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SUMMING POMERON TREES

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It is argued that multi-Pomeron exchange diagrams not involving Pomeron loops play a dominant rôle in the presently accessible energy range. These diagrams are explicitly summed for a Pomeron pole with intercept larger than one (in the limit $\alpha' = 0$). S channel unitarity is automatically satisfied (but not saturated).

For particle-particle scattering we obtain phenomenological features in agreement with the data $[\sigma_T, \sigma_{el}, (d\log\sigma_{el}/dt)|_{t=0}]$ all increase to a finite limit, and σ_{el}/σ_T can be made arbitrarily small. Our results are also applied to nucleus-nucleus scattering.

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I. -

In this paper we will discuss the scattering amplitude arising from the sum of an arbitrary number of Pomeron exchanges in the framework of the Gribov calculus, neglecting, however, Pomeron loops. This implies diagrams as those of Fig. 1. The intercept of this unrenormalized Pomeron, larger than the one in our view, will appear as a parameter of the model. Let us first motivate our interest in this approach.

The theory of the Pomeron with unit renormalized intercept has been widely studied in the last years ¹⁾, using renormalization group techniques. Asymptotically, it yields to cross-sections rising as $(\log s)^\eta$, η being a critical index. The scale which governs the approach of such asymptopia is so large ²⁾ that the critical behaviour is not applicable at present machine energies. This is mainly due to the smallness of the triple Pomeron coupling λ as obtained from data on large diffractive production compared to the critical value λ^* ^{*)}. The consideration of two, or more, Pomeron couplings to the particle lines does not improve the situation. Indeed this inclusion ⁴⁾ increases even more the energy scale needed in order to reach the critical asymptotic behaviour.

At present energies, the relevant expansion is given by a perturbative treatment with a Pomeron whose intercept, renormalized but not critical, is larger than 1. This excess, call it $\bar{h}$, is also determined by λ and its rôle is that of guaranteeing the unit intercept at infinite energies (i.e., the critical phenomenon). The smallness of λ implies a too small value of $\bar{h}$ ($\bar{h} \sim 0.01$) which is insufficient therefore to explain the experimental rise of cross-sections ³⁾. Therefore the critical Pomeron theory has troubles in accounting for the observed rise of cross-sections. The inclusion of the simplest non-leading terms (in the critical limit) as for instance, $g\phi^4$ or two Pomeron couplings to external particles seems not to improve this situation.

^{*)} As discussed in Refs. 2), 3), it could happen that λ is large and that the small effective number obtained from diffraction production of large masses is due to a complicated shadowing mechanism. This way out presents the inconvenience that the Pomeron pole is unobservable and therefore the Regge pole approach is unwarranted.

We are therefore naturally led to disentangle the Pomeron intercept as observed at accelerator energies ^{*)} [call it $\alpha(0) = 1 + \mu$] from the value $\bar{\alpha}(\lambda)$ it should have if the theory were asymptotically critical. This input Pomeron of intercept larger than one will be, of course, modified by the diagrams as those of Fig. 1, which can be interpreted as absorptive corrections. On the other hand, Pomeron loop corrections will renormalize the intercept and are therefore also important. Even more, this renormalization can well have as effect to lower the intercept below one ⁵⁾. But if λ is small this will begin to show at extremely high energies. In the investigation of a simplified model, this effect has the form ⁵⁾

$$s^{\mu} \left(1 + \frac{\lambda^2}{\mu^2} s^{\mu} \right)^{-2} \quad (1)$$

in the one-Pomeron exchange amplitude.

