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ON THE $2P_{3/2} - 2S_{1/2}$ ENERGY DIFFERENCE IN VERY LIGHT MUONIC SYSTEMS

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I will talk about some measurements that we hope it will be possible to perform in a near future, and present the results¹⁾ of an experiment undertaken recently at the CERN Synchro-cyclotron by a group-collaboration from the Pisa-Bologna-Saclay-CERN laboratories.

This experimental line started from the realization that by performing, on a simple muonic system (and in particular let us consider the cases of μP , μD , μHe , and μHe), an accurate measurement of the energy difference between the 2S and the 2P levels, we can measure a quantity which is essentially determined by the electronic vacuum polarization of the electromagnetic field acting between the muon and the nucleus²⁾.

Let us spend a little time on treating this general fact in more detail. If we write

$$V(r) = V_C(r)[1 + \beta(r)] , \quad (1)$$

the potential between two (distinct and spinless) particles of unitary (and point-like) charges, and assume that their masses are much bigger than the electron mass, then if $V_C(r)$ is the familiar Coulomb potential (r being the distance between the two charged particles), for values of $r \geq 10^{-13}$ cm, according to QED $\beta(r)$ is a small correction which depends, apart from r , only on the two quantities α (fine structure constant = $1/137$) and λ_e (the reduced Compton wavelength of the electron = 3.8×10^{-11} cm).

Generally, under these particular conditions, $\beta(r)$ is a correction entirely due to the electronic vacuum polarization of the electromagnetic field acting between the two point-like charges (Uehling term). The correction $\beta(r)$ can be calculated by developing it in a power series of the parameter α and using the general prescription of quantum electrodynamics; the dependence of $\beta(r)$ on r , considering only terms up to α^2 (inclusive), is represented in Fig. 1³⁾. Looking at Fig. 1 we can make the following remarks:

- i) $\beta(r)$ in the region considered here ($r \geq 10^{-13}$ cm) is not more than 8‰;
- ii) $\beta(r)$ is a "short-range" correction which for $r > \lambda_e$ goes very quickly to zero.

Therefore considering that the Bohr radius of a light muonic atom is in value very near to λ_e (in Fig. 1, a_μ^B and a_e^B are the Bohr radii of the μP and eP systems, respectively), it is clear that the *energy difference* S_μ between the 2S and 2P levels in a muonic atom will be quite sensitive to the magnitude of the "short-range" electron vacuum polarization correction $\beta(r)$. Moreover, since the muon mass is so large compared to the electron mass, corrections introduced by the muon radiation reaction

Fig. 1

(Lamb shift terms) are greatly reduced and, in fact, for the case of a muonic atom, they contribute to S_μ for a very small quantity.

Concluding a precise measurement of S_μ gives an almost unique opportunity to check the correction $\beta(r)$ directly.

Let us now go to the real cases. The expected energy differences S_μ between the various 2S and 2P levels for the four above-mentioned simpler muonic systems μP , μD , $(\mu^3\text{He})^+$ and $(\mu^4\text{He})^+$ are shown in Tables 1, 2, 3, and 4, respectively; in all cases considered here the 2P levels are always less bound than the 2S ones.

Table 1 (see Ref. 4)

Contribution to the 2S-2P splitting for the μP muonic atom.

The states are indicated as $^{2F+1}L_j$, where $F = j + \text{nuclear spin } I$. Energies are given in units of $\alpha^2\text{Ry} = 0.13461 \text{ eV}$. The disappearance rate of the 2P state is $1.2 \times 10^{11} \text{ sec}^{-1}$: in the given units the width of the 2P level is $0.0006 \alpha^2\text{Ry}$ (which for λ causes a linewidth of $20 \text{ \AA}$).

