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HYDROGEN ATOM ON NULL-PLANE AND MELOSH TRANSFORMATION

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The null-plane dynamics of hydrogen-like atoms is studied in approximations depending on c , the velocity of light, being large. Neglecting terms in the Hamiltonian of order c^{-3} (relative to electron rest energy) a symmetry $SU(2)_W$ appears which is analogous to the $SU(6)_W$ of hadron classification. This symmetry, if accurate, would dictate zero ground state magnetic moment. The symmetry is broken by terms of third order, which can, however, be transformed away by the appropriate approximation to the Melosh transformation. There then emerges a better symmetry, $SU(2)_M$, broken only at fourth order. The ground state magnetic moment acquires its usual non-relativistic value. The symmetry $SU(2)_M$ corresponds to a subgroup of a symmetry $[U(2) \times U(2)]_{FW}$ which appears in the old Foldy-Wouthuysen approach when spin-orbit coupling is neglected. As well as "current" and "constituent" pictures, "classification" pictures are distinguished; it is to one of the latter that the Melosh transformation transforms.

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We consider here the null plane dynamics of hydrogen-like atoms. It is hoped thereby to throw some light on the Melosh transformation ¹⁾. The arguments of Melosh (we have in mind his paper ¹⁾ rather than his thesis ²⁾) ^{*}) were developed only for free particle wave packets. The next most simple case is that of interaction with an external field. That is what we consider here, the nucleus being treated as a fixed force centre.

The starting point is the ordinary Dirac equation

$$p_0 \psi = (\beta mc^2 + c \vec{\alpha} \cdot \vec{p} + v) \psi \quad (1)$$

where ψ is a function of (t, x, y, z) and

$$p_0 \equiv i\hbar \frac{\partial}{\partial t} \quad \vec{p} \equiv -i\hbar \frac{\partial}{\partial \vec{x}} \quad (2)$$

In what follows we will put $\hbar = c = m = 1$. But we also wish at certain points to regard c as large and to work to limited accuracy in c^{-1} . It will then be appropriate to regard p and v as small quantities of first and second order, respectively. We take v to be a Coulomb-type potential, the unit matrix with respect to Dirac indices. Similar results are obtained with a Lorentz-scalar-type, containing a factor β .

To go over to null-plane dynamics, we introduce

$$\hat{\psi}(t, x, y, z) = \psi(t-z, x, y, z) \quad (3)$$

so that

$$p_0 \hat{\psi} = (\beta + \vec{\alpha}_1 \cdot \vec{p}_1 + \alpha_3 (\hbar - p_0) + v) \hat{\psi} \quad (4)$$

^{*}) The various arguments are outlined in Ref. 3). An argument for some such transformation, going beyond free particles, is outlined in Ref. 4).

where $\perp$ indicates restriction to x, y directions, and we have followed convention in using η , rather than p_3 , to denote z differentiation on the null-plane :

$$-i \frac{\partial \hat{\Psi}}{\partial z} = \eta \hat{\Psi} \quad (5)$$

We will later use $p_3 \hat{\Psi}$ rather for the combination

$$(\eta - p_0) \hat{\Psi} \equiv p_3 \hat{\Psi} \quad (6)$$

Consider now specifically a good representation of the Dirac matrices in which

$$\alpha_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \quad \beta = \begin{pmatrix} & \sigma_3 \\ \sigma_3 & \end{pmatrix} \quad \vec{\alpha}_1 = \begin{pmatrix} & -\vec{\sigma}_1 \\ -\vec{\sigma}_1 & \end{pmatrix} \quad (7)$$

Decomposing $\hat{\Psi}$ into good and bad components

$$\hat{\Psi} = \begin{pmatrix} \hat{\Psi}_g \\ \hat{\Psi}_b \end{pmatrix} \quad (8)$$

the Dirac equation (1) becomes

$$2p_0 \hat{\Psi}_g = (\eta + v) \hat{\Psi}_g + (\sigma_3 - \vec{\sigma}_1 \cdot \vec{p}_1) \hat{\Psi}_b \quad (9)$$

$$\eta \hat{\Psi}_b = v \hat{\Psi}_b + (\sigma_3 - \vec{\sigma}_1 \cdot \vec{p}_1) \hat{\Psi}_g \quad (10)$$