It is therefore interesting to investigate whether there are other transient "critical phenomena" which could play a rôle at our present-day energies, before the Pomeron loop corrections set in. This can be the case if, for instance, the Pomeron coupling to external particles G is a parameter quite larger than the triple Pomeron coupling λ (we shall discuss later the meaning of this comparison). Then all terms in $(G\lambda)^n$ must be summed even if a perturbation in λ^2 is still valid. This possibility has been advanced by Schwimmer ⁶⁾ in the case of particle scattering by a nucleus ($G \propto A^{1/3}$) and we shall indeed follow closely his argumentation. In particular we shall make use of the fact that the sum over tree diagrams is provided by the classical solution of the interacting Pomeron Lagrangian. What actually happens at ultra high energies for a Pomeron, described by that Lagrangian, is a very interesting question which is, however, beyond the scope of this paper. We believe that our method is applicable in a transient region that includes present (and future !) machine energies ²⁾. In the following, we shall use the word asymptotic having always in mind that our arguments are applicable before reaching a super high energy scale.

^{*)} Data of σ_{tot} are well accounted for by $\mu \sim 0.1$ ³⁾.

II. -

Let us now provide a quantitative support to our statement that effects of order $G\lambda$ are larger than those of order λ^2 not only for nuclei but also in proton-proton scattering.

A typical $G\lambda$ effect is given by the triple Pomeron diagram. By comparing this with the measured cross-sections for diffractive production of large masses (normalized to the total cross-section) we obtain approximately $G\lambda Y \sim 0.05 Y$ where

$$Y \equiv \log s$$

A typical λ^2 effect is the one-loop contribution to the Pomeron intercept which we called $\bar{h}$ before. This produces a correction to the total cross-section which has been evaluated in Refs. 2), 3) and has the approximate value $\bar{h}Y \sim 0.01 Y$ which is safely smaller than the typical $G\lambda$ correction.

We shall further simplify the problem by setting $\alpha' = 0$. This approximation is much less reliable in our case than in the nuclear case of Ref. 6). Our justification relies on the fact that the shrinkage (i.e., $\alpha'Y$) is not the dominant contribution to the diffraction slope for not too high values of Y we are interested in. Since it will turn out that in our approach, even in the $\alpha' = 0$ limit, $(d \log \sigma / dt)|_{t=0}$ shrinks considerably for the present values of Y (although it approaches a finite limit) we can argue that α'_{pole} can in fact be much smaller than the observed value 0.2-0.25. The approximation $\alpha'Y \ll (d \log \sigma / dt)|_{t=0}$ can therefore be valid for $Y \lesssim 25$ (i.e., for $s \lesssim 10^{10} \text{ GeV}^2$!)

III. -

The Lagrangian describing our dynamical problem can then be written as

$$\mathcal{L} = \frac{1}{2} p \frac{\partial^2}{\partial y^2} q + \mu p q - \lambda p q (p+q) + q \zeta_1 + p \zeta_2 \quad (2)$$

The field variables p and q - which depend on the rapidity y and impact parameter $\vec{b}$ - are formally related to the usual Gribov fields ψ^+ and ψ by $p = i\psi^+$, $q = i\psi$. This change of variables eliminates the usual i in the triple Pomeron interaction. The responsibility of the minus signs of the Pomeron cuts is now assigned to the propagator and it stems from the fact that the commutator of p and q has a sign opposite to the canonical one. G_1 and G_2 in Eq. (2) are the (real) Pomeron sources provided by the colliding particles; they depend on $\vec{b}$, y as well as on the impact parameters and rapidities of each source. We will consider

$$g_1 = g_1(\vec{b}) \delta(y) \quad , \quad g_2 = g_2(\vec{b} - \vec{B}) \delta(Y - y) \quad (3)$$

where $Y = \log s$ is the total rapidity and g_1 and g_2 are the impact parameter distributions of both colliding particles centred around $\vec{b} = 0$ and $\vec{b} = \vec{B}$, respectively. As in Ref. 6), the partition function

$$Z(g_1, g_2) = N \int dp dq e^{-\int \mathcal{L} dy d\vec{b}} \quad (4)$$

[normalized to $Z(0,0) = 1$], is the elastic S matrix under the assumption, typical of an eikonal, that the Pomerons interact with the sources in an uncorrelated way, i.e.,