Transition	Vacuum polar.		Muonic Lamb shift	Fine struct.	Hyper-fine struct.	Nuclear size ± 0.0006	Total in $\alpha^2\text{Ry}$ ± 0.0006	λ in $\text{\AA}$ $\pm 20 \text{ \AA}$
	α	α^2						
$^1S_{1/2} \rightarrow ^3P_{1/2}$	1.5225	0.0112	-0.0049	0	0.1417	-0.0258	1.6447	56000
$^1S_{1/2} \rightarrow ^3P_{3/2}$	1.5225	0.0112	-0.0049	0.0625	0.1135	-0.0258	1.6790	54860
$^3S_{1/2} \rightarrow ^1P_{1/2}$	1.5225	0.0112	-0.0049	0	-0.0845	-0.0258	1.4185	64930
$^3S_{1/2} \rightarrow ^3P_{1/2}$	1.5225	0.0112	-0.0049	0	-0.0280	-0.0258	1.4750	62450
$^3S_{1/2} \rightarrow ^3P_{3/2}$	1.5225	0.0112	-0.0049	0.0625	-0.0563	-0.0258	1.5092	61030
$^3S_{1/2} \rightarrow ^5P_{3/2}$	1.5225	0.0112	-0.0049	0.0625	-0.0337	-0.0258	1.5318	60130

Table 2 (see Ref. 5)

Contribution to the 2S-2P splitting for the μD muonic atom.

The states are indicated as for Table 1. Energies are given in units of $\alpha^2\text{Ry} = 0.14182 \text{ eV}$. The width of the 2P level is $0.0006 \alpha^2\text{Ry}$ (natural linewidth of λ is $20 \text{ \AA}$).

Transition	Vacuum polar.		Muonic Lamb shift	Fine and hyperf. struct.	Nuclear size ± 0.009	Nuclear polar. ± 0.002	Total in $\alpha^2\text{Ry}$ ± 0.009	λ in $\text{\AA}$ ± 300
	α	α^2						
$^2S_{1/2} \rightarrow ^2P_{1/2}$	1.6051	0.0175	-0.0060	0.0192	-0.202	0.003	1.437	60960
$^2S_{1/2} \rightarrow ^2P_{3/2}$	1.6051	0.0175	-0.0060	0.0846	-0.202	0.003	1.502	58300
$^2S_{1/2} \rightarrow ^4P_{1/2}$	1.6051	0.0175	-0.0060	0.0336	-0.202	0.003	1.451	60350
$^2S_{1/2} \rightarrow ^4P_{3/2}$	1.6051	0.0175	-0.0060	0.0902	-0.202	0.003	1.508	58080
$^4S_{1/2} \rightarrow ^2P_{1/2}$	1.6051	0.0175	-0.0060	-0.0240	-0.202	0.003	1.394	62850
$^4S_{1/2} \rightarrow ^2P_{3/2}$	1.6051	0.0175	-0.0060	-0.0413	-0.202	0.003	1.459	60030
$^4S_{1/2} \rightarrow ^4P_{1/2}$	1.6051	0.0175	-0.0060	-0.0096	-0.202	0.003	1.408	62210
$^4S_{1/2} \rightarrow ^4P_{3/2}$	1.6051	0.0175	-0.0060	0.0467	-0.202	0.003	1.464	59800
$^4S_{1/2} \rightarrow ^6P_{3/2}$	1.6051	0.0175	-0.0060	0.0506	-0.202	0.003	1.468	59650

With reference to these tables the following remarks have to be made:

- a) The different errors quoted include only the uncertainty introduced by the errors with which the various form factors are experimentally known. No errors due to the inaccuracy or approximations of the theoretical calculations are included; according to Di Giacomo⁴⁾ these are on the level of the natural linewidth Γ .

For the cases of Tables 2, 3, and 4 the quoted uncertainties⁵⁻⁷⁾ are always much bigger than the respective natural linewidths; for the case of Table 1 (μP system)

Table 3 (see Ref. 6)

Contribution to the 2S-2P splitting for the $(\mu^3\text{He})^+$ muonic ion. The states are indicated as for Table 1. Energies are given in units of $\alpha^2\text{Ry} = 0.14438$ eV. The disappearance rate of the 2P state is $2 \times 10^{12} \text{ sec}^{-1}$; in the given units the width of the 2P level is 0.0096 (which for λ causes a natural linewidth of 8 Å).

