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THE ORIGIN OF A HIGH-ENERGY SCALE IN THE POMERON CALCULUS

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ABSTRACT

The Gribov-Reggeon calculus, whose perturbative expansion is expected to converge at low energy, is known to give rise to critical phenomena at ultra-high energies. The aim of this paper is to investigate the transition between these two regimes and to determine the scale with which this transition happens. This is done by solving exactly the renormalization group equation in a model theory. The propagator shows a singularity whose position determines the scale in question. The dynamical origin of this scale -- whose evaluation situates it beyond present machine energies -- is discussed, and so the dual relation between the perturbative and the critical expansions.

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An attractive approach to high-energy hadronic collisions is provided by the theory of the interacting Pomerons, which, as is well known, is described ¹⁾ by an equivalent field theory in the space of the rapidity ($Y = \ln s/s_0$) and impact parameter. The conjugate variables are, respectively, $E \equiv 1 - J$ and the two-dimensional transverse momentum vector k ($t = -k^2$). The strong coupling solution of the theory gives ²⁾ at high energy

$$\frac{d\sigma(s,t)}{dt} \sim (\ln s)^{\eta_*} F(t[\ln s]^{\nu_*}) \quad (1)$$

where η_* and ν_* are critical exponents which can be evaluated with techniques similar to those of statistical mechanics like the ϵ expansion ²⁾ or the high temperature expansion ³⁾.

The question we want to answer in this paper is what is the energy scale which determines the validity of the superasymptotic expansion of Eq. (1).

It has been often advocated ^{2),4)} that the development in "bare" Pomeron pole and cuts could be more relevant at laboratory energies. This was done by recognizing the expansion parameter in the critical development and in the perturbative one. But our aim being the understanding of the transition between the two regimes, we need an evaluation of the non-leading terms. In particular, the non-leading terms of the critical expansion will be the ones that prevent the leading contribution of Eq. (1) to be a correct sum over the Pomeron iterations at low energies. This analysis will, at the same time, provide a better understanding of the theory at the present-day energies.

We will use the renormalization group techniques which tell us how the theory develops as soon as the variables are sufficiently large so as to have overcome all explicit scales. It is therefore convenient to normalize the theory to the observation of the experimental data in a given range of energies. This is achieved in E space by defining the parameters of the theory (for instance, the coupling constant) at a value E_1 to be chosen in the range in which we are interested. This is the standard procedure of a renormalized field theory. As a by-product, the ultra-violet infinities in E disappear. Moreover, in general, the definition of the effective parameters, on the basis of the measured values, takes into account the main effects of the large E values in the integrals of the perturbation expansion. As an example, consider the radiative correction to the inverse propagator $\Gamma^{(1,1)}(E)$ which may be written in the form

$$\Sigma(E) = \int \frac{dE'}{E' - E} \varrho(E')$$

The region $E' \gg |E_1|$ has the same effect as the corrections due to secondary trajectory loops. They both redefine the parameters of the Pomeron propagator.

To be more definite, denoting as usual by $\Gamma^{(n,m)}$ the amputated Green functions with n incoming m outgoing Pomerons, the normalization conditions will be:

$$\begin{aligned} \Gamma^{(1,1)} \Big|_{\substack{E=-E_1 \\ k=0}} &= \bar{g} \quad , \quad \frac{\partial \Gamma^{(1,1)}}{\partial k^2} \Big|_{\substack{E=-E_1 \\ k=0}} = \bar{\alpha}' \\ \frac{\partial \Gamma^{(1,1)}}{\partial E} \Big|_{\substack{E=-E_1 \\ k=0}} &= 1 \quad , \quad \Gamma^{(1,2)} \Big|_{\substack{E=-E_1 \\ k=0}} = -E_1 + \bar{h} \end{aligned} \quad (2)$$

In the case of $\Gamma^{(1,2)}(E, k; E^{(1)}, k^{(1)}, E^{(2)}, k^{(2)})$, the normalization is at the symmetric point $k = k^{(1)} = k^{(2)} = 0$, $E = 2E^{(1)} = 2E^{(2)} = -E_1$. $\bar{\alpha}'$ is the effective slope, and $\bar{h}$ determines the effective intercept. The theory is assumed to be at critical values of the parameters, which means that for $E \rightarrow 0$, $\Gamma^{(1,1)} \rightarrow 0$. This condition will imply that $\bar{h}$ has a particular value which depends on $\bar{g}$. Technically $\bar{h} = \bar{h}(\bar{g})$ defines a critical line.