$$Z(g_1, g_2) = \sum_{m,n} \Gamma^{(m,n)} \frac{g_1^m g_2^n}{m! n!} \quad (5)$$

where $\Gamma^{(m,n)}$ are the Green functions. The sum of the tree diagrams we are interested in is given by the classical solution of the problem, namely

$$S(Y, \vec{B}) = [\exp - A_{\min}(g_1, g_2)] [\exp A_{\min}(0,0)] \quad (6)$$

where $A = \int \mathcal{L} dy d\vec{b}$ is the action and $A_{\min}$ is the minimal action. Let us first notice that after having set $\alpha' = 0$, the problem is really uni-dimensional. This means that the equations of motion will determine the evolution of the co-ordinates p and q in y (y will be often called "time"), while the dependence in $\vec{b}$ will be trivially given by the sources.

IV. -

We shall start by solving the classical problem given by the Lagrangian of Eq. (2) without sources. The equations of motion are

$$\dot{p} \equiv \frac{\partial p}{\partial t} = - \frac{\partial H}{\partial q}, \quad \dot{q} = \frac{\partial H}{\partial p} \quad (7)$$

where

$$H = -\mu pq + \lambda pq(p+q) \quad (8)$$

which individuates H as a constant of motion. Let us find the solution characterized by $H=E$. Solving $H(p,q)=E$ gives $p=p(q,E)$ which, inserted into the explicit equation (7) for $\dot{q}$, gives

$$y = y(q) = \int \frac{dy}{dq} dq = \frac{1}{\lambda} \int \frac{dx}{x \sqrt{\left(\frac{\mu}{\lambda} - x\right)^2 + \frac{4E}{\lambda x}}} \quad (9)$$

Some care must be exerted due to the multivaluedness of $y(q)$. If we call $q'(E)$ and $q''(E)$, with $q'' > q'$, the zeros of $\dot{q}(E,q)$, the period is given by

$$\Omega(E) = 2 (y(q'') - y(q')) \quad (10)$$

We can therefore write the solution of the mechanical problem as

$$y_2 - y_1 + n \Omega = \pm (y(q_2) - y(q_1)) \quad (11)$$

where y_1 and y_2 are the actual "times" corresponding to co-ordinates q_1 and q_2 while $y(q)$ is the specific function given by Eq. (9) and n is an arbitrary.

Before we discuss in detail our results, let us remark the important rôle which is played by the fixed points of the problem. These are defined by $\dot{p} = \dot{q} = 0$ and they are :

$$(0,0) \quad (\mu/\lambda, 0) \quad (0, \mu/\lambda) \quad (\mu/\lambda, \mu/\lambda)$$

where the notation (p,q) is used.

The first three fixed points correspond to $E=0$ and are saddle points. The fourth one is stable and corresponds to $E = (-\mu^3/27\lambda^2)$. They are represented by a dot in Fig. 2.

In Fig. 2 are also shown typical orbits in the q, p plane where the arrows individuate the sense of the motion of increasing y . The orbits can be classified into three families depending on the sectors they lie in the (p, q) plane when subdivided by the three straight lines $p=0$, $q=0$, $p+q=\mu/\lambda$ shown in Fig. 2. These three families are represented in Fig. 2 by

- i) dashed lines, they correspond to $E > 0$;
- ii) dotted-dashed lines, they correspond to $E < 0$;
- iii) solid lines, they correspond to $(-\mu^3/27\lambda^2) < E < 0$.

For $(-\mu^3/27\lambda^2) < E < 0$ the orbit belongs to family ii) or iii) depending on the initial condition.

The orbits degenerate for $E=0$ in the three straight lines discussed before. They identify therefore a triangle defined by the first three fixed points.

The period of the orbits is a function of E ; for $E \rightarrow 0$, it tends to infinity as $-\log|E|$. Indeed it is easy to see from Eqs. (9) and (11) that for E small one passes near the fixed points with a vanishing speed. Therefore one takes a long time y in the vicinity of those fixed points.

