A NULL TECHNIQUE FOR CALIBRATING POLARIZATION VALUES

IN π p SCATTERING

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ABSTRACT

The purpose of this investigation is to find regions of energy and angle for π p elastic scattering in which the proton polarization P reaches its maximum value of unity. For incident pion beam energies below 900 MeV, five such regions exist in π^+ p and six such regions exist in π^- p, as shown by simple graphical techniques. This suggests that crude polarized target experiments measuring the Wolfenstein parameters R and A will precisely determine the proton polarization because of the quadratic relation $P^2 + R^2 + A^2 = 1$.

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1. INTRODUCTION

A general null technique to determine polarization standards precisely, has been applied at low energies¹⁾. In the present report we show that the proton polarization P is unity in the π^+p system near $T_\pi = 484$ MeV and $\theta_{cm} = 115^\circ$. Also, we show that measurements of the Wolfenstein parameters²⁾ R and A , with an accuracy of ± 0.07 , will lead to an absolute determination of P with an accuracy of ± 0.005 . This accuracy can be used to calibrate polarized proton targets with greater precision than previously obtained. We develop and discuss a new graphical technique for finding regions of energy and angle in which the polarization becomes unity for spin $0 + \frac{1}{2} \rightarrow 0 + \frac{1}{2}$ interaction. Consequently, in addition to the region of energy and angle listed above, four more regions where $|P| = 1$ for π^+p scattering and six regions where $|P| = 1$ for π^-p scattering are tabulated for $0 \leq T_\pi \leq 900$ MeV.

2. THE NULL TECHNIQUE

The null technique for determining polarization values was discussed and applied in Ref. 1. It utilizes the relation $P^2 + R^2 + A^2 = 1$ which holds for any spin $0 + \frac{1}{2} \rightarrow 0 + \frac{1}{2}$ interaction, where P is the proton polarization (or analysing power), and where R and A are "triple scattering parameters" defined by Wolfenstein²⁾ [more generally designated polarization transfer coefficients³⁾]. In πp elastic scattering, the Wolfenstein parameters are measured by appropriate combinations of proton target spin orientations in the reaction plane and the subsequent measurement of polarizations for the recoil proton. If R and A are measured to be zero with an accuracy of e.g. ± 0.07 , one can deduce that $|P|$ is unity with a precision of $\pm (0.07)^2 \approx \pm 0.005$. The reason for the small uncertainty on determining $|P|$ in this manner is the quadratic relation that exists between P , R , and A .

A graphical technique for finding regions in which $|P| = 1$ so that the null technique can be applied is the subject of the next section.

3. A GRAPHICAL TECHNIQUE

The graphical technique described here is a variation of the idea first suggested by Plattner and Bacher⁴⁾. Simply stated, we form the complex number $Z(E, \theta) = R + iA$, so that $P^2 + R^2 + A^2 = P^2 + |Z|^2 = 1$. If one can show that $Z(E, \theta) = 0$ for a particular energy and angle, then one has found a point where $|P| = 1$.

Showing that $Z(E, \theta) = 0$ consists of plotting Z trajectories for specific energies on the complex plane using angle as a dependent variable. For all energies, $Z(E, 0) = +1$ and $Z(E, \pi) = -1$, as will be shown in Section 4. Observing that all Z trajectories emanate from a single point and terminate at a single point, one looks for two trajectories passing on either side of the origin. Since Z is constructed with physical observables that vary smoothly with energy and angle, one can deduce that for some intermediate energy a trajectory exists which passes through the origin for at least one angle, i.e., $Z(E, \theta) = 0$.

As an example, we note that as the bombarding energy approaches zero, R approaches $\cos \theta$ and A approaches $-\sin \theta$ as a consequence of geometry and the fact that only s-waves contribute to the scattering amplitude. Thus, the $E = 0$ trajectory is shown in Fig. 1 as a semicircle in the lower half of the complex plane. If, for some $E_1 \neq 0$, we find a trajectory that passes on the other side of the origin as illustrated in Fig. 1 by the dashed trajectory, we can conclude that for some E' , where $0 < E' < E_1$, there exists a trajectory that passes through the origin. At that point, $R = A = 0$, and $|P| = 1$.

