Archives

MULTIPLICITY DISTRIBUTIONS IN A BOOTSTRAP MODEL
INCLUDING MULTIPLY DIFFRACTIVE EVENTS

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FOREWORD

The main ideas presented in this article have been developed in collaboration with Philippe SALIN, who died on 7th May, 1974. It is the last work in which, though already ill, he was actively engaged. We dedicate this paper to his memory.

A B S T R A C T

We show how to include multiply diffractive events in a bootstrap model, such that the cross-section grows, but does not violate the Froissart bound. We exploit the consequences for the multiplicity distribution. Furthermore it is shown that non-asymptotic effects are important for an understanding of the correlations. A quantitative analysis is made, which shows that experimentally observable effects, connected with the multiply diffractive events, are expected at presently available energies.

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1. INTRODUCTION AND PRESENTATION OF THE MODEL

Experiment shows that the multiplicity distribution inside the missing mass M in a reaction $p + p \rightarrow p + M$ looks very similar to that observed in a hadron-hadron collision at an energy $\sqrt{s} \approx M$ ¹⁾. In particular, the charged multiplicity distribution $P_{n_c}^{(MM)}(M^2)$, defined for n_c odd, remarkably interpolates the corresponding distribution in a pp collision $P_{n_c}^{(pp)}(s \approx M^2)$, defined for n_c even²⁾.

The present experimental observation can be formulated as

$$\sigma_{n_c+1}(x, s) = \sigma_p(x, s) P_{n_c}(s(1-x)) \quad (1)$$

observed for $0.5 \lesssim x \leq 1 - M_0^2/s$, where $x_0 \sigma_p(x, s)$ is the invariant cross-section for producing a proton at a given value of the Feynman variable x (with the corresponding scaled energy x_0), M_0^2 is a threshold of a few GeV^2 , σ_{n_c+1} is the total cross-section for a final state with n_c particles in addition to the proton observed at x . The main point is that $P_{n_c}(M)$ appears to be a universal probability distribution for producing n_c particles when the energy is M . Here, we propose to generalize the "experimental" formula (1) and write

$$\sigma_{n+1}(s) = \int_0^{1 - M_0^2/s} dx \sigma_L(x, s) P_n(s(1-x)) \quad (2)$$

where $\sigma_{n+1}(s)$ is the total cross-section for producing $n+1$ hadrons (we do not consider any selection rule here), $x_0 \sigma_L(x, s)$ is the invariant cross-section for producing a leading particle with scaled momentum x , and $P_n[s(1-x)]$ is the probability that the recoil cluster contains n hadrons. Here and in the following, we integrate over transverse momenta. For simplicity we assume that there always exists exactly one leading particle at some x , $0 \leq x \leq 1$, and consider M_0^2 as an effective inelastic threshold. Thus we get the normalization condition

$$\sigma_{i_n}(s) = \int_0^{1 - M_0^2/s} dx \sigma_L(x, s) \quad (3)$$

and the simple recursion formula for the universal probability distribution

$$P_{n+1}(s) = \frac{1}{\sigma_{i_n}(s)} \int_0^{1 - M_0^2/s} dx \sigma_L(x, s) P_{n+1}(s(1-x)) \quad (4)$$

The information in Eq. (4) can be equivalently summarized in a homogeneous integral equation for the generating function $G(t,s) = \sum_n t^n P_n(s)$:

$$G(t,s) = \frac{t}{\sigma_{in}(s)} \int_0^{1 - m_0^2/s} dx \sigma_L(x,s) G(t, s(1-x)) \quad (5)$$

Note that only the ratio σ_L/σ_{in} enters in Eq. (5). We are led to this equation directly from our interpretation of Eq. (1). An equation similar to Eq. (5) has also been derived within the bootstrap model for inclusive spectra proposed by Krzywicki and Petersson³⁾ and by Finkelstein and Peccei⁴⁾. In those works, however, only the case when the leading-particle spectrum is exactly scaling, and sufficiently regular near $x = 1$, was treated⁵⁾.

We show, in Section 2, how one can include diffraction, i.e. a singularity of σ_L near $x = 1$ of the type $1/(1-x)$. This is important, because of the experimental evidence for such a behaviour of the proton spectrum together with the observation given by Eq. (1). It should be pointed out that, in our "nested box model" (NBM), the system recoiling from a diffractively produced proton is again supposed to behave as an ordinary hadron system, and in itself include a diffractive part, and so on. Thus, the model is quite different from two-component models⁶⁾, and more in the spirit of the original recursion conjecture by Krzywicki and Petersson³⁾.

Furthermore, σ_L may have an explicit s -dependence, coming from non-asymptotic effects. In Section 3, we discuss this and argue that the behaviour of σ_L near $x = 0$ is important for the multiplicity distribution. We point out the connection with the proton spectrum near $x = 0$, and then build a more realistic leading-particle spectrum, which allows us to make a comparison of the NBM with pp data. In the concluding remarks (Section 4), we point out the importance of the ISR region for our model. This energy range might be a transition region for the multiplicity distribution: the average multiplicity is predicted by the NBM to level off slightly and the correlations should have important non-asymptotic energy dependence.