V. -

We have discussed the solutions of the classical action given by Eq. (2) for the zero source case [i.e., $G_1 = G_2 = 0$ in Eq. (2)]. We found that they are characterized by a continuous parameter E , i.e., $q_E(y)$ and $p_E(y)$. Let us now go back to our original problem, the solutions of Eq. (2) with G_1 and G_2 given by Eqs. (3).

It is easy to verify, from the equations of motion (7), (8), that the expressions

$$q(y) = \frac{q_E(y)}{q_E(Y)} g_2 \Theta(Y-y), \quad p(y) = \frac{p_E(y)}{p_E(0)} g_1 \Theta(y) \quad (12)$$

are solutions of our problem where E must be thought of as a function of g_1 , g_2 and Y . We have to find, therefore, the orbits for which $p_E(0) = g_1$ and $q_E(Y) = g_2$ and we are of course interested in the case of large Y .

For every orbit which will be characterized by a value of E , we must then calculate the corresponding value of the action and retain the minimal one which - when introduced in Eq. (6) - will give us the scattering amplitude. The y integral over the Lagrangian can be written as

$$\mathcal{A} \equiv \int \mathcal{L} dy = \int p \dot{q} dy - EY + g_1 q(0) + g_2 p(Y) \quad (13)$$

Taking into account the expression (12) we find

$$\mathcal{A} = \int_c p_E dq_E - EY + g_1 q_E(0) = \int_c p_E dq_E - EY + p_E(0)q_E(0) \quad (14)$$

where c is the path along the orbit. The integral in the right-hand side of Eq. (14) represents the area subtended by the path c in the p, q plane. The analytic expression for $\mathcal{A}$ in terms of q_1, q_2 and Y is rather complicated. We shall therefore exploit the simple geometrical interpretation to find first the expressions for $\mathcal{A}$ for $Y \rightarrow \infty$, as function of g_1 and g_2 . We shall then discuss how this asymptotic value is approached.

For $Y \rightarrow \infty$, we see that if we want orbits with a limited number of periods [i.e., n limited in Eq. (11)] we must have $E \rightarrow 0$ to force the integral in Eq. (9) to be very large. The logarithmic divergence in E of that integral shows indeed that $E \propto e^{-Y}$. As a first result, for such orbits the term EY will contribute negligibly to the action of Eq. (14).

To find $A(Y \rightarrow \infty, g_1, g_2)$ let us first discuss the case in which g_1 and g_2 are larger than μ/λ . This implies that we must look for orbits of the first kind in the classification we made before. For large Y the system can evolve along a path as that depicted in Fig. 3. Of course, another possibility would imply the addition of an extra period, with a slightly larger value of E , and so on. For $Y \rightarrow \infty$, the path of Fig. 3 will approach the hypotenuse of the triangle plus the two segments in the abscissa and ordinate axes between the vertices of the triangle and $p=g_1$ and $q=g_2$. In this case $q(0) \sim 0$ and the area under the path [the integral in Eq. (14)] is the area of the triangle, i.e.,

$$A(Y \rightarrow \infty) = \frac{1}{2} \frac{\mu^2}{\lambda^2} \quad (15)$$

independently of g_1 and g_2 as long as they are both larger than μ/λ . It is easy to show by using geometrical arguments that if one does an extra cycle one increases the action by μ^2/λ^2 . The minimum action for the case g_1, g_2 greater than μ/λ is therefore $\frac{1}{2}\mu^2/\lambda^2$.

If at least one of g_1 and g_2 is smaller than μ/λ we will find, for large Y , two possible paths, apart from extra cycles. The example in which both g_1 and g_2 are smaller than μ/λ is shown in Fig. 4. Let us calculate the action for these two possibilities. The source term in Eq. (14) is different from zero in only one case. Nevertheless, it is easy to realize that for both cases the integral minus the source term represents the area subtended by the corresponding curve, as shown in Fig. 4, where the surfaces are shaded vertically and horizontally, respectively.

