MULTIPLICITY CORRELATIONS IN \( \bar{p}p \) - COLLISIONS AT 540 GeV

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SUMMARY
Multiplicity correlations of long range in pseudorapidity are found to be stronger than at ISR energies. The analysis gives no evidence for intrinsic long range correlations. The data are consistent with a physical picture involving random emission of small clusters along the rapidity plateau. The average cluster size is about 2 charged particles, the same as at ISR energies.

INTRODUCTION
The experiment has been carried out by the UA5 collaboration using its streamer chamber system as described in the literature and by the previous speaker, D. Ward [1]. At the CERN SPS Collider one has available a rapidity plateau which is long compared with the correlation range of about \( \pm 1 \) unit of rapidity typical for the decay of small mass resonances. We observe two-particle short range correlations of the mentioned type but concentrate here on our observation and analysis of the longer range correlations between the number of charged particles, \( n_F \) and \( n_B \), falling into two selected rather large regions, one forward (F) and one backward (B), chosen symmetric around 90° in the center-of-mass system (\( \eta = 0 \)). The pseudorapidity, \( \eta = -\ln \tan \theta/2 \), where the angle \( \theta \) is the polar angle with the respect to the beam axis, is the variable used to label the charged particles.
Our sample consists of about 4000 minimum bias events. The trigger requires at least one particle into each hemisphere, which leads to an almost complete elimination of single diffraction events. The triggering efficiency for non single diffractive inelastic events is about 95%. The streamer chambers cover the rapidity range \(-5 \leq \eta \leq 5\). The data presented refer to the range \(|\Delta \eta| \leq 4\) where the geometrical acceptance is greater than 85%.

Long range correlations were first reported by S. Uhlig et al \([2]\) at ISR. Our results in preliminary versions have been given last year and are now submitted for publication \([3]\).

OBSERVATIONS
The problem concerns the fluctuations of the number of particles in the F- and B-regions. If these variables were independent of each other the correlation strength would be zero. At ISR energies a positive correlation was found \([2]\) which increases with energy within the ISR energy range. The following two ways to measure the correlation strength are equivalent: (1) one computes the average of \(n_B\) at fixed \(n_F\) and finds the slope \(b\) in a straight line fit, \(<n_B(n_F)> = a - b \cdot n_F\), (2) one computes the correlation coefficient of \(n_F\) and \(n_B\), involving mainly the mean value of the product of the two factors \(n_F(i) - <n_F>\) and \(n_B(i) - <n_B>\) where \(n_F(i), n_B(i)\) are the observed number of charged tracks in event \((i)\) falling into the selected F and B regions. A straight line fit with unit weight to all events gives \(b = \text{cov}(n_F, n_B)/\text{var}n_F\) which illustrates above mentioned equivalence. We present here the results for three definitions of the regions F and B.

(A) \(F = (0 \leq \eta \leq 4)\) and \(B = (-4 \leq \eta \leq 0)\) which means that the two regions are in contact but non-overlapping.

(B) \(F = (1 \leq \eta \leq 4)\) and \(B = (-4 \leq \eta \leq -1)\) which means that a gap of size \(\Delta \eta = 2\) has been introduced between the two non-overlapping regions.

(C) \(F = (0 \leq \eta \leq 1)\) and \(B = (-1 \leq \eta \leq 0)\) which corresponds to the size of the observed range of the two-particle correlation function.
In all three cases the F and B regions are symmetric. We consider also the initial state to be symmetric since we do not identify particles, only count the number of charged particles.

The scatterplot Fig. 1a, shows the two-dimensional distribution, $F(n_F, n_B)$, of events in case A above (no gap), while Fig. 1b shows how remarkably well the data fit to a straight line. The least squares fit gives a slope of $b = 0.54 \pm 0.01$. The value drops to $b = 0.41 \pm 0.01$ when a gap of size $\Delta \eta = 2$ is introduced (case B above). We interpret this drop as mainly due to a decoupling of the F and the B regions from effects due to clusters (such as $\rho^0 \rightarrow \pi^+ \pi^-$) produced near $\eta = 0$ and emitting one (or more) particles into each region simultaneously.