2. THE INCLUSION OF MULTIPLY DIFFRACTIVE EVENTS IN THE BOOTSTRAP SCHEME

2.1 How to include diffraction-comparison with MPM

In fact, Eq. 2 is quite similar in form to the multiperipheral equation

$$\sigma_{n+1}(s) = g^2 \int_0^{1 - m_0^2/s} dx K(x) \sigma_n(s(1-x)) \quad (6)$$

in which the important property is the scaling behaviour of the kernel K , leading to the well-known power law behaviour of the total cross-section, $\sigma \propto s^{B(g^2, K)}$. Whatever B is, one may always identify Eqs. (4) and (6) by choosing

$$\sigma_L(x, s) = g^2 K(x) [s(1-x)]^{B(g^2, K)} \quad (7)$$

i.e. if the s -dependence of the leading particle cross-section is a factorized power law dependence. As a simple illustration, take the Chew-Pignotti⁷⁾ case $K(x) = (1-x)^{1-2\alpha}$, $\alpha < 1$, with $g^2 = 2 - 2\alpha$ chosen in such a way as to build up a constant cross-section [$B(g^2, K) = 0$]. Then, up to an arbitrary normalization constant, the leading-particle cross-section in the equivalent NBM is $\sigma_L(x, s) = (2 - 2\alpha)(1-x)^{1-2\alpha}$: we have just generated a triple-Regge term of the RRP type for σ_L .

The very interesting new observation here is that the NBM formulation (4) is no longer equivalent to the MPM one (6) if one tries to include diffraction. In MP models, one has to set $K(x) \simeq 1/(1-x)$ near $x = 1$, and this leads to the famous Finkelstein-Kajantie disease⁸⁾, i.e. either to violation of the Froissart bound or to the first of the many decoupling theorems. Experimentally, a spectrum $K \sim 1/(1-x)$ without decoupling seems to well represent the data⁹⁾. In the present formulation, one takes for the leading-particle cross-section $\sigma_L(x, s)$ a distribution of such a type, and just requires that the normalization condition (3) does not violate the Froissart bound. If one restricts oneself to the scaling case, σ_L can be chosen as singular as $-\log(1-x)/(1-x)$. Once it is chosen, one solves Eq. (5) which generates the multiplicity distribution.

Let us again look at a simple example, namely $\sigma_L(x, s) = 1/(1-x)$ (uncoupled zero slope triple Pomeron), so that the inelastic cross-section behaves like $\log(s/M_0^2)$. The solution to Eq. (5) is easily found to be

$$G(t, s) = \left[\log \left(\frac{s}{M_0^2} \right) \right]^{t-1} \quad (8)$$

i.e. a Poisson law with average multiplicity $\log [\log (s/M_0^2)]$. Recalling that the solution to the MP equation with $K(x) = 1/(1-x)$ is a Poisson law with $\langle n \rangle = g^2 \log s$ and cross-section s^{g^2} , which violates the Froissart bound, it is amusing to notice that the equivalence between the two models can be artificially enforced by considering g^2 as an effective coupling $g^2(s)$, vanishing asymptotically like $\log (\log s) / \log s$ ^{*)}. The other thing we clearly see in this example is, of course,

*) We wish to thank J. Weis for this remark.

that multiplicities tend to be small when diffraction is turned on, as an obvious reflection of the singular behaviour of σ_L near $x = 1$ which favours low-mass recoil clusters, and thus large gaps in rapidity and low densities. This is analogous to a Chew-Pignotti model with an input trajectory α_{in} close to one, $\alpha_{in} = 1 - \epsilon$, where the multiplicity, though still logarithmic, acquires a small 2ϵ coefficient in front of the $\log s$. Indeed, in the NBM, the more σ_L is singular near $x = 1$, the lower is the average multiplicity. One easily derives the following illustrative hierarchy from Eq. (5), valid at $s \rightarrow \infty$

$$\sigma_L = \frac{1}{(1-x)^{1-2\epsilon}} \longrightarrow \langle n \rangle \propto 2\epsilon \log s \quad (9a)$$

$$\sigma_L = \frac{1}{(1-x)(a - \log(1-x))^2} \longrightarrow \langle n \rangle \propto \frac{\log s}{\log(\log s)} \quad (9b)$$

$$\sigma_L = \frac{1}{1-x} \longrightarrow \langle n \rangle \propto \log(\log s) \quad (9c)$$

$$\sigma_L = \frac{1}{\sqrt{3}(1-x)^{3/2}} \longrightarrow \langle n \rangle = \text{const.} \quad (9d)$$

Cases (9a) and (9c) have already been discussed. Case (9b) corresponds to a PPP term with decoupling at $t = 0$, and case (9d) to a typical PPR term (old-fashioned diffraction).