For $Y \rightarrow \infty$, the paths approach the side of the triangle and, therefore, the actions are given by the area of the fractions of the triangle bounded by the line $p=g_1$ in one case and $q=g_2$ in the other. Therefore

$$A = A_1 = g_1 \left(\frac{\mu}{\lambda} - \frac{g_1}{2} \right) \text{ for the first case, and} \quad (16)$$

$$A = A_2 = g_2 \left(\frac{\mu}{\lambda} - \frac{g_2}{2} \right) \text{ for the second.}$$

The minimal action is then given by

$$A_{\min} = A_1 \theta(g_2 - g_1) + A_2 \theta(g_1 - g_2) \quad (17)$$

It is clear that any extra cycle will add an extra area of the triangle to the action. If one of the sources is larger than μ/λ we shall find the

If one of the sources is larger than μ/λ we shall find the path of Fig. 5 (the case shown is $g_1 > \mu/\lambda$, $g_2 < \mu/\lambda$). The arguments developed above can be paralleled and therefore Eqs. (16), (17) are again obtained for this case. As expected, the result of Eq. (17) joins that of Eq. (15) when the smallest of the g approaches μ/λ .

VI. -

We have therefore found the values of the classical minimal action when $E \sim 0$, which is the situation for $Y \sim \infty$. Let us now discuss how this limiting result is approached and the corresponding scale. We will first show that the approach to the "asymptotic" action is from above for $E > 0$ (outside the triangle) and from below for the case $E < 0$ (inside the triangle) discussed before.

Let us define

$$F(E) = \int_{q_1(E)}^{q_2(E)} p_E(q) dq \quad (18)$$

where q_1 is the value of q for $y=0$ which depends, of course, on the value of E . Then

$$\frac{dF}{dE} = Y(E) - p_1 \left. \frac{\partial q_1}{\partial E} \right|_{p_1} \quad (19)$$

where Eqs. (9) and (11) have been used (for $n=0$) for the term obtained by differentiating Eq. (18) inside the integration sign.

We shall first discuss the $E > 0$ case, i.e., paths which are outside the triangle. For different values of E we shall find different paths, $Y(E)$ is the "time" taken in following the path and $q_1(E)$ the initial value of q [the path being determined by $p_1 = p(y=0)$, $q_2 = q(y=Y)$].

For increasing values of E , $Y(E)$ decreases and the path moves away from the side of the triangle, as shown in Fig. 6. Indeed $Y(E)$ is a decreasing function of E , as shown in Fig. 7, and $E_{\max}$ is the value of

E for which the path shrinks to the point (p_1, q_2) so that zero time ($Y \sim 0$) is needed to follow it.

Integrating Eq. (19) and remembering that $q_1(E=0) = 0$

$$F(E) = F(0) + \int_0^E Y(E') dE' - p_2 q_1 \quad (20)$$

Using Eq. (14)

$$\mathcal{A} = F(E) - E - E Y(E) + p_1 q_1 = F(0) + \int_0^E Y(E') dE' - E Y(E) \quad (21)$$

Then, apart from a constant $F(0)$, the action $\mathcal{A}$ is the area shaded in Fig. 7 which is obviously an increasing function of E or, analogously, a decreasing function for increasing Y . For $Y \sim 0$ $\mathcal{A}(0) = p_1 q_2$ and from there $\mathcal{A}$ decreases towards its asymptotic value (15) [or (17) if one g is smaller than μ/λ] for $Y \rightarrow \infty$.