The general solution of the second equation is

$$\hat{\Psi}_b = \eta^{-1} (v \hat{\Psi}_b + (\sigma_3 - \vec{\sigma}_1 \cdot \vec{p}_1) \hat{\Psi}_g) + F(t, x, y) \quad (11)$$

where F is an arbitrary function independent of z , and η^{-1} is an integral operator with respect to z , with kernel

$$(z | \eta^{-1} | z') = \frac{1}{2} \text{sign}(z - z') \quad (12)$$

Assuming that all components of $\hat{\psi}$ vanish for z large, $F=0$. Then

$$\hat{\psi}_b = (\eta - v)^{-1} (\sigma_3 - \vec{\sigma}_1 \cdot \vec{p}_1) \hat{\psi}_g \quad (13)$$

where

$$(\eta - v)^{-1} = \eta^{-1} + \eta^{-1} v \eta^{-1} + \dots \quad (14)$$

whence

$$p_0 \hat{\psi}_g = \left[\frac{1}{2} \eta + \frac{1}{2} v + \frac{1}{2} (\sigma_3 - \vec{\sigma}_1 \cdot \vec{p}_1) (\eta - v)^{-1} (\sigma_3 - \vec{\sigma}_1 \cdot \vec{p}_1) \right] \hat{\psi}_g \quad (15)$$

This, according to whether it is the first- or second-quantized theories under consideration, is the null-plane Schrödinger equation for the wave function $\hat{\psi}_g$ or the null-plane equation of motion for the Heisenberg operator $\hat{\psi}_g$. We rewrite it

$$p_0 \hat{\psi}_g = \hat{h} \hat{\psi}_g \quad (16)$$

where

$$\begin{aligned} \hat{h} = & \frac{1}{2} (\eta + \eta^{-1}) \\ & + \frac{1}{2} \vec{p}_1^2 \eta^{-1} + \frac{1}{2} v + \frac{1}{2} \eta^{-1} v \eta^{-1} \\ & + \frac{1}{2} \eta^{-1} [\sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1, v] \eta^{-1} \\ & + \frac{1}{2} \eta^{-1} \vec{\sigma}_1 \cdot \vec{p}_1 v \vec{\sigma}_1 \cdot \vec{p}_1 \eta^{-1} + \frac{1}{2} \eta^{-1} v \eta^{-1} v \eta^{-1} \\ & + \dots \end{aligned} \quad (17)$$

where the terms written explicitly include all those of less than fifth order in small quantities. Note that for such considerations η ($\approx p_0$) has to be counted as containing a part of zero order.

Externally applied electric and magnetic fields can be included by the minimal substitutions in the original Dirac equation

$$\vec{p} \rightarrow \vec{p} - \frac{e}{c} \vec{A} \quad p_0 \rightarrow p_0 - \frac{e}{c} \phi$$

Particularly simple, because they do not change Eq. (10) expressing bad components in terms of good, are potentials such that

$$\vec{A}_1 = 0 \quad \phi = -A_3$$

The corresponding increment in the null-plane Hamiltonian is, without approximation

$$\delta \hat{h} = -e A_3 \quad (18)$$

With

$$A_3 = \mathcal{H}_x \gamma$$

corresponding to a uniform magnetic field $\mathcal{H}_x$ in the x direction and a certain uniform electric field in the y direction,

$$\delta \hat{h} = -m_x \mathcal{H}_x \quad (19)$$

$$m_x = +e\gamma \quad (20)$$

Thus static dipole magnetic moments are given by the appropriate matrix elements of $\vec{e}_y$, evaluated between good components of null-plane wave functions. The fact that (18) implies also an applied electric field does not matter when parity conservation is assumed, so that static electric dipole moments vanish.