The normalization point $-E_1$ (usually called $\bar{E}$ in the literature) is, in principle, arbitrary but negative (i.e., $E_1 > 0$), in order that the various Green functions be real. Actually E_1 must be sufficiently small so as to individuate a sufficiently large rapidity range $Y_1 = 1/E_1$ in which the Pomeron appears as the dominant contribution. We know that the Pomeron needs a rapidity gap of the order of 2 to manifest itself clearly, therefore $E_1 < \frac{1}{2}$.

We will take $Y_1 \sim 3$ ($E_1 \sim 1/3$) which is the typical rapidity gap in which the triple-Pomeron contribution has been identified and measured ⁵⁾. Let us stress that in our (renormalized) approach we have to consider Y_1 as the typical experimental rapidity gap used to determine the parameters of the theory (as $\bar{g}$ and $\bar{\alpha}'$), since in this way the effect of higher-order corrections has been substantially taken into account by the renormalization procedure ^{*}). We therefore identify $\bar{g}$ with the triple-Pomeron

^{*}) Let us consider, as an example, the radiative correction to the propagator, $\Sigma(E)$, whose transform in the rapidity space we will call $\Sigma(Y)$. The normalization conditions (2) being, on the whole, renormalized propagators, $\Sigma(E)$ is subject to the two conditions $\Sigma(-E_1) = 0$,

coupling constant as extracted from the experimental data. We will discuss in a moment the relevant numerical values.

We would point out, nevertheless, that the determination of $\bar{g}$ is not exempt of ambiguity. To extract its value, one must assume that the single diffractive events are indeed dominated by the triple-Pomeron graph and that, for instance, the multiple Pomeron exchanges among the particle lines are negligible. This assumption is perhaps reasonable due to the fact that the obtained value of $\bar{g}$ is small compared, for instance, to the critical coupling constant g_* which we will discuss later on.

Let us now illustrate how we shall proceed in order to estimate the scale which gives the onset of the asymptotic result of Eq. (1). Due to the fact that for $E < E_1$ ($Y > Y_1$) we have overcome all ad hoc scales, we expect the Green functions of the theory which depend on E, k^2 and the parameters $\bar{g}, \bar{h}$ and $\bar{\alpha}'$ to satisfy the renormalization group equation

$$\left(E \frac{\partial}{\partial E} + \beta \frac{\partial}{\partial g} + w \frac{\partial}{\partial h} + \gamma \frac{\partial}{\partial \alpha'} - 1 - \gamma \right) \Gamma = 0 \quad (3)$$

*) Contd.

$(\partial \Sigma / \partial E)_{E = -E_1} = 0$. Since

$$\Sigma(E) = \int_0^\infty dY e^{EY} \tilde{\Sigma}(Y)$$

we see that

$$\int_0^\infty dY e^{-E_1 Y} \tilde{\Sigma}(Y) = 0 \quad \text{and} \quad \int_0^\infty dY Y e^{-E_1 Y} \tilde{\Sigma}(Y) = 0$$

which we can read in the following way: given a function $f(Y)$ which approximates $e^{-E_1 Y}$ (i.e., it is smooth, it is different from zero in the range and $0 < Y < 1/E_1$, it is normalized by $\int_0^\infty dY f(Y) = 1/E_1$, then the integrals