The demonstration that the limit $Y \rightarrow \infty$ is approached from below for g_1 and g_2 smaller than μ/λ (inside the triangle) can be done in an absolutely analogous way. It approaches from below because the correction has the opposite sign due to the fact that E is negative. The only difference is that the argument cannot be pushed - as before - to small values of Y ($Y \sim 0$). Indeed for large Y , $Y(E)$ is a monotonic function as before. For small Y , however, $Y(E)$ is not necessarily monotonic; indeed, depending on g_1 and g_2 , it can have more complicated behaviours. A case of particular interest is that in which g_1 and $g_2 \ll \mu/\lambda$. Then, as Y decreases from infinity, $|E|$ first increases and then decreases again. Indeed, for

$$g_1, g_2 < \frac{\mu}{\lambda} e^{-\mu Y} \quad (22)$$

the path is as depicted in Fig. 8. This is the situation in which the perturbative solution

$$q = g_2 e^{-\mu(Y-y)} \theta(Y-y), \quad p = g_1 e^{\mu y} \theta(y) \quad (23)$$

is valid for all $0 < y < Y$. This gives

$$A_{\min} \approx g_1 g_2 e^{\mu Y} \quad (24)$$

The way in which the asymptotic results are approached can be intuitively visualized. For $Y \sim 0$, $A \sim g_1 g_2$ while for $Y \sim \infty$ the larger g is substituted by μ/λ . The smaller one turns also into μ/λ if it is larger than that value. Equation (24), in particular, allows us to estimate the scale of Y with which the asymptotic behaviour, Eqs. (16) and (17), is approached. Indeed the characteristic rapidity, call it $\tilde{Y}$, is given by

$$\frac{\mu}{\lambda} \sim g_{\min} e^{\mu \tilde{Y}}$$

where $g_{\min}$ is the smaller of g_1 and g_2 .

VII. -

Let us now discuss the physical picture emerging from our results.

We have obtained the minimal action as function of g_1, g_2 and Y where g_1 and g_2 still depend on the impact parameter $\vec{b}$ over which we should integrate. We rewrite Eq. (6)

$$\log S(Y, B) = \int d^2b A_{\min}(Y, g_1(\vec{b}), g_2(\vec{B}-\vec{b})) \quad (25)$$

where we used $A_{\min}(Y, 0, 0) = 0$.

Let us discuss the hadron-hadron scattering by considering a Gaussian parametrization - with radius a - of the hadron impact parameter distribution, i.e.,

$$g_i(\vec{b}) = g_0 e^{-b^2/a^2} \quad (26)$$

Let us discuss different kinematical regions :

$$(A) \quad B^2 \gg 2 a^2 \left(\mu Y + \log \frac{1}{\mu} g_0 \right) \quad (27)$$

In the integral over $\vec{b}$ let us separate the region in which $|\vec{b}| \sim |\vec{B}|/2$ from the rest. In the latter region, one of the g is very small and we can use the result (17) for $R_{\min}$. In the second region, both g_1 and g_2 are very small and we find ourselves in the perturbative case of Eq. (23). By performing explicitly the $\vec{b}$ integration, one sees that the last region is the one that gives the leading contribution (leading power of $s = e^Y$) and

$$\log S \simeq -g_0^2 e^{\mu Y - B^2/2a^2} = -g_0^2 s^{\mu} e^{-B^2/2a^2} \quad (28)$$

$$(B) \quad B^2 \leq 2 a^2 \left(\mu Y + \log \frac{1}{\mu} g_0 \right) \quad (29)$$

$$i) \quad g_0 < \mu/\lambda \quad (30)$$

The result of Eq. (17) is the relevant one for this case. Again the region $b \sim B/2$ is seen to dominate the integral of Eq. (25) and the result is

$$\log S = -g_0 \frac{\mu}{\lambda} e^{-B^2/4a^2} \quad (31)$$

For $\vec{B}$ fixed and increasing Y [always in the region defined by Eq. (29)], this asymptotic result is reached from below, as discussed previously.