When small terms of third and higher order are neglected in (17), i.e., when only the first two lines are retained, $\hat{h}$ does not involve $\vec{\sigma}$. So we have a kind of spin independence. The infinitesimal symmetry operation on $\hat{\psi}_g$ is

$$\delta \hat{\psi}_g = -i \vec{\epsilon} \cdot \vec{\sigma} \hat{\psi}_g \quad (21)$$

where $\vec{\epsilon}$ is an arbitrary constant vector. We denote this symmetry by $SU(2)_W$, for it is the analogue here of the $SU(6)_W$ (of good null-plane charges) of the particle physicists ^{*}). Only in the case

$$\epsilon_x = \epsilon_y = 0$$

is this operation simply related to a rotation, because it is only for rotations about the z axis that passage to the null-plane does not complicate things. The rotation about the z axis remains simply

$$\delta \hat{\psi}_g = -i \omega (x p_y - y p_x + \frac{1}{2} \sigma_3) \hat{\psi}_g \quad (22)$$

In particular for the ground state, of zero orbital angular momentum, the z component of W spin can be identified with the z component of total angular momentum.

The ground state magnetic moment is then given by the W_z spin flip matrix elements of $\mathcal{M}_x$. In the $SU(2)_W$ symmetric approximation this magnetic moment is therefore zero, for $\mathcal{M}_x$ in (20) is an $SU(2)_W$ scalar. This is a perfect echo of the particle physics situation where $SU(6)_W$ symmetry dictates zero magnetic moments for nucleons ⁶⁾. Note, moreover that the null result does not depend directly on the symmetry

^{*}) For reviews and references, see Refs. 3) and 5).

of the Hamiltonian, but only on the assumed transformation properties of the state in question, i.e., on the identification of the " $SU(2)_W$ of good null-plane charges" with the "classification" $SU(2)_W$.

The $SU(2)_W$ symmetry is broken, in (17), at the third order of small quantities. Consider now the Melosh transformed Hamiltonian

$$\hat{h}_M = u \hat{h} u^{-1} \quad (23)$$

$$u = \frac{1 + \sqrt{h^2} - \sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp}{\sqrt{(1 + \sqrt{h^2})^2 + \vec{p}_\perp^2}} \quad (24)$$

$$= 1 - \frac{1}{2} \sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp + \dots \quad (25)$$

where the terms ... are of second and higher order ^{*)}. From (17)

$$\begin{aligned} \hat{h}_M &= \hat{h} - [\frac{1}{2} \sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp, \frac{1}{2} v + \frac{1}{2} h^{-1} v h^{-1}] + o(c^{-4}) \\ &= \hat{h} - \frac{1}{2} h^{-1} [\sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp, \frac{1}{2} h v h + \frac{1}{2} v] h^{-1} + o(c^{-4}) \\ &= \frac{1}{2} (h + h^{-1}) + \frac{1}{2} p_\perp^2 h^{-1} + \frac{1}{2} v + \frac{1}{2} h^{-1} v h^{-1} \\ &\quad + \frac{1}{2} h^{-1} [\sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp, \frac{1}{2} v - \frac{1}{2} h v h] h^{-1} \\ &\quad + o(c^{-4}) \end{aligned} \quad (26)$$

Now

$$h v h = v + \{v(h^2 - 1) + [h, v] h\} \quad (27)$$

^{*)} Note that it is only in these higher order terms that the difference ³⁾ between first and second Melosh transformations appears.

The terms in curly brackets are of higher order, because in zero order $\eta^2 = 1$, and if we had retained $\hbar$, m and c

$$i[h, v] = \frac{\partial v}{\partial z} \rightarrow \frac{\hbar}{mc} \frac{\partial v}{\partial z} \quad (28)$$

Thus the spin dependent term in $\hat{h}_M$, potentially of third order, is actually of higher order. To this approximation there is again an $SU(2)$ symmetry. We denote it by $SU(2)_M$.