$$\int_0^\infty dY f(Y) \tilde{\Sigma}(Y)$$

and

$$\int_0^\infty dY Y f(Y) \tilde{\Sigma}(Y)$$

are small compared to the absolute value of $\Gamma^{(1,1)}(E = -E_1)$ and $[\partial \Gamma^{(1,1)} / \partial E]_{E = -E_1}$, which are the quantities to be corrected by $\Sigma(E)$ and $\partial \Sigma / \partial E$. If the unperturbed propagator $G(Y) = 1$, the correction to it will be

$$\int_0^Y dx_1 \int_0^{x_1} dx_2 \tilde{\Sigma}(x_1 - x_2) = Y \int_0^Y dx_2 \tilde{\Sigma}(x_2) - \int_0^Y dx_2 x_2 \tilde{\Sigma}(x_2)$$

Therefore, for $Y \sim 1/E_1$ the correction is small since the above formula corresponds to take $f(Y) = \theta(1/E_1 - Y)$. This example shows explicitly that to the normalization energy $E = -E_1$ it corresponds a (positive) rapidity interval $Y_1 = 1/E_1$ for which the correction to the unperturbed Green function is small.

where the functions β , ω , φ and η (the last one related to the wave function renormalization) can be obtained in terms of a perturbative development. Let us concentrate our attention on the inverse propagator $\Gamma^{(1,1)}(E, k^2)$ which we shall call $\Gamma(E, k^2)$ in the following. Its behaviour for $E \rightarrow 0$ is controlled by the knowledge of the theory at the value $g = g_*$ given by $\beta(g_*) = 0$. This critical point behaviour is at the origin of the asymptotic result of Eq. (1)

We are, however, interested in all $E < E_1$ and we need therefore the full knowledge of the functions β , ω , φ and η . This implies the solution of the theory, something which we are unable to do. We shall then consider for these functions their approximate expression as obtained by calculating up to one-loop diagrams ^{*)}. But then, with these expressions, we will solve Eq. (3) for all values of E . From the explicit form of $\Gamma(E, k^2)$ obtained that way, we will be able to recognize the region of validity of a critical point development. This means how the critical value $\lim_{E \rightarrow 0} \Gamma(E, k^2)$ -- which coincides with that one of the whole theory -- is approached.

This model is explicitly solvable for any dimension D of the transverse space $\vec{k}$, therefore we will work directly at $D=2$. At the one-loop order

$$\begin{aligned} \beta &= \frac{g}{2} - g^3 H & \eta &= g^2 \gamma \\ \gamma &= d'(1 + g^2 B) & \omega &= h(1 + \gamma) - \gamma E_1 \end{aligned} \quad (4)$$

H , γ , B are explicit constants, proportional to $1/E_1$ the inverse of the normalization point, whose actual value -- even if easily computable -- is irrelevant. We notice that the expression of ω in terms of η is actually valid at all orders. The solution of Eq. (3) is obtained through the auxiliary functions

$$\begin{aligned} g^2(E) &= \frac{g_*^2}{1 + (1 - \frac{g_*^2}{g^2}) \frac{E}{E_1}} & , \quad d'(E) &= -d' \frac{E_1}{E} \left(-\frac{E_1}{E} \frac{g^2(E)}{g_*^2} \right)^{g_*^2 B} \\ h(E) &= -h \frac{E_1}{E} \left(-\frac{E_1}{E} \right)^{1 - \frac{g_*^2}{g^2(E)}} \left(\frac{g^2(E)}{g_*^2} \right)^{g_*^2 \gamma} \left[1 + \frac{r g_*^2 E_1}{h} \int_1^{\frac{E}{E_1}} \frac{dx}{x} \left(\frac{g^2}{g_*^2} \left(\frac{1}{x} - 1 \right) + 1 \right)^{-1 - g_*^2 \gamma} \right] \end{aligned} \quad (5)$$

*) In Ref. 4), Bronzan is quoted to have proposed a similar model for the unrenormalized version of the theory (unpublished).