Fig. 2 shows the UA5 values for the correlation strength parameter $b$ together with published results from the R701 experiment at ISR $^{22}$. The increase with energy is seen to continue. To be noted is that $b \neq 0$ even in the case when a gap is inserted, fig. 2c, and increases with energy. This observation constitutes the main reason for introducing the term long-range correlation in rapidity space. The almost zero value at the lowest ISR energy seems to be accidental. Our analysis offers a rather simple explanation for the existence of the effect and the increase of its strength with energy.

ANALYSIS

The presence of correlations means that the two-dimensional distribution of events (Fig. 1a) does not factorize, $F(n_F, n_B) \neq f(n_F) \cdot f(n_B)$. In other words $n_F$ and $n_B$ are not independent. We find it advantageous to transform the problem into the variables $n_S = (n_F + n_B)$ and $z = (n_F - n_B)$, i.e. rotating the coordinate system of Fig. 1a by $45^\circ$. The marginal distribution of the combined multiplicity $n_S$, obtained by projecting all events in the scatterplot onto the new $n_S$-axis, does not seem by itself to contain information relevant to our problem. However, we will find the first two moments to be of decisive interest. These are the mean $\langle n_S \rangle = 16.0 \pm 0.2$ and the variance $D_S^2 = 78.0 \pm 1.8$ for the case with a gap between the
F- and B-regions (case B). The two-dimensional distribution describing the densities of events in the scatterplot, such as Fig. 1a, is studied by us at each fixed n_S and is formally denoted f_S(n_F) [at fixed n_S the use of the variables n_F or z = (2n_F - n_S) is equivalent]. This set of functions describes the distribution of events with any n_F at fixed n_S. The distributions f_S(n_F) must trivially be symmetric with a mean \( \langle n_F \rangle = \frac{1}{2} n_S \). All odd moments must vanish. It turns out that the correlation strength parameter b is related to the second moments, \( d_S^2(n_F) \), by the following identity

\[
b = \frac{\frac{1}{4} D_S^2 - \langle d_S^2(n_F) \rangle}{\frac{1}{4} D_S^2 + \langle d_S^2(n_F) \rangle}
\]

where \( \langle \cdot \rangle \) denotes an average value over the marginal n_S-distribution.

The proof of this relation rests on the following steps: (1) the least squares fit gives

\[
b = \frac{\text{cov}(n_F, n_B)}{\text{var} n_F} = \frac{\sum_i (n_F(i) - \langle n_F \rangle)(n_B(i) - \langle n_B \rangle)}{\sum_i (n_F(i) - \langle n_F \rangle)^2}
\]

(2) the sums over all events (i = 1 --- M, in our case M ~ 4000) are carried out in two steps: (i) over all events with a given fixed value of n_S, (ii) over the marginal n_S-distribution.

The variances, \( d_S^2(n_F) \), depend on the shape of the f_S(n_F) distributions which in turn are sensitive to the kind of physical process (or processes) which dominate. The discussion is summarized in Table 1. We cannot exclude that a suitable mixture of processes with intrinsic long range correlations can describe the data but we do not find evidence for a dominance of any single such process. The data can be described as due to a random emission of clusters of an average size (k) close to 2. The random process is described by a binomial distribution in the number of clusters (C) falling into F with probability \( p_F = \frac{1}{2} \). The variance is expected to be

\[
d_S^2(n_F) = \frac{1}{4} k^2 C = \frac{1}{4} k \cdot n_S,
\]

if all clusters have the same size k. The more realistic case of a mixture of sizes is treated in the Appendix. As an example we take \( n_S = 12 \) which means 6 clusters in case the sizes are all \( k = 2 \).
6 clusters (i.e. all 12 particles) in the F-region is $2^{-6} = \frac{1}{64}$ whereas if, instead of clusters, particles were randomly emitted the probability for all 12 particles to be in F is only $2^{-12} = \frac{1}{1024}$. Our observed number of events with all 12 particles in one region is 4 events out of 185 events as expected for the model with random cluster emission.