Finally, for

$$ii) \quad g_0 > \frac{\mu}{\lambda} \quad (32)$$

the result (17) is valid for

$$B^2 > 4a^2 \log\left(\frac{1}{\mu} g_0\right) \equiv 4\bar{b}^2 \quad (33)$$

while for $B^2 < 4\bar{b}^2$ there is a region of b defined by

$$\left(\vec{b} + \frac{\vec{B}}{2}\right)^2 < \bar{b}^2 \quad ; \quad \left(\vec{b} - \frac{\vec{B}}{2}\right)^2 < \bar{b}^2$$

in which both g_1 and g_2 are larger than μ/λ and therefore the result of Eq. (15) should be used. The contribution of that region is proportional to the overlap of two discs of radius $\bar{b}$ and with centres separated by $\vec{B}$, i.e.,

$$\log S = - \frac{\mu^2}{\lambda^2} \left[\arcsin \sqrt{1 - \frac{B^2}{4\bar{b}^2}} - \frac{B}{2a} \sqrt{1 - \frac{B^2}{4\bar{b}^2}} \right] \quad (34)$$

and it is the dominant one if $\bar{b}^2/a^2 \gg 1$ and $B \ll \bar{b}$. And, as discussed before, this result is approached from above for increasing Y .

The resulting picture is therefore a grey disc of radius increasing from $\sqrt{2}a$ at low Y to an asymptotic value $2a$.

For $g_0 < \mu/\lambda$, the asymptote is reached from below for all values of B . The increase towards the asymptote starts with the power law s^μ to slow down when the asymptotic value is approached. The larger B is, the larger is the rapidity needed to approach the asymptote.

For $g_0 > \mu/\lambda$, the pattern discussed before is still valid for large B [Eq. (33)]. For small B we find a flatter asymptotic distribution in B which is reached from above, i.e., which decreases for increasing rapidity.

To have a rough estimate of parameters, let us notice that present ISR data on pp scattering show little s dependence at $B \sim 0$. This would suggest $g_0 \sim \mu/\lambda$. The impact parameters around $B \sim 1F$, where, at ISR energies the perturbative approach is certainly applicable, increase with energy

with a power growth compatible with $\mu \sim 0.1$. These results are - order of magnitude-wise - compatible with $g_0 \lambda \sim 0.05$ obtained in Section II. Furthermore, from the actual value $S(B \sim 0) \sim 1/4$, one obtains the estimate $\mu/\lambda \sim 1.5$.

To summarize, our Pomeron which starts with intercept larger than one (by $\mu \sim 0.1$) converts itself with increasing Y into a constant grey distribution. This is due to the Pomeron iterations calculated here, which are of an absorptive nature.

This can occur even before the onset of important loop renormalization effects thanks to the smallness of λ .

VIII. -

Let us now turn our attention to the inclusive distributions. Following the usual prescriptions ⁷⁾, the only tree diagrams that contribute to the one-particle spectrum are those which can be separated into two disconnected tree diagrams by cutting an intermediate Pomeron line (cf. Fig. 9). This implies the joining of two one-sided tree diagrams of the type calculated in Ref. 6). Asymptotically and in the central region

$$\frac{d\sigma}{dy} \sim \gamma \left(\frac{\mu}{\lambda}\right)^2 ; \quad \sigma_T \bar{n} \approx \gamma \frac{\mu^2}{\lambda^2} \log s$$

where γ represents the coupling of the observed particle in Fig. 9.

For the two-particle spectrum the two contributions of Fig. 10 should be considered. Figure 10a gives a bounded $d^2\sigma/dy_1 dy_2$, and a contribution to $\langle n^2 \rangle$ which is limited by $c \log^2 s$. Figure 10b, on the contrary, behaves like $\gamma^2 (\mu^2/\lambda^2) e^{\mu(Y_1 - Y_2)}$, and therefore provides the dominant term in the correlation function $f_2(s) \simeq \gamma^2 (\mu^2/\lambda^2) s^\mu$.

Analogously, $f_n(s) \simeq (\mu^2/\lambda^2) \gamma^n s^{(n-1)\mu}$. These features have some resemblance with properties of the eikonal : there, however, f_n behaves like $s^{n\mu}$, and the multiplicity is power behaved.