Let us remark that to the given approximation the same result obtains for a Lorentz-scalar potential such that instead of (1)

$$p_0 \hat{\Psi} = (\beta (mc^2 + v) + c \vec{\alpha} \cdot \vec{p}) \Psi \quad (1bis)$$

The null plane Hamiltonian is then

$$\begin{aligned} \hat{h} = & \frac{1}{2} (h + h') \\ & + \frac{1}{2} \vec{p}_\perp^2 h' + \frac{1}{2} v h' + \frac{1}{2} h' v \\ & + \frac{1}{2} [h' \sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp, v] \\ & + \frac{1}{2} v h' v \end{aligned} \quad (17bis)$$

After transformation (23)

$$\begin{aligned} \hat{h}_M = & \frac{1}{2} (h + h') + \frac{1}{2} \vec{p}_\perp^2 h' + \frac{1}{2} v h' + \frac{1}{2} h' v \\ & + \frac{1}{4} \{ \sigma_3 \vec{\sigma}_\perp \cdot \vec{p}_\perp, [h', v] \} + o(c^{-4}) \end{aligned} \quad (26bis)$$

The spin dependent term is again of order c^{-4} , one order smaller than in (17bis), because

$$[h', v] = h' [v, h] h' = h' i \frac{\partial v}{\partial z} h'$$

and again we have implicitly an extra factor c^{-1} as in (28).

The transformed magnetic moment operator acquires a spin-dependent term :

$$u m_x u^{-1} = e\gamma + \frac{1}{2} e\sigma_x + \dots$$

where the terms not written are of higher order. This supplies the conventional non-zero non-relativistic value, $e\hbar/2mc$, for the hydrogen atom ground state magnetic moment.

The symmetry $SU(2)_M$ is again broken, at the fourth order, by spin-dependent terms. It might be possible, perhaps by adding interaction dependent terms, to cast these fourth order terms into some particularly illuminating form. But they cannot be transformed away, for it is at this order that spin-orbit splitting appears in the energy level spectrum. Thus if we require the Melosh transformation only to make manifest a certain symmetry, in so far as there is such a symmetry, we have no motivation for adding interaction dependence to it.

For the free particle case it has already been observed ^{3),*)} that the Melosh transformation is a null plane version of the Foldy-Wouthuysen transformation. We will confirm that this is again the case here, so that the Melosh symmetry $SU(2)_M$ again derives from a subgroup of a symmetry $[U(2) \times U(2)]_{FW}$.

The Foldy-Wouthuysen transformation ⁷⁾ is designed to take the Dirac Hamiltonian

$$h_D = \beta + \vec{\alpha} \cdot \vec{p} + v \quad (29)$$

into a form

$$h_{FW} = e^{iS} h_D e^{-iS} \quad (30)$$

which commutes with β

$$[\beta, h_{FW}] = 0 \quad (31)$$

*) This observation was also made by K. Mahanthappa.

In the no interaction case

$$e^{iS} = e^{iS_0} = \frac{1 + \sqrt{1 + \vec{p}^2} + i\vec{r} \cdot \vec{p}}{\sqrt{(1 + \sqrt{1 + \vec{p}^2})^2 + \vec{p}^2}} \quad (32)$$

$$= 1 + \frac{1}{2} i\vec{r} \cdot \vec{p} - \frac{1}{8} \vec{p}^2 + o(c^{-3}) \quad (33)$$

and

$$e^{iS_0} (\beta + \vec{\alpha} \cdot \vec{p}) e^{-iS_0} = \beta \sqrt{1 + \vec{p}^2} \quad (34)$$

So that

$$\begin{aligned} e^{iS_0} h_0 e^{-iS_0} &= \beta \sqrt{1 + \vec{p}^2} + v \\ &\quad + [\frac{1}{2} i\vec{r} \cdot \vec{p}, v] \\ &\quad - \{\frac{1}{8} \vec{p}^2, v\} - \frac{1}{4} i\vec{r} \cdot \vec{p} v i\vec{r} \cdot \vec{p} \\ &\quad + o(c^{-5}) \end{aligned} \quad (35)$$