The trajectories are plotted by generating values of R and A as a function of angle using results of phase-shift analysis. Showing *existence* of $|P| = 1$ points is insensitive to a particular choice of phase shifts in the low energy region. However, extrapolating to an exact energy and angle where $|P| = 1$ depends on the particular phase shifts. For this reason, the graphical technique is used only to indicate energies and angles where experimental values of R and A are to be determined.

In concluding this section, we point out the convenience of making Z trajectories which utilize as dependent variables angles that range from 0° to 180° . Since R and A are defined as laboratory observables, the

scattering angles will have this full range only when the spin $\frac{1}{2}$ particle is bombarding a heavier spin 0 target particle [e.g., the ${}^4\text{He}(\vec{p}, \vec{p}){}^4\text{He}$ interactions reported in Ref. 1]. To apply the graphical technique to the present problem, we find analogous forms of R and A which are functions of the centre-of-mass angle so that phase-shift calculations can be related to Z trajectories. This is the object of the next section.

4. THE COORDINATE SYSTEM

Polarization transfer experiments require the utilization of a set of two coordinate systems -- one to describe polarization in the initial channel and one to describe polarization in the final channel. For present purposes, we must describe two *sets* of coordinate systems -- one to relate scattering amplitude values to laboratory observables, and one to relate scattering amplitude values to Z trajectories.

We begin by writing down the scattering matrix for an interaction with spins $0 + \frac{1}{2} \rightarrow 0 + \frac{1}{2}$ in the form

$$M = g + ih \vec{\sigma} \cdot \hat{n} , \quad (1)$$

where the σ 's are the Pauli spin matrices, $\hat{n}$ is the normal to the reaction plane, and g and h are the spin non-flip and spin flip amplitudes, respectively, which are functions of the energy E and the centre-of-mass scattering angle θ . Four observables expressed in a single coordinate system are:

$$\begin{aligned} I_0 &= \frac{1}{2} \text{Tr}(MM^\dagger) = |g|^2 + |h|^2 \\ I_0 P &= \frac{1}{2} \text{Tr}(MM^\dagger \sigma_y) = 2 \text{Im}(gh^*) \\ I_0 K_X^X &= \frac{1}{2} \text{Tr}(M \sigma_X M^\dagger \sigma_X) = |g|^2 - |h|^2 \\ I_0 K_Z^X &= \frac{1}{2} \text{Tr}(M \sigma_Z M^\dagger \sigma_X) = -2 \text{Re}(gh^*) , \end{aligned} \quad (2)$$

where I_0 is the differential cross-section for unpolarized protons, P is the polarization parameter, K_X^X and K_Z^X are polarization transfer coefficients, the y-axis is oriented along $\hat{n}$, and the x- and z-axes are as yet unspecified. M contains three independent numbers which are $|g|$, $|h|$, and the relative phase between them. Consequently, a quadratic relation exists among the observables namely, $P^2 + (K_X^X)^2 + (K_Z^X)^2 = 1$, which can be easily verified from Eqs. 2. By parametrizing g and h in terms of phase shifts in the usual manner, we can find I_0 , P, K_X^X and K_Z^X in terms of phase shifts.