A closer study of the expectations for this physical picture has been made by a Monte Carlo simulation. The simple assumptions made are summarized in Fig. 3. In an event with C clusters these are positioned at random along the pseudorapidity axis. Their sizes are $k_i (i = 1 \ldots C)$ so that the number of charged particles is given by

$$n_{ch} = \sum_{i=1}^{C} k_i.$$ 

The experimental distribution of $n_{ch}$ was used to generate a distribution in C once the k-distribution was selected. Average values of k near k=2 are of interest and the k-distribution was chosen to follow a Poisson distribution in the range k=1, \ldots, 5. The $k_i$ cluster products were assigned positions on the pseudorapidity axis in the neighbourhood of the cluster itself using the two-particle correlation function. In this way some leakage of cluster products will occur. A cluster in the F (or B) region will sometimes appear smaller than generated. Also clusters outside the considered region will at times leak particles into the region. This compensation will generally not occur in the same event so that fluctuations (variances) will be influenced whereas mean values will remain the same (if the net leakage is zero). The observed variances are given in Fig. 4 together with curves obtained by the M.C. simulations. The agreement is excellent provided the average cluster size is about 2 charged particles. When the sizes of the F- and B-regions are reduced from $\Delta \eta = 3$ to $\Delta \eta = 2$ and $\Delta \eta = 1$, respectively, the increased leakages lead to reductions of the ratio $d_s^2\langle n_F \rangle / \langle n_S \rangle$ in the data and in the M.C. in the same way. One notices also that the plotted ratio is rather independent of the total multiplicity $n_S$ (up to $n_S \sim 30$). We interpret this to support the assumption in M.C. that large and small multiplicities are produced by the same cluster sizes. This determination of the average (or effective) cluster size is also independent of the gap size once it is larger than about two units as seen in Fig. 5.
The M.C. events were used to compute the correlation strength $b$ and the results for the long range correlation, given in Fig. 2, are in good agreement with data. The M.C. simulation was repeated at ISR energies with again good agreement provided the average cluster size was set equal to about 2 charged particles.

**Energy dependence of the correlation strength ($b$)**

The physical picture as presented reproduces the energy dependence well. The reason for this is seen from the identity formula. Assuming random cluster emission the formula reads

$$b = \frac{D^2 / \langle n \rangle_s - k_{\text{eff}}}{D^2 / \langle n \rangle_s + k_{\text{eff}}}$$

where the effective cluster size $k_{\text{eff}}$ is related to the average cluster size by

$$k_{\text{eff}} = \langle k \rangle + d_k^2 / \langle k \rangle$$

(see Appendix) in the limit of no leakages. It is a fairly good approximation to consider $k_{\text{eff}}$ to be an energy independent constant. The energy dependence of $b$ is then totally due to the $D^2 / \langle n \rangle$ ratio which increases approximately linearly with $\langle n \rangle$. Since the particle density in the plateau region increases approximately linearly with $\ln s \ (\sqrt{s} = \text{c.m. energy})$ [1] one finds that it is possible to represent the correlation strength parameter by

$$b = \frac{\ln s - B_1}{\ln s + B_2}$$

where the constants $B_1$ and $B_2$, however, depend on the size in rapidity of the $F$- and $B$- regions, and on the size of the rapidity gap between them.