IX. -

As noted by Schwimmer⁶⁾, the nuclear scattering can be a relevant field of applications of these tree Pomeron diagrams. Indeed, the value of λ is always small but G is expected to increase as $A^{1/3}$. Therefore for large A it could seem even more reasonable to neglect the loop corrections (of order λ^2) - at least up to some energy - considering, however, the expansion $G\lambda$ to all orders. On the other hand, the assumption of uncorrelated sources can be more questionable, at least in the nucleus fragmentation region.

If nevertheless^{*)} we simply consider the nucleus as an uncorrelated source (of Pomerons) of strength and radius increasing as $A^{1/3}$, i.e.,

$$G = g_0 A^{1/3} \theta(a^2 A^{2/3} - \vec{b}^2)$$

we find that for A sufficiently large the relevant result is that of Eq. (15) - with the asymptotic limit approached from above - for all b for which both G_1 and G_2 are not zero. We would therefore find for nucleus-nucleus scattering the following picture.

At asymptotic energies ($Y \gg (1/\mu) \log(\lambda/\mu) g_0 A^{1/3}$) the nucleus-nucleus scattering is given by the geometrical overlapping of two grey discs of radius $aA^{1/3}$ and $aA_2^{1/3}$

$$\log S(B) \sim - \frac{\mu^2}{2\lambda^2} \int d^2\vec{b} \theta(a^2 A_1^{2/3} - \vec{b}^2) \theta(a^2 A_2^{2/3} - (\vec{B} - \vec{b})^2)$$

When Y decreases the disc becomes increasingly blacker. If at large but fixed Y we increase A "indefinitely", we find a situation analogous to that discussed in Fig. 6 where p_1 and q_2 are so large that "time" Y is not sufficient to approach the fixed point. In the limit of very large A the action approaches $p_1 q_2$ if we are in the overlapping region of the two discs, i.e.,

*) A more refined analysis of the nuclear case is under preparation by L. Bertocchi and one of us (R.J.).

$$\log S \simeq - A_1^{1/3} A_2^{1/3} g_0^2 \int d^2b \, \theta(a^2 A_1^{2/3} - b^2) \theta(a^2 A_2^{2/3} - (B-b)^2)$$

which is the usual eikonal. The pattern of approach to the asymptotic greyness is of the type

$$g_0 A^{1/3} e^{\mu Y} \left(1 + g_0 A^{1/3} \frac{\mu}{\mu} (e^{\mu Y} - 1) \right)^{-1}$$

suggested by Schwimmer ⁶⁾.

For the one-particle inclusive distribution the two fragmentation regions proportional to $A_1^{1/3}$ and $A_2^{1/3}$ are joined when $Y \rightarrow \infty$ by an A independent plateau. When Y is small the two fragmentation regions get in contact with each other, and $\bar{n}$ is proportional to $A^{1/3}$, as it should happen, since in this limit we recover the eikonal. In the $Y \rightarrow \infty$ limit the correlations are power behaved and A independent as in the case of particle-particle scattering.

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FIGURE CAPTIONS

- Figure 1 Multi-Pomeron exchange diagrams summed in the present paper.
- Figure 2 Orbits in the p, q plane representing three types of solutions of Eq. (7). The points at infinity are, in one solution, identified by the same letter.
- Figure 3 A solution with $E > 0$.
- Figure 4 The two solutions of class iii) involving no extra cycles.
- Figure 5 Solution with $E > 0$, but $g_2 < \mu/\lambda$.
- Figure 6 A class of solutions with g_1 and $g_2 > \mu/\lambda$ and different values of Y (and consequently of E).
- Figure 7 The explicit dependence of $Y(E)$ on E for the solutions of Fig. 6.
- Figure 8 Solution for small g_1, g_2, Y .
- Figure 9 The diagram that determines the inclusive one-particle production.
- Figure 10 The two possible contributions to the two-particle inclusive distribution.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

(a)

(b)

FIG. 10