Transition	Vacuum polar.		Muonic Lamb shift	Fine and hyperf. struct.	Finite size ± 0.27	Total in $\alpha^2\text{Ry}$ ± 0.27	λ in Å ± 300
	α	α^2					
$^1S_{1/2} \rightarrow ^3P_{1/2}$	11.372	0.079	-0.096	-0.673	-2.76	7.922	10840
$^1S_{1/2} \rightarrow ^3P_{3/2}$	11.372	0.079	-0.096	0.468	-2.76	9.063	9475
$^3S_{1/2} \rightarrow ^1P_{1/2}$	11.372	0.079	-0.096	0.399	-2.76	8.994	9548
$^3S_{1/2} \rightarrow ^3P_{1/2}$	11.372	0.079	-0.096	0.133	-2.76	8.728	3839
$^3S_{1/2} \rightarrow ^3P_{3/2}$	11.372	0.079	-0.096	1.266	-2.76	9.861	8709
$^3S_{1/2} \rightarrow ^5P_{3/2}$	11.372	0.079	-0.096	1.160	-2.76	9.755	8803

Table 4 (see Ref. 7)

Contribution to the 2S-2P splitting for the $(\mu^4\text{He})^+$ muonic ion. The states are indicated as L_j. Energies are given in units of $\alpha^2\text{Ry} = 0.145687$ eV. In these units the width of the 2P level is 0.0096 (which for λ causes a natural linewidth of 8 Å).

Transition	Vacuum polar.		Muonic Lamb shift	Fine struct.	Nuclear polar.	Finite size ± 0.0597	Total in $\alpha^2\text{Ry}$ ± 0.0597	λ in Å ± 47
	α	α^2						
$S_{1/2} \rightarrow P_{1/2}$	11.4347	0.0789	-0.0982	0	0.0213	-1.9678	1.3795	7359
$S_{1/2} \rightarrow P_{3/2}$	11.4347	0.0789	-0.0982	1.0000	0.0213	-1.9678	1.5251	8136

the quoted inaccuracy, due to the r.m.s. experimental error, is comparable to the line-width Γ .

b) The uncertainty introduced by the experimental errors on the r.m.s. are smaller than the vacuum polarization term in α^2 only for the cases of the μP and $(\mu^4\text{He})^+$ muonic systems.

c) As expected, the contribution due to the muonic Lamb shift is in all cases a very small part of the total, which is mainly determined by the vacuum polarization term in α .

d) The fine and hyperfine structure contributions are also small compared to the total.

Let us now go on to some experimental considerations. So far, the method envisaged for measuring the $2P \rightarrow 2S$ energies level difference with good accuracy is the following: it consists of sending short pulses of almost monochromatic electromagnetic

radiation on a muonic system previously prepared in a 2S state, and varying the radiation wavelength until a $2S \rightarrow 2P$ transition takes place; since the 2P level is highly unstable (for instance the 2P level lifetime in a muonic helium system is $\tau_{2P} \approx 5 \times 10^{-13}$ sec), the $2S \rightarrow 2P$ induced transition will quickly be followed by a fast muonic X-ray (~ 8 keV) emission which is therefore used to signal the transition and identify the "resonance" wavelength.

In order to perform this type of measurement, various different problems have first to be solved; in the following we will first state and then discuss the two most relevant ones, and give the answers when these are available.

1) Formation of a muonic system in a 2S state (metastable) and in a medium so that it lives for a long enough time to perform an experiment on it. Let us first talk about the formation of a muonic metastable 2S state.

At the CERN muon channel of the 600 MeV Synchro-cyclotron, measurements have been done of the fraction $F(^4\text{He})$ of metastable muonic systems $(\mu^-^4\text{He})_{2S}^+$ initially formed per negative muon stopped in a gaseous ^4He target (7-10 atm, 300°K); it has been found experimentally that $F(^4\text{He}) \approx 4\%$. This value agrees well with the result of a theoretical calculation. No experiment has been done using a gaseous ^3He target, but one expects that in this case the situation will be similar to that obtained with a ^4He target.

There are no measurements on the fraction $F(\text{H}_2)$ or $F(\text{D}_2)$ of metastable muonic systems $(\mu^-P)_{2S}$ and $(\mu^-D)_{2S}$ initially formed when negative muons are stopped in a gaseous hydrogen or deuterium target, respectively. It has to be said, however, that simple theoretical estimates indicate that most probably $F(\text{H}_2)$ and $F(\text{D}_2)$ will at least be as big as $F(^4\text{He})$.