The most common set of coordinate systems used to describe polarized target experiments in the laboratory consists of one in the incident channel with the z-axis aligned parallel with the incident (pion) momentum $\hbar\vec{k}_i$, and one in the final channel with a z'-axis aligned parallel with the outgoing momentum $\hbar\vec{k}_R$ of the recoil proton (see Fig. 2). (All coordinate systems described here are right-handed coordinate systems with the y-axis aligned parallel with $\vec{k}_i \times \vec{k}_0$, where $\hbar\vec{k}_0$ is the outgoing momentum of the scattered pion.) This set of coordinate systems indicate a simple geometrical relationship between the Wolfenstein parameters for experiments with polarized targets and the K's as described above:

$$\begin{aligned} R_T &\equiv K_X^{x'} = K_X^x \cos \theta_R - K_Z^x \sin \theta_R \\ A_T &\equiv K_Z^{x'} = K_X^x \sin \theta_R + K_Z^x \cos \theta_R \end{aligned} \quad (3)$$

where θ_R is the laboratory recoil angle of the proton between $\vec{k}_i$ and $\vec{k}_R$ and where the subscript "T" is added to distinguish these polarized target parameters from R and A in Section 3.

The other set of coordinate systems describes polarization transfer experiments in the centre-of-mass system with helicity frames of reference for the proton before and after the interaction (see Fig. 3). The different systems in Fig. 2 and Fig. 3 are distinguished by an appropriate number of primes. Note that since $x''' = -x$ and $z''' = -z$, we have $K_{x'''}^{x'''} \equiv K_X^x$ and $K_{z'''}^{x'''} \equiv K_Z^x$, and it will be convenient to drop the triple prime notation on these K's. We define r and a which are analagous to R and A, except that they are defined in the centre-of-mass coordinate system and are functions of the centre-of-mass scattering angle which ranges, of course, from 0° to 180° . That is,

$$\begin{aligned} r &\equiv K_{x'''}^{x'''} = K_X^x \cos \theta + K_Z^x \sin \theta \\ a &\equiv K_{z'''}^{x'''} = -K_X^x \sin \theta + K_Z^x \cos \theta \end{aligned} \quad (4)$$

The form of Eqs. 3 and Eqs. 4 preserves the quadratic nature mentioned earlier, namely

$$P^2 + (K_X^x)^2 + (K_Z^x)^2 = P^2 + R_T^2 + A_T^2 = P^2 + r^2 + a^2 = 1 \quad (5)$$

Rotation invariance and phase-shift analysis require that the spin flip amplitude vanishes for $\theta = 0^\circ$, $\theta = 180^\circ$, and for all θ when the bombarding energy approaches zero. Whenever the spin flip amplitude vanishes, $K_X^X = 1$, $K_Z^X = 0$, $r = \cos \theta$, and $a = -\sin \theta$. Thus, r and a are exactly analogous to R and A as discussed in Section 3, and are suitable for constructing Z trajectories. Now, we define $z = r + ia$. In particular, we have $z(E, 0) = 1$ and $z(E, \pi) = -1$ for any value of the energy E .

Concluding this section, the spin flip and spin non-flip amplitudes are calculated from phase shifts and related to I_0 , P , K_X^X and K_Z^X according to Eqs. 2. The Wolfenstein parameters R and A are defined in the framework of commonly used polarized target laboratory coordinate systems in Eqs. 3 and labelled R_T and A_T . The graphical technique as described in Section 3 can be applied to polarized target experiments by defining parameters analogous to R and A in polarized beam experiments. These are the polarization transfer coefficients r and a , which are defined in Eqs. 4, and which are explicit functions of the centre-of-mass scattering angle θ . It was shown in Eq. 5 that $P^2 + r^2 + a^2 = 1$, so that one can construct $z = r + ia$ and search for trajectories which pass through the origin. The results of these searches for $|P| = 1$ regions in π^+p and π^-p scattering are shown in the next section.