where $g_*^2 = 1/2H$ is the fixed point for the coupling constant given by the zero of β . The result for $\Gamma^{(1,1)}$ is then, for $k^2 = 0$

$$\Gamma(E, 0) = \left(-\frac{E_1}{E}\right)^{1+g_*^2\gamma} \left(\frac{\bar{g}^2}{g_*^2}\right)^{g_*^2\gamma} [-E_1 + h(E)] \quad (6)$$

If $k^2 \neq 0$, the dependence on k^2 will appear through a function of the combination $\alpha'(E) \cdot k^2$, as is well known. Notice that, in general

$$h(E) \underset{E \rightarrow 0}{\sim} \left(-\frac{E_1}{E}\right)^{1+g_*^2\gamma}$$

and therefore Γ does not go to zero for $E \rightarrow 0$. The condition $\Gamma(0, 0) = 0$ - unit intercept at infinite energy - implies

$$\bar{h} \equiv \bar{h}(\bar{g}) = \bar{g}^2 \gamma E_1 \int_0^1 \frac{dx}{x} \left[\frac{\bar{g}^2}{g_*^2} \left(\frac{1}{x} - 1\right) + 1 \right]^{-1-g_*^2\gamma} \quad (7)$$

In such a case

$$h(E) \underset{E \rightarrow 0}{\rightarrow} \frac{g_*^2 \gamma}{1+g_*^2 \gamma} E_1$$

We note that $\bar{h}(\bar{g})$ is not analytic ^{*)} for $\bar{g} = 0$, indeed

$$\bar{h} \underset{\bar{g} \rightarrow 0}{\rightarrow} \gamma E_1 \bar{g}^2 \ln g_*^2 / \bar{g}^2$$

Let us analyze the explicit result of Eq. (6). $\Gamma(E, 0)$ has a singularity for $E = E_2$ where

$$E_2 = \frac{E_1}{g_*^2 / \bar{g}^2 - 1} \quad (8)$$

If $E < E_2$ the asymptotic expansion [in $(E/E_2)^n$], which gives Eq. (1) as the first term, is meaningful. If on the other hand, $E > E_2$, then the usual perturbation expansion is convergent. In terms of the rapidity, the boundary of the superasymptotic region is given by

$$Y_2 = \frac{g_*^2}{E_1 \bar{g}^2} - 1/E_1 \quad (9)$$

*) This branch point of the Pomeron intercept for vanishing coupling constants has been noticed in Ref. 4).

It is only for $Y > Y_2$ that we expect the result of Eq. (1) to be the leading terms of a convergent development in inverse powers of Y . For $Y < Y_2$ we find instead, that the appropriate development is in terms of Y/Y_2 .

Let us now evaluate Y_2 . The values of $\bar{g}$ and g_* depend, of course, on the convention used to define the coupling constant. In particular, they have the dimension of E . It is therefore convenient to rewrite Y_2 in terms of an adimensional parameter as, for instance, the critical index $n_* = \bar{g}_*^2 Y$. Then

$$Y_2 = \frac{Y_*}{\bar{g}_*^2 Y E_1} - \frac{1}{E_1} \quad (10)$$

The quantity $\bar{g}_*^2 Y E_1$ is directly related to measurable quantities in the Pomeron development. For instance, the diffractive production of two large masses (double diffractive events) behave for large Y as $\bar{g}_*^2 Y E_1 Y \log Y \sigma_{ND}$ where σ_{ND} is the non-diffractive contribution to the cross-section identified with the unperturbed Pomeron. The comparison of the triple-Pomeron contribution with the experimental results of Ref. 5) for the diffractive production of a single large mass state *) gives $\bar{g}_*^2 Y E_1 \sim 0.01$.