As regards the lifetime of these metastable muonic systems, once they have been formed, let us first look at those channels and their rates through which the metastable 2S muonic state will spontaneously disappear, i.e. at the limiting conditions of zero density; these are given in Table 5 for only the two most important cases of $(\mu^-P)_{2S}$ and $(\mu^-^4\text{He})_{2S}^+$ muonic systems.

It is, however, clear that to obtain the total disappearance rate $\lambda^{2S}(p)$ -- p being the pressure of the gas target at 300°K -- we have to add to the rates of Table 5 the disappearance rates of the metastable 2S state via Stark mixing and Auger transitions, which will take place owing to the inevitable collisions experienced by the metastable muonic systems against the neighbouring atoms of the medium (at pressure p) in which they have been formed.

The value of the total disappearance rate $\lambda^{2S}(p)$ for the case of the muonic metastable state $(\mu^-^4\text{He})_{2S}^+$ has been experimentally determined at CERN⁸⁾ for the cases in which the 2S systems were formed in a gaseous target at densities corresponding to pressures within 7-50 atm, 300°K. The results indicate that with this

Table 5

Decay channels (present and corresponding rates for $(\mu\text{P})_{2\text{S}}$ and $(\mu^4\text{He})_{2\text{S}}^+$ muonic systems in eV^{-1} ; M1 transitions from 2S to 1S are neglected⁸).

Process	$(\mu\text{P})_{2\text{S}}$	$(\mu^4\text{He})_{2\text{S}}^+$
Muon decay	4.54×10^5	4.54×10^5
Muon capture	80	45
Two-quantum decay to the 1S level	1.66×10^3	1.06×10^5

range of pressure the 2S-state lifetime is at least 1.4 μsec long. Also here one expects that for the $(\mu^{-3}\text{He})_{2\text{S}}^+$ the situation will be very similar to the one found for the $(\mu^{-4}\text{He})_{2\text{S}}^+$ system.

No data exist for the total disappearance rate of muonic metastable atoms $(\mu^{-\text{P}})$ or $(\mu^{-\text{D}})$ formed in a gaseous target; estimates show that it is conceivable that already at a pressure of 1 atm the 2S-state lifetime could be as short as a fraction of a nanosecond.

2) The almost monochromatic electromagnetic radiation pulses must be adjustable in their wavelength values around the values shown in Tables 1 to 4, at least within the errors indicated there; and moreover, each pulse must contain sufficient energy to be able to induce the selected $2\text{S} \rightarrow 2\text{P}$ transition at an experimental detectable level.

For the cases of the muonic system $(\mu^{-4}\text{He})_{2\text{S}}^+$ and $(\mu^{-3}\text{He})_{2\text{S}}^+$ given in Tables 3 and 4, the range of wavelength needed is easily covered by dye lasers.

As a result of a compromise between the existing experimental and physics interests, and using the general method outlined above, a Pisa-Bologna-Saclay-CERN Collaboration performed this type of experiment at CERN (as part of a first step) searching for the $2\text{S}_{1/2} \rightarrow 2\text{P}_{3/2}$ transition on the muonic metastable 2S system $(\mu^{-4}\text{He})_{2\text{S}}^{+9,1}$. In what follows we will briefly mention some details of this experiment and give the first result, together with some conclusions drawn from it.

In Fig. 2 a simplified view of the set-up is shown. The negative muon beam entering the target (filled with ^4He gas) was bunched in pulses, a few milliseconds wide, at the same repetition rate as that of the ruby laser (0.25 Hz). The electromagnetic pulses were produced by an infrared dye laser (excited by a Q-switched ruby laser) which could be tuned using a diffracting grating inserted in the infrared dye laser cavity⁹.

The main characteristics of the lasers are given in Table 6. The energy per pulse ensured that for the experimental conditions chosen we would have a probability for the $2\text{S}_{1/2} \rightarrow 2\text{P}_{3/2}$ transition, at the resonance wavelength, of at least 15%. The target

Fig. 2 Very simplified view of the whole apparatus. M = CH₂ moderators; 1,3,4,5 = plastic scintillators; 2 = anticoincidence Čerenkov counter; T = invar steel tank; V = useful volume for stopping muons; A₁-A₈ = Na(Tl) counters; L.G. = light-guide used to inject the infrared radiation into the target; TC = optical telescope; PC = optical fibre supplying the energy-monitoring signal.