The third term, of third order, anticommuting with β , can be eliminated by a supplementary transformation

$$iS_1 = \frac{1}{2} \beta [\frac{1}{2} i\vec{r} \cdot \vec{p}, v] \quad (36)$$

Then

$$\begin{aligned} e^{iS_1} e^{iS_0} h_0 e^{-iS_0} e^{-iS_1} &= \beta \sqrt{1 + \vec{p}^2} + v \\ &\quad - \frac{1}{8} \{\vec{p}^2, v\} - \frac{1}{4} i\vec{r} \cdot \vec{p} v i\vec{r} \cdot \vec{p} \\ &\quad + o(c^{-5}) \end{aligned} \quad (37)$$

Spin dependence enters here only at the fourth order, in the last explicitly written term. If such terms are neglected H_{FW} has a symmetry

generated by Dirac matrices which commute with β :

$$1, \quad \beta, \quad \vec{\sigma}, \quad \vec{\sigma}\beta$$

We will see that Melosh's $SU(2)_M$ corresponds to the subset

$$\beta\sigma_1, \quad \beta\sigma_2, \quad \sigma_3 \quad (38)$$

which commutes also with σ_3 . In the good representation they are, in terms of Pauli matrices

$$\begin{pmatrix} \sigma_1 & \\ & -\sigma_1 \end{pmatrix}, \quad \begin{pmatrix} \sigma_2 & \\ & -\sigma_2 \end{pmatrix}, \quad \begin{pmatrix} \sigma_3 & \\ & \sigma_3 \end{pmatrix} \quad (39)$$

Consider for a moment the case of zero interaction. The free Foldy-Wouthuysen transformation can be written

$$\sqrt{(1 + \sqrt{1 + \vec{p}^2})^2 + \vec{p}^2} \quad \phi = (1 + \sqrt{1 + \vec{p}^2} + i\vec{r} \cdot \vec{p}) \psi \quad (40)$$

or in terms of good and bad components

$$\sqrt{(1 + \sqrt{1 + \vec{p}^2})^2 + \vec{p}^2} \quad \phi_g = (1 + \sqrt{1 + \vec{p}^2} - \sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1) \psi_g - \sigma_3 p_3 \psi_b$$

This can be rewritten on the null-plane as

$$\sqrt{(1 + \sqrt{1 + \vec{p}^2})^2 + \vec{p}^2} \quad \hat{\phi}_g = (1 + \sqrt{1 + \vec{p}^2} - \sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1) \hat{\psi}_g - \sigma_3 p_3 \hat{\psi}_b$$

with the understanding that now

$$p_3 \equiv (p_- - p_0) \equiv \frac{1}{4} \left(\frac{\partial}{\partial z} + \frac{\partial}{\partial t} \right) \quad (41)$$

We could eliminate the time derivative by introducing the Hamiltonian, but then in the interacting case one would have to be careful about operator ordering, so we will not do so immediately. Using (13) to eliminate the bad components

$$\sqrt{(1+\sqrt{1+\vec{p}^2})^2+\vec{p}^2} \hat{\phi}_g = (1 + \sqrt{1+\vec{p}^2} - p_3 h^{-1} - \sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1 (1 - p_3 h^{-1})) \hat{\psi}_g$$

so that

$$\begin{aligned} & \left(\sqrt{(1+\sqrt{1+\vec{p}^2})^2+\vec{p}^2} \right)^{-1} (1 - p_3 h)^{-1} \sqrt{(1+\sqrt{1+\vec{p}^2})^2+\vec{p}^2} \hat{\phi}_g \\ & = u \hat{\psi}_g \end{aligned} \quad (42)$$