In our model -- one-loop calculation in the realistic $D=2$ case -- $\mu^* = 0.12$, $\nu_* = 1.06$. With such values for μ^* and $E_1 = 1/3$ -- as discussed before -- we obtain $Y_2 \sim 9$. Before discussing this result, let us note that the critical exponents μ_* and ν_* have also been evaluated with the ϵ expansion ²⁾ [$\mu_* = 0.16$, $\nu^* = 1.08$ at $O(\epsilon)$ and $\mu_* = 0.38$, $\nu^* = 1.18$ at $O(\epsilon^2)$] and with a high temperature expansion ³⁾ ($\mu^* = 0.2$, $\nu^* = 1.2$). The similar ball park for these values indicate that our model could also be quantitatively reasonable. Anyhow, the other evaluations of μ^* being slightly higher than ours, would imply -- if used in Eq. (10) -- values of Y_2 even higher than the one we obtained.

The value $Y_2 \sim 9$ implies $s_2 = s_0 \log Y_2 \sim 10^4$ and -- as said before -- this is probably a too low evaluation of the scale s . This means that we expect

*) This corresponds to $G_{PPP}(0) \sim 4 \text{ mb/GeV}^4$ in the expression

$$\frac{d\sigma}{dt dM^2} = G_{PPP}(t) \frac{M^2}{s} \left(\frac{s}{M^2} \right)^{2\alpha(t)}$$

for large M^2 . Two recent analyses of pp and pd data -- communicated, respectively, by F. Sanners and S. Olsen at the 1975 Rencontre de Moriond -- indicate a smaller value for $G(0)$, i.e., $G_{PPP}(0) \sim 2 \text{ mb/GeV}^4$. This result would decrease by a factor 2 our evaluation of $\bar{g}/g_*$ and increase therefore by a factor Y_2^4 approximately our evaluation of Y_2 . This would give a scale $s_2 = e^{Y_2^2}$ beyond any possible reach.

the critical point behaviour of Eq. (1) to be completely unsuited for present-day laboratory energies. As said before, in this region the convergent development is in $\log s$. It coincides with the perturbative expansion of a renormalized field theory in which the unperturbed Pomeron has an effective intercept $1 + \bar{h}$ ($\bar{h} > 0$) -- in the range $Y \sim 1/E_1$ --, and an effective non-vanishing triple coupling $\bar{g}$ at $t=0$.

Of course, in order to construct completely the answer of the theory, one has to include the contributions in which more Pomerons couple to the external particle lines ^{*)}, and perhaps also consider four and more Pomeron interactions. These couplings will not alter the superasymptotic behaviour ⁷⁾ but will introduce other new dynamical scales besides the one which we have discussed.

Let us comment on our result. We have found, on the basis of a specific model, that there are two languages -- dual to each other -- to describe the same reality in different kinematical regions. One of them is represented by the Pomeron pole plus its iteration which provide the correct description below a certain energy s_2 . Above it, this development diverges and the correct description appears to be the critical development. Analogously, this development diverges (non-leading terms are crucial!) below the transition energy s_2 .

It is reasonable to expect this duality to be a general feature of the theory. If at "low" energies the Regge pole is a meaningful description, it is obvious that somehow the non-leading critical terms should have stopped the critical behaviour to have any sense, irrespectively of the critical exponents. The existence of a radius of convergence, due to the singularity, can well be a result of the simplified model. If the two developments should represent asymptotic series, the interconnection between the two dual languages can be even more loose.

We found a rather large value of the transition energy s_2 . Being a model-dependent result, the actual value $s_2 \sim 10^4$ should not be taken very seriously. We consider it rather as a lowest guess. Indeed s_2 could turn out to be much larger than that, if some new data (see footnote on page 6) on diffractive production is confirmed. The origin of the large value of s_2 is that -- by identifying $\bar{g}$ and Y_1 with the experimental triple-Pomeron coupling and Pomeron rapidity gap -- we obtained a value of

*) Let us remark that the Pomeron calculus, in its original version of an effective field theory with few parameters, seems to have difficulty in explaining the observed rise of the total cross-section ⁶⁾.

$\bar{g}$ rather smaller than g_* (critical value). The actual expression of s_2 depends, of course, on the explicit form of $\beta(g)$ used. Nevertheless, if we keep the identification of $\bar{g}$, it is clear that the rapidity Y should become pretty large before the effective coupling approaches the rather larger critical value g_* *).