Table 6

Characteristics of the dye laser used in the present apparatus

Dye	HITC a)
Solvent	DMSO b)
Molar concentration (moles/litre)	$\sim 5 \times 10^{-5}$
Average ruby pumping energy (J)	1.2
Average pulsed infrared output energy (mJ)	300
Infrared power at 8150 Å (MW)	~ 15
Radiation pulse duration (nsec)	~ 20
Bandwidth of radiation (Å)	~ 6

a) 1,3,3,1',3',3'-hexamethyl-2,2'-indotricarbocyanine iodide: purchase from K. & K. Labs., Plainview, N.Y., USA.

b) Dimethylsulfoxide: purchased from Carl Bittman A.G., Basel, Switzerland.

is surrounded by eight NaI(Tl) crystals in order to detect, after a muon stopped in the target (with or without infrared light entering in it), any X-rays of energy between 4 and 14 keV and at any time t (from the muon stop signal initial time) contained in an interval of about 2 μsec . The main difficulty in performing this experiment was represented by the high rate of accidental X-ray background.

In Fig. 3 is shown the results¹⁾ for a series of exploratory runs in which the wavelength interval 8090-8160 $\text{\AA}$ was covered (about 8×10^5 flashes to the ruby laser): a peak appears in the distribution, and this can be well fitted by a Lorentzian curve with $\Gamma = 8 \text{ \AA}$. A fit of the data with a simple straight line (case of no effect) gives a χ^2 corresponding to a confidence level of about 4.5%. Taking the peak as evidence that the $2S_{1/2} \rightarrow 2P_{3/2}$ transition has been induced by the infrared light we obtain for the resonance wavelength

$$\lambda = 8117 \pm 5 \text{ \AA} . \quad (2)$$

Fig. 3

Comparing the value (2) with the theoretical predictions, we conclude the following:

i) The difference DS between the theoretical and the experimental transition energy (see Table 4) for the $2S_{1/2} \rightarrow 2P_{3/2}$ line in the muonic $(\mu^- \text{He})^+$ ionic system is $DS_\mu = (0.016 \pm 0.060)\alpha^2 \text{ Ry}$, and is compatible with zero within the errors. This indicates that in this case, within the quoted accuracy, there is agreement between the QED prediction as far as concerns the electronic vacuum polarization and the experiment. A better understanding of this result is obtained by comparing it to other results which are most relevant with respect to tests of QED; this is done in Table 7¹⁰⁾.

ii) On the other hand, if we assume QED to be valid comparing the result (2) with theoretical values (Table 4), we obtain for the r.m.s. charge radius of the ^4He nucleus the value $\langle r^2 \rangle^{1/2} = 1.644 \pm 0.005 \text{ fm}$, where the error³⁾ contains also the uncertainty in the theoretical calculation¹¹⁾.

Table 7 (see Ref. 10)

Tests of the vacuum polarization effects S_{vp}

Experiment	Exp. precision $\Delta S/S$ (ppm)	$\frac{S_{vp}}{S}$ (Th.)	$\frac{S_{vp}}{S_{vp}}$ (1%)	Mean momentum transfer
Lamb shift in hydrogen	60	2.5×10^{-2}	2-3	αm_e
Hyperfine structures in hydrogen and muonium	4 ^{a)}	α^2	100	αm_e
$(\mu^4\text{He})^+ \Delta E(2P_{3/2} - 2S_{1/2})$	6000	1.2	5	αm_e
$g_e - 2$	3	10^{-4}	30	m_e
Hyperfine structure in positronium	25	$\frac{1}{3} \frac{\alpha}{\pi}$ b)	40	$2m_e$
High-Z μ -mesic atoms	50	5×10^{-3}	10	$\frac{Z\alpha m_\mu}{n}$
$g_\mu - 2$	27	$2 \frac{\alpha}{\pi}$	7	m_μ

a) This figure represents the theoretical uncertainty.

b) Contribution of the vacuum polarization in the annihilation channel.

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- 11) Taking for the r.m.s. charge radius of ^4He the weighted average of the existing experimental results (see Ref. 1), one obtains $\langle r^2 \rangle^{1/2} = 1.650 \pm 0.025$ fm.