where u is given by (24), and we have used (15) in the sequence

$$\begin{aligned} p_3 &= h - p_0 = \frac{1}{2}h - \frac{1}{2}h^{-1} - \frac{1}{2}h^{-1}\vec{p}_1^2 \\ \sqrt{1+\vec{p}^2} &= \sqrt{1 + \vec{p}_1^2 + \left(\frac{1}{2}h - \frac{1}{2}h^{-1} - \frac{1}{2}h^{-1}\vec{p}_1^2\right)^2} \\ &= \sqrt{\left(\frac{1}{2}h + \frac{1}{2}h^{-1} + \frac{1}{2}h^{-1}\vec{p}_1^2\right)^2} \\ &= (1 - p_3 h^{-1}) \sqrt{h^2} \end{aligned}$$

It follows from (42) that application of the matrices (39) to the Foldy-Wouthuysen ϕ corresponds to the application of the Pauli matrices to the Melosh transformed $\hat{\psi}$.

If now the potential v is added there will be extra terms. These arise from the extra term (36) in the Foldy-Wouthuysen transformation, in the elimination of bad components using (13), and in the elimination of p_0 using (15). The modification of (42) can be written in the form of an extra factor

$$\hat{\psi}_g \rightarrow (1 + A + \vec{B}_1 \cdot \vec{\sigma}_1 + C \sigma_3) \hat{\psi}_g$$

which would imply a modification of (26)

$$\begin{aligned} \hat{h}_M \rightarrow \hat{h}_M &+ \frac{1}{2}[A, h + h^{-1}] + \frac{1}{2}[\vec{B}_1 \cdot \vec{\sigma}_1, h + h^{-1}] \\ &+ \frac{1}{2}[C \sigma_3, h + h^{-1}] + o(c^{-4}) \end{aligned}$$

where we have noted that A, B and C, containing v, are of second or higher order, and so need only be combined with zero and first order parts of $\hat{h}_M$ when working to third order. The extra term generated by A is $\vec{\sigma}$ independent, and so $SU(2)_M$ symmetric. For reasons of reflection and rotation invariance, $\vec{B}$ and C must contain not only v, but also some small momenta. Since, moreover

$$[v, h] = 0 (c^{-3})$$

[as explained in connection with (27)] the B and C terms in the modified h_M are of fourth order. Thus the extra interaction terms which would bring the Melosh transformation closer to the interacting Foldy-Wouthuysen transformation can be omitted when defining $SU(2)_M$ to third order.

It is clear from this relationship with the Foldy-Wouthuysen transformation that the Melosh transformation again ³⁾ simplifies the description of rotations in null-plane dynamics. Thus the superiority of $SU(2)_M$ over $SU(2)_W$ can be attributed again to greater compatibility with "angular conditions".

The remaining remarks will be specific to the second quantized theory, in which ψ and $\hat{\psi}$ above are q number fields. Assuming the usual null-plane anticommutators, in particular (good components only understood)

$$2 \{ \hat{\psi}(t, \vec{x}), \hat{\psi}^+(t, \vec{y}) \} = \delta(\vec{x} - \vec{y}) \quad (43)$$

the equation of motion (16) becomes

$$\frac{\partial \hat{\psi}}{\partial t} = [\hat{\psi}, -i\hat{H}] \quad (44)$$

with

$$\hat{H} = 2 \int d\vec{x} \hat{\psi}^+(t, \vec{x}) \hat{h} \hat{\psi}(t, \vec{x}) \quad (45)$$

where $\hat{h}$ is the differential operator (17). The approximate $SU(2)_W$ symmetry operation (21) is then expressed by

$$\delta \hat{\psi} = [\hat{\psi}, -i 2 \int d\vec{x} \hat{\psi}^\dagger \vec{\epsilon} \cdot \vec{\sigma} \hat{\psi}] \quad (46)$$

whose generators approximately commute with the Hamiltonian

$$[\hat{H}, 2 \int d\vec{x} \hat{\psi}^\dagger \vec{\sigma} \hat{\psi}] = 2 \int d\vec{x} \hat{\psi}^\dagger [\hat{h}, \sigma] \hat{\psi} = o(c^{-3}) \quad (47)$$