5. RESULTS

The polarization transfer coefficients r and a were computed as defined in Eqs. 4 and Eqs. 2, using phase shifts for g and h that were taken from Ref. 5. Trajectories for $z = r + ia$ were plotted. Each time trajectories for two consecutive energies indicated that $z = 0$ for some intermediate energy, a linear interpolation of the phase shifts was made to determine that energy. For example, Fig. 4 indicates that $z = 0$ at least once somewhere between $E_{cm} = 1416$ MeV and $E_{cm} = 1444$ MeV. Figure 5 shows the interpolation process between those two energies which determine that $z = 0$ for $E_{cm} = 1438$ MeV and $\cos \theta = -0.42$. All regions for which $z = 0$ ($|P| = 1$) that were found between incident pion kinetic energies of $T_{lab} = 0$ and $T_{lab} = 900$ MeV are given in Table 1 for π^+p and Table 2 for π^-p . The precision is of the order of 1 to 2 MeV in E_{cm} and about 0.01 in $\cos \theta$. A larger uncertainty is due to the phase shifts themselves

and to the linear interpolation. The accuracy is sufficient to use Tables 1 and 2 as guides to regions in which experimental values of R_T and A_T should be determined. Applications in this manner were discussed in Section 2 as the null technique.

6. CONCLUSION

Polarization values in π^+p and π^-p elastic scattering reach their maximum value of unity many times for $T_\pi < 900$ MeV. A graphical technique is developed that locates regions of energy and angle in which this occurs. The results are given in Tables 1 and 2.

The graphical technique is not considered precise enough to be used as a polarization standard. However, it is sufficiently accurate to indicate regions in which values of R_T and A_T are likely to vanish simultaneously. At such points very precise experimental determinations of $|P| = \sqrt{1 - R_T^2 - A_T^2}$ can be made. Used in this way, the null technique can yield polarization values without requiring an accurately calibrated polarization analyser or large counting statistics.

The authors have learned of a LAMPF proposal to measure R_T and A_T at $T_\pi = 410, 491, \text{ and } 533$ MeV for both π^+p and π^-p scattering⁶⁾. The centre-of-mass angular range will span 85° - 160° for π^- and 105° - 160° for π^+ . The counting statistics on R_T and A_T will be 8%. At the suggestion of the authors, the $T_\pi = 483.5$ MeV, $\theta = 115^\circ$ point of Table 1 has been incorporated into the LAMPF proposal and the experiment is in progress.

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Table 1

Points at which $|P| = 1$ in π^+p scattering

T_{lab} (MeV)	P_{lab} (MeV/c)	E_{cm} (MeV)	$\cos \theta_{\text{cm}}$	θ_{cm}
66.8	152.1	1134.5	0.95	18°
483.5	607.2	1438.4	-0.42	115°
596.2	722.4	1510.1	-0.43	115°
809.5	938.8	1637.3	-0.28	106°
892.5	1022.6	1684.2	-0.75	139°

Table 2

Points at which $|P| = 1$ in π^-p scattering

T_{lab} (MeV)	P_{lab} (MeV/c)	E_{cm} (MeV)	$\cos \theta_{\text{cm}}$	θ_{cm}
389.9	510.7	1376.0	0.72	44°
496.0	620.1	1446.5	-0.32	109°
502.1	626.3	1450.3	0.03	88°
508.2	632.6	1454.5	0.53	58°
591.1	717.3	1507.0	-0.22	103°
856.9	986.6	1664.3	0.25	76°

Figure captions

- Fig. 1 : Plots of $Z(E, \theta) = R + iA$ where R and A are Wolfenstein parameters. For some $E' < E_1$, there exists a point $Z(E', \theta') = 0$, where $|P| = 1$.
- Fig. 2 : Laboratory coordinate systems for πp scattering.
- Fig. 3 : Centre-of-mass coordinate systems for πp scattering.
- Fig. 4 : Trajectories of $z = r + ia$ for $E_{\text{cm}} = 1416 \text{ MeV}$ and $E_{\text{cm}} = 1444 \text{ MeV}$ in $\pi^+ p$ elastic scattering.
- Fig. 5 : Interpolation of z trajectories to find $z = 0$ at $E_{\text{cm}} = 1438 \text{ MeV}$ and $\cos \theta = -0.42$. Numbers in parenthesis indicate $\cos \theta$, where θ is the pion centre-of-mass scattering angle.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

$\pi^+ p$

KEY:

1416 ●—●

1438 +—+

1444 ×—×

Fig. 5