**CONCLUSIONS**

We observe both short range ($\Delta \eta \sim 1$) and longer range correlations in charged particle multiplicities. The strength of the long range (forward-backward) correlation increases with energy and $b \approx 0.5$ is observed at the SPS collider energy. The data are consistent with a physical picture involving random emission of clusters along the rapidity axis in the plateau region. The average cluster size is
about 2 charged particles, the same as at ISR energies. It seems natural to assume that the clusters are small mass particles and resonances ($\pi, \eta, \rho, \omega, K, K^* \ldots$) or small groups of them. These are then copiously produced and constitute the dominant part of minimum bias events. This hypothesis is supported by our published observation that the average number of $\gamma$-rays increases strongly with the number of charged particles in the events [4].

Table 1. Summary of discussion of various hypothetical physical processes

\[ b_{\text{observed}} = 0.41 \ \text{(case } 1 \leq |\Delta \eta| \leq 4) \]

<table>
<thead>
<tr>
<th>Physical picture</th>
<th>Shape of $f_S(n_F)$</th>
<th>$d^2 S(n_F)$</th>
<th>$b$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRINSIC CORRELATION</td>
<td>peaked at $\frac{1}{2} n_S$</td>
<td>small</td>
<td>large</td>
<td>Not observed</td>
</tr>
<tr>
<td>(a) 2 fireballs with strongly correlated sizes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) 1 large + 1 small fireball</td>
<td>double peaked</td>
<td>large</td>
<td>small (or neg.)</td>
<td>Not observed</td>
</tr>
<tr>
<td>II. UNCORRELATED sizes of 2 fireballs</td>
<td>broad</td>
<td>0</td>
<td>Not observed</td>
<td></td>
</tr>
<tr>
<td>III. Mixture of above</td>
<td></td>
<td></td>
<td>Not excluded</td>
<td></td>
</tr>
<tr>
<td>IV. NO INTRINSIC CORRELATION</td>
<td>Binomial with $p = \frac{1}{2}$</td>
<td>$\frac{1}{4} n_S$</td>
<td>0.66</td>
<td>Too high b</td>
</tr>
<tr>
<td>(a) Random emission of particles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Random emission of clusters of fixed size $k$</td>
<td>Binomial in clusters</td>
<td>$\frac{1}{4} \cdot k \cdot n_S$</td>
<td>0.42</td>
<td>O.K. but unrealistic</td>
</tr>
<tr>
<td>(c) Random emission of clusters of mixed sizes</td>
<td>Binomial in clusters</td>
<td>$\frac{1}{4} k_{\text{eff}} \cdot n_S$</td>
<td>0.42</td>
<td>Agrees well</td>
</tr>
<tr>
<td></td>
<td></td>
<td>if $k_{\text{eff}} = 2.0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX

The effective cluster size, \( k_{\text{eff}} \)

Consider \( C \) clusters in an event with sizes \( k_i \) \( (i = 1 \ldots C) \) charged particles. Both \( C \) and \( k_i \) are stochastic variables. The resulting number of charged particles, \( n_S \), is given by

\[
n_S = \sum_{i=1}^{C} k_i
\]

From this one gets the first two moments of the \( n_S \)-distribution:

\[
\bar{n}_S = \bar{C} \cdot \bar{k}
\]

\[
\text{var} n_S = \bar{k}^2 \text{var} C + \bar{C}\text{var} k
\]

This problem is a case of branching processes where cluster \( i \) branches into \( k_i \) particles. Next, we introduce a binomial branching into forward and backward regions with probabilities \( p_F \) and \( p_B \) for each cluster. We assume no leakage so that all \( k_i \) particles from cluster \( i \) remains in the same region as the cluster itself. In this case the first two moments of the \( n_F \)-distribution are given by:

\[
\bar{n}_F = \bar{C} \cdot p_F \cdot \bar{k}
\]

\[
\text{var} n_F = p_F^2 \bar{k}^2 \text{var} C + \bar{C} \cdot \bar{k}^2 p_F(1 - p_F) + \bar{C} \cdot p_F \text{var} k
\]

the variance of the binomial distr.