It could, of course, happen that $\bar{g}$ is not small as compared to g_* and what is wrong is its identification with the experimental triple Pomeron. In that case the diffractive production of large masses should not be dominated by the triple Pomeron diagram, but be described by the sum of many multi-Pomeron exchanges. This possibility, of course, exists: it implies that even at accelerator energies the Pomeron cuts have a fundamental role and shadow completely the Pomeron pole. The unpleasant feature of this possibility is that it makes it unclear why the factorizable Regge pole description has any meaning.

We have used the Regge poles to build the infra-red phenomenon as the atoms are used to build a ferromagnet. The scale we studied is the boundary of the critical domain. To recuperate the microscopic structure of the ferromagnet below that length, is equivalent to recuperate the Reggeons below our scale.

It is indeed interesting to note how, in the scheme we have in mind ⁸⁾, the different scales are dynamically generated in an analogous way. The low-energy basic scale (call it μ) is seen, for instance, in $\langle k_T^2 \rangle$. This is of the order of a fraction of a GeV and could be the average mass of particles in a multiperipheral description of high-energy events. These give rise to a Regge description.

How quickly the Regge regime is reached depends, however, not only on the basic scale μ , but also on the coupling constant in the multiperipheral chain. This determines the coefficient of $\log s$ in the average multiplicity and the scale s_0 ($\sim 1 \text{ GeV}^2$) in the $(s/s_0)^\alpha$ expansion.

To recognize clearly the Pomeron, we need, however, energies considerably higher than 1 GeV. Indeed a rapidity gap ($\log s/s_0$) of at least 2 is needed. This implies $s > s_1 \sim 10 (\text{GeV})^2$. This value is determined not only by s_0 , but also by the strength of the Pomeron couplings as compared to couplings of the non-leading Reggeons.

) Let us recall that while $\bar{g}$ is a measure of the strength of an interaction, g_ has a "geometrical" origin. As in statistical mechanics, it depends on the dimensionality of the system (space-time) and not on the actual form or strength of the elementary interaction.

Energies of the order of s_1 are those in which the Pomeron can be well identified and its parameters determined. These parameters are then the ones that determine the onsetting of the infra-red phenomena -- and therefore the new scale s_2 -- as discussed in this paper.

We see how this dynamical succession of scales, in which no ad hoc new scale is introduced, not only is natural but is also necessary for the consistency of the theoretical picture. Indeed, if power laws are not just invented but interpreted in terms of a dynamical mechanism (exchanges, for instance), dynamics itself will suggest how this mechanism will be iterated or repeated in terms of parameters which are understandable and measurable. This will show how a simple picture, even if appropriate in a certain (energy) region, can cease to be valid if we leave that region, and how a new one becomes more suitable. Where this transition takes place, represents of course a dynamically obtained scale.

* * *

REFERENCES

- 1) V.N. Gribov, JETP (Soviet Physics) 26, 414 (1968).
- 2) A.A. Migdal, A.M. Polyakov and K.A. Ter Martirosyan, Phys. Letters 48B 239 (1974); and ITEP-102 (1973);
H.D.I. Abarbanel and J.B. Bronzan, Phys. Rev. D9, 2397 (1974).
- 3) J. Ellis and R. Savit, CERN Preprint TH.1974 (1975).
- 4) R.L. Sugar and A. White, FNAL Pub-74/59-THY (1974).
- 5) CHLM (ISR) and USA-USSR (FNAL) Collaborations. Cf., for instance, F.K. Loebinger, Proceedings of the XVII International Conference on High-Energy Physics, London 1974, p. I 71 (1974).
- 6) A. Capella and J. Kaplan, Phys. Letters B52, 448 (1974);
A. Capella, J. Kaplan and Tran Thanh Van, Orsay Preprint, to be published.
- 7) D. Calucci and R. Jengo, Nuclear Phys. B84, 413 (1975).
- 8) D. Amati, Phys. Letters 48B, 253 (1974).