There are now two different styles of using the Melosh transformation. It could be used to define new field variables to simplify the expression of the same Hamiltonian $\hat{H}$; this was the style of Ref. 3). Here, instead, we will retain the original field variables but pass to a new Hamiltonian associated with a new representation in Hilbert space. For any state Ψ associated with the original Hamiltonian (45), we define a new representation Φ

$$\Psi = e^{iY} \Phi \quad (48)$$

which is associated with a new Hamiltonian

$$e^{-iY} \hat{H} e^{iY} = \hat{H}_n \quad (49)$$

With

$$Y = 2 \int d\vec{x} \hat{\psi}^\dagger \sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1 \hat{\psi} + \dots \quad (50)$$

$$e^{-iY} \hat{\psi} e^{iY} = \hat{\psi} + \sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1 \hat{\psi} + \dots \quad (51)$$

so that

$$\hat{H}_n = 2 \int d\vec{x} \hat{\psi}^\dagger (h + [h, \sigma_3 \vec{\sigma}_1 \cdot \vec{p}_1] + \dots) \hat{\psi} = 2 \int d\vec{x} \hat{\psi}^\dagger h_n \hat{\psi} \quad (52)$$

with $\hat{h}_M$ given by (26). The approximate symmetry $SU(2)_M$ is then expressed by

$$\begin{aligned} [\hat{H}_M, 2 \int d\vec{x} \hat{\psi}^\dagger \vec{\sigma} \hat{\psi}] &= 2 \int d\vec{x} \hat{\psi}^\dagger [\hat{h}_M, \vec{\sigma}] \hat{\psi} \\ &= 0 \quad (c^{-4}) \end{aligned} \quad (53)$$

To obtain the symmetry operators in this simple form

$$2 \int \hat{\psi}^\dagger \sigma \hat{\psi} d\vec{x} \quad (54)$$

we are obliged to adopt complicated forms for other quantities of interest, e.g., currents

$$j_\mu \rightarrow e^{-iY} j_\mu e^{iY}$$

Thus we may refer to the ψ and ϕ pictures as the "current" and "classification" (referring to classification of states with respect to a symmetry group) pictures, respectively, naming the aspects which they respectively simplify. The "current" and "classification" pictures are connected by the Melosh transformation.

We could also introduce a "constituent" picture. In the plane-wave expansion

$$\hat{\psi}(0, \vec{x}) = \sum a u e^{i\vec{p}_\perp \cdot \vec{x}_\perp + i\eta z} \quad (55)$$

the operators a can be interpreted as bare electron destruction and creation operators (for positive and negative η , respectively). Suppose now we work in the Fock space generated from the bare vacuum by these creation operators. Then both ψ and ϕ will contain not only one electron but also particle-antiparticle pairs. These are generated from the one-electron state in particular by the action of the pair parts in the lowest order interaction term

$$2 \int d\vec{x} \psi^\dagger \psi \quad (56)$$

- which is common to both "current" and "classification" pictures. We could choose to eliminate these pair terms in the interaction by unitary transformation :

$$\psi = e^{iX} \psi' \quad \phi = e^{iZ} \phi' \quad (57)$$

We could then say that ψ' and ϕ' belong to "constituent" pictures, minimizing the number of constituents. We do not wish to develop this, but only to make two remarks. The first is that we would expect these pairs, restored by x and y if they have been eliminated, to show up in an appropriate kind of "deep inelastic scattering", and so to be analogous to pairs of "partons". We would have no such expectation for the fictitious "pairs" generated ⁸⁾, in the equal-time formalism, by the Foldy-Wouthuysen transformation - generated even from the vacuum and even in the absence of interaction. The second remark is that "constituent" and "classification" pictures are designed for different purposes, and can be introduced separately, and will not in general be the same unless this has been deliberately arranged. The Melosh transformation serves not to introduce the "constituent", but rather the "classification", picture.

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