
<td>18.44 ± 0.62</td>
<td>1.99 ± 0.16</td>
</tr>
<tr>
<td>8</td>
<td>344</td>
<td>8573</td>
<td>0.106</td>
<td>7.61 ± 0.40</td>
<td>2.48 ± 0.27</td>
</tr>
<tr>
<td>10</td>
<td>92</td>
<td>2284</td>
<td>0.0282</td>
<td>2.03 ± 0.20</td>
<td>3.56 ± 0.64</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>199</td>
<td>0.0025</td>
<td>0.18 ± 0.06</td>
<td>2.57 ± 1.41</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3250</td>
<td>81062</td>
<td>1.000</td>
<td>72 ± 4</td>
<td>1.72 ± 0.12</td>
</tr>
</tbody>
</table>
$W = \langle n \rangle / D$ defined by charged particle number $n$

- $pp$
- $\bar{p}p$, @ Present 15 GeV/c

- multi-regge model
- statistical model

$P_{lab}$ (GeV/c)

Fig. 1
Fig. 2
Fig. 3

a) $P_{lab} = 15\, \text{GeV/c}$

b) $P_{lab} = 6.9\, \text{GeV/c}$

c) $P_{lab} = 3.7\, \text{GeV/c}$

(qw) $^u_0$

--- multi-range model

--- statistical model

--- experimental data
Fig. 4
DISCUSSION

- Malhotra:

You have explained the value of \( \langle n \rangle/D = 2 \) as a consequence of two components in pp collisions. At high energies, pp and \( \bar{p}p \) are the same and, therefore, at 15 GeV one can expect three components to contribute to \( \bar{p}p \) interactions.

- Dao:

It seems to me that pp data at high energies look similar to the low-energy \( \bar{p}p \) data.

- Šimáčk:

I would like to point out that the small value of \( \langle n \rangle/D \) in \( \bar{p}p \) interactions compared to pp interactions may well be due to zero-charge in the initial state in \( \bar{p}p \) collisions.

- Vandermeulen:

I find that the main interest of Orfanidis and Rittenberg's model lies in the coefficients for the different charge configurations for each multiplicity. The model does not refer to isospin, but emphasizes the role of charge. By excluding the fireballs with \( |Q| > 1 \) in the linear chain decay, the model imposes alternate production of \( \pi^+ \) and \( \pi^- \) whereas no constraint exists for \( \pi^0 \)'s. Therefore, the production of neutrons is favoured. The result of the counting is \( n(\pi^0)/n(\pi^-) = \sqrt{2} \), whilst \( n(\pi^+)/n(\pi^-) = 1 \) is obtained in several models based on statistical isospin conservation (e.g. the Cerulus coefficients). The experimental situation seems to be closer to \( \sqrt{2} \) than to 1.

On the other hand, the dynamical content of the model is rather limited. It does not claim to account for the energy variation of the magnitude of the cross-sections. The multiplicity distribution is assumed to be Gaussian with values of the mean and standard deviation chosen to fit the data.

The assumption of a single cluster cannot account for the forward-backward asymmetry in the emission of charge.

- Fields:

About the ordering according to rapidity (Alexander et al.), I see a difficulty with resonance production mixing the order, especially for high multiplicity events.