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MULTIPLE EXCHANGE AND HIGH ENERGY FIXED ANGLE SCATTERING

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A B S T R A C T

Contrary to a recent claim, made by Fishbane and Simmons, we prove that the "eikonal Schiff approximation" gives the correct leading asymptotic behaviour for fermion-fermion elastic scattering in vector gluon field theories, in the high energy fixed angle limit.

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For fermion-fermion elastic scattering with Abelian vector gluon exchanges, some progress has recently been made in understanding the high energy fixed angle behaviour. Cardy ¹⁾ conjectured that, for generalized ladder exchanges, a suitable "eikonal" ansatz should give the correct leading asymptotic behaviour. In subsequent work ²⁾, the assumptions made by Cardy on what should be the dominant large momentum flow were shown to be correct, order by order. A consequence of this "eikonalization" is that important cancellations take place between the leading contributions of ordinary ladder graphs and those of crossed ladder graphs. As a result of such cancellations, in the leading logarithm approximation, the dominant contribution, to the sum of all graphs of a given order in the coupling constant g , is due to vertex corrections to one-gluon exchange. In the leading logarithm summation (LLS) to all orders in g , one then expects a behaviour, for the elastic scattering amplitude $\mathcal{M}(s, \theta)$ as $s \rightarrow \infty$ with fixed centre-of-mass scattering angle θ ,

$$\mathcal{M}(s, \theta) \approx f(\theta, g) \mathcal{M}_{\text{Born}}(s, \theta) G^2(s) s^{h(\theta, g)}, \quad (1)$$

with $f(\theta, g)$ and $h(\theta, g)$ independent of s and with $G(s)$ the elastic form factor in the LLS,

$$G(s) \approx \text{const} \times \exp \left[-\frac{g^2}{16\pi^2} \log^2 s \right]. \quad (2)$$

Equations (1) and (2) identify an important mechanism for fixed angle high energy elastic scattering, i.e., the particle tends to acquire the large momentum transfer through a single "hard" gluon exchange, the remaining exchanges being "soft". A mechanism of this kind had been advocated by Schiff ³⁾ as the one leading to an eikonal approximation appropriate for high energy, fixed-angle scattering.

In a recent paper, Fishbane and Simmons ⁴⁾ calculated the leading high energy fixed-angle behaviour of the elastic scattering amplitude in order g^4 . They concluded that the Schiff eikonal approximation gives the wrong answer, contrary to the results of Refs. 1), 2). Our purpose is to show that the arguments of Ref. 4) are incomplete, and that their result confirms the one of Refs. 1), 2).

The result of the calculations of Ref. 4) is as follows : as $s \rightarrow \infty$ with s/t fixed, the amplitude $\mathcal{M}_2(s, t)$ corresponding to the sum of the two graphs shown in Fig. 1 behaves as follows ^{*)} :

$$\mathcal{M}_2(s, t) = - \frac{i g^4}{8\pi^4} \frac{1}{t} \log^2 \frac{s}{m^2} \sum_{i=1}^3 A_i + O(g^4 \log \frac{s}{m^2}), \quad (3)$$

where

$$A_1 \equiv \bar{u}(p_3) \gamma^2 \gamma^5 u(p_1) \bar{u}(p_4) \gamma_2 \gamma^5 u(p_2), \quad (4)$$

$$A_2 \equiv \bar{u}(p_3) \not{x}_2 u(p_1) \bar{u}(p_4) \not{x}_1 u(p_2) \frac{t}{su}, \quad (5)$$

$$A_3 \equiv \bar{u}(p_3) \not{x}_2 \gamma^5 u(p_1) \bar{u}(p_4) \not{x}_1 \gamma^5 u(p_2) \frac{s-u}{su}, \quad (6)$$

with

$$s \equiv (p_1 + p_2)^2, \quad t \equiv (p_1 - p_3)^2, \quad u \equiv (p_1 - p_4)^2.$$

On the other hand, the graphs of Fig. 2 give

$$\mathcal{M}_3(s, t) = - \frac{g^2}{2\pi} \log^2 \frac{s}{m^2} \mathcal{M}_{\text{Born}}(s, t) + O(g^4 \cdot 1), \quad (7)$$

where

$$\begin{aligned} \mathcal{M}_{\text{Born}}(s, t) &= i \frac{g^2}{4\pi} \frac{1}{t} \bar{u}(p_3) \gamma^2 u(p_1) \bar{u}(p_4) \gamma_2 u(p_2) \\ &\equiv i(g^2/4\pi t) A_4. \end{aligned} \quad (8)$$

^{*)} There is a difference of an over-all factor of two in the right-hand side of Eq. (3) relative to that of Eq. (2.18) of Ref. 4). This factor is due to the neglect, in Ref. 4), of some paths of dominant momentum flow. With the Feynman parameters α_i , $\sum_{i=1}^4 \alpha_i = 1$, labelled as in Fig. 1, in Ref. 4) only the paths corresponding to $\alpha_4 \approx 1, \alpha_1 \approx 0$ are kept, while equal leading contributions arise from the paths corresponding to $\alpha_1 \approx 1, \alpha_4 \approx 0$.

In order that (1) be correct to order g^4 in the leading logarithm approximation, it is clearly necessary and sufficient that, as $s \rightarrow \infty$ with s/t fixed,

$$\sum_{\text{spins}} \left| \sum_{i=1}^3 A_i \right|^2 / \sum_{\text{spins}} |A_4|^2 \rightarrow 0. \quad (9)$$

We will now show that Eq. (9) holds, the limit being approached at least as fast as $1/s$. The proof involves a straightforward calculation based on well-known trace theorems [see, e.g., Ref 5), pp. 103-106]. One obtains

$$\sum_{\text{spins}} A_1 A_1^* = 8(s^2 + u^2) + O(s), \quad (10a)$$

$$\sum_{\text{spins}} A_2 A_2^* = 4t^2 + O(s), \quad (10b)$$

$$\sum_{\text{spins}} A_3 A_3^* = 4(s-u)^2 + O(s), \quad (10c)$$

$$\sum_{\text{spins}} (A_1 A_2^* + A_1^* A_2) = -8(s+u)^2 + O(s), \quad (10d)$$

$$\sum_{\text{spins}} (A_1 A_3^* + A_1^* A_3) = -8(s-u)^2 + O(s), \quad (10e)$$

$$\sum_{\text{spins}} (A_2 A_3^* + A_2^* A_3) = O(s), \quad (10f)$$

$$\sum_{\text{spins}} A_4 A_4^* = \sum_{\text{spins}} A_1 A_1^* = 8(s^2 + u^2) + O(s). \quad (11)$$

Upon adding Eqs. (10a-f), the result (9) immediately follows. Therefore, the "leading" contribution calculated by Fishbane and Simmons, see Eq. (3), is actually non-leading relative to (7) by at least one power of $s^{-\frac{1}{2}}$. This result is obtained much more transparently if the techniques of Ref. 2) are used to perform the loop integrals, before any reordering of γ matrices

takes place. The resulting γ algebra is then trivial for these low order diagrams, and greatly simplifies the discussion of the higher order diagrams. Similar considerations apply to the calculation, done in Ref. 4), to order g^4 in a non-Abelian vector gluon theory [see also Ref. 6].

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FIGURE CAPTIONS

Figure 1 Ladder and crossed ladder graphs in order g^4 . The α_i 's label the Feynman parameters associated with the various internal lines.

Figure 2 Vertex corrections to the Born graph.

Fig .1

Fig. 2