If one in (4) eliminates the moments \( \bar{C} \) and \( \text{var} C \) using (1) and (2) one obtains the desired result:

\[
\text{var} n_F = \frac{1}{4} \frac{D_S^2}{S} - \frac{1}{4} \bar{n}_S (\bar{k} + \text{var} k / \bar{k})
\]

where \( p_F = \frac{1}{2} \) and \( \text{var} n_S \equiv \frac{D_S^2}{S} \) have been substituted.

As a special case the simple formula for the case of fixed size clusters \( (k) \) is recovered if \( \text{var} k = 0 \) is assumed, namely

\[
\text{var} n_F = \frac{1}{4} \frac{D_S^2}{S} + \frac{1}{4} k \cdot \bar{n}_S
\]

Formula (5) is of the same form with the replacement \( k \rightarrow k_{\text{eff}} = \bar{k} + \text{var} k / \bar{k} \). The distribution of cluster sizes is not known. Were it Poisson the result is \( k_{\text{eff}} = \bar{k} + 1 \). We have truncated the Poisson distribution by setting the probability to zero outside the interval \( 1 \leq k \leq 5 \), when \( \bar{k} \) is near 2. In this case \( \text{var} k / \bar{k} \sim 0.5 \) and we believe the truncation to be somewhat more realistic.

The \( \text{cov}(n_F, n_B) \) is finally obtained from the relation \( n_S = n_F + n_B \) through

\[
\text{cov}(n_F, n_B) = \text{var} n_F + \text{var} n_B + 2 \text{cov}(n_F, n_B)
\]

and the symmetry requirement \( \text{var} n_F = \text{var} n_B \). Thus

\[
\text{cov}(n_F, n_B) = \frac{1}{2} \frac{D_S^2}{S} - \text{var} n_F = \frac{1}{4} \frac{D_S^2}{S} - \frac{1}{4} k_{\text{eff}} \cdot \bar{n}_S
\]

Thus the identity relation for the correlation slope \( b \) is

\[
B = \frac{D_S^2 - k_{\text{eff}} \cdot \bar{n}_S}{D_S^2 + k_{\text{eff}} \cdot \bar{n}_S}
\]
FIGURE CAPTIONS

Fig. 1a) The two-dimensional distribution of events (scatterplot) with the two coordinate systems used. The area of a ring is proportional to the number of events. These data correspond to the case with no gap between the F- and B-regions.

1b) The linear relation between the average of $n_F$ (at fixed $n_F$) and $n_F$. The slope $b$ is a measure of the correlation strength.

Fig. 2 The energy dependence of the correlation slope $b$.

Fig. 3 Illustration of the Monte Carlo simulation. The C clusters in an event with random sizes ($k_i$) are assigned random positions in pseudorapidity.

Fig. 4 The observed dispersions $d_S(n_F)$ and the Monte Carlo results (lines) for two assumed average sizes ($k$) of the clusters. The F- and B-intervals are 3, 2, and 1 units of pseudorapidity, respectively. In all three cases the gap is two units.

Fig. 5 The effective cluster size, $k_{\text{eff}}$, is shown to be independent of the gap size once it is larger than 2 units of pseudorapidity.

REFERENCES


Fig. 1

\[ \langle n_b(n_f) \rangle = a + b \cdot n_f \]

Fig. 2

(a) Full range

\[ 0.54 \pm 0.01 \]

(b) Central region

\[ 0.48 \pm 0.02 \]

(c) Full range with gap

\[ 0.4 \pm 0.01 \]
MONTE CARLO SIMULATION

C = 7 clusters

Fig. 3

\[
\frac{\Delta \sigma(n_p)}{n_5} \quad k = 3
\]

\[
\Delta \eta = 3
\]

\[
0 \quad 10 \quad 20 \quad 30
\]

\[
n_5
\]

Fig. 4

\[
\kappa_{eff}
\]

\[
\Delta \eta_F = 3, 2, 1
\]

\[
\text{GAP SIZE, } \delta \eta
\]

Fig. 5