As far as correlations are concerned, the Mueller moment $f_2 = \langle n(n-1) \rangle - \langle n \rangle^2$ is asymptotically constant in cases (9a), (9c) and (9d), and very weak in case (9b) (numerical exploration). From these examples, we infer that purely diffractive models in the NBM framework lead to small average multiplicities and weak correlations. The next thing to explore now is the behaviour of the multiplicity distribution when a regular term is added to a diffractive one in the leading-particle spectrum.

2.2 Multiplicity distribution in simple examples

In this section, we shall restrict ourselves to the scaling case where σ_L does not depend on s .

When $\sigma_L(x)$ is not too singular near $x = 1$ ^{*)}, the upper limit of integration in Eq. (5) can be set equal to 1, and the solution is (to the order $1/s$) that of the simpler equation

*) Mathematically, when $\int_0^1 dx \sigma_L(x) [\log(1-x)]^k$ exists for any k ⁽⁵⁾.

$$G(t, s) = \frac{1}{\sigma_{in}(s)} \int_0^1 dx \sigma_L(x) G(t, s(1-x)) \quad (10)$$

where now σ_{in} is a constant. This case has already been studied in the literature⁵⁾. We just recall that the solution of Eq. (10) is of the short range type, namely

$$G(t, s) \equiv \exp \left[\sum_q (t-1)^q - \frac{f_q(s)}{q!} \right] = \exp [b(t) \log s + c(t)] \quad (11)$$

where $b(t)$ is determined by the model, and $c(t)$ arbitrary. Thus all the Mueller coefficients f_q are linear in $\log s$. In the special case $\sigma_L = (1-x)^{\lambda-1}$, $b(t) = \lambda$. In order to perform a qualitative investigation of what happens when σ_L contains both a diffractive term and a regular one, we choose to study the case

$$\sigma_L(x) = L + \frac{1}{1-x} \quad (12)$$

which is not too academic if we consider the shape of the proton spectrum for $x \gtrsim 0.2$ in pp collision. One may also look at the term L as representing a triple-Regge term RRP, with an intercept 0.5 for the secondary trajectory R . We postpone the discussion of the small x behaviour to the next section.

The inelastic cross-section from Eqs. (3) and (12) is, to the order $1/s$

$$\sigma_{in}(s) = L + \log \left(\frac{s}{M_0^2} \right) \quad (13)$$

and Eq. (5) for G reads (to that order)

$$G(t, s) = \frac{t}{L + \log \left(\frac{s}{M_0^2} \right)} \int_0^{1 - M_0^2/s} dx \left(L + \frac{1}{1-x} \right) G(t, s(1-x)) \quad (14)$$

It happens that Eq. (14) can be solved to the order $1/s$ by deriving a second-order differential equation for G with respect to the variable $z = L + \log(s/M_0^2)$, and recognizing it as the equation for confluent hypergeometric functions. The combination of these functions which satisfies Eq. (14) is rather lengthy and not

particularly transparent. Therefore, we prefer to present here the results of numerical computations for $\langle n \rangle$ and f_2 . Setting $Y = \log(s/M_0^2)$, the equations for $\langle n \rangle = B_1(Y)$, $\langle n(n-1) \rangle = B_2(Y)$, derived from Eq. (14) are

$$B_1(Y) = 1 + \frac{1}{L+Y} \int_0^Y dy (L e^{y-Y} + 1) B_2(y) \quad (15)$$

$$B_2(Y) = 2(B_1(Y) - 1) + \frac{1}{L+Y} \int_0^Y dy (L e^{y-Y} + 1) B_2(y) \quad (16)$$

They are of the Volterra type and thus can be numerically solved^{*)} by replacing them by two triangular systems of linear equations for the values of B_1 and B_2 at N points of the interval $[0, Y]$. One hundred points per unit of Y ensures a good precision.

The results are shown in Fig. 1 for $\langle n \rangle$ and Fig. 2 for $f_2 = B_2 - B_1^2$. Four curves have been drawn for each of the two functions. Two of them correspond to the limiting cases $L = 0$ (pure diffraction $\langle n \rangle = \log Y$, $f_2 = 0$) and $L = \infty$ (no diffraction $\langle n \rangle = Y$, $f_2 = 0$). The other two have been drawn for $L = 10$ and 15 . Recall that all the functions are defined up to an additive constant. Some comments are in order:

- i) The inelastic cross-section is proportional to $Y = \log(s/M_0^2)$. A 10% increase in the ISR range corresponds to $L \simeq 15$, which explains one of our choices.
- ii) Fig. 1 clearly shows the bend-over of the average multiplicity when diffraction is present. For $L = 15$, the departure from linearity in Y is already important. It can be shown that the (very) asymptotic limit of $\langle n \rangle$ is $\log Y$ (the solution with $L = 0$), the next order term being $-L/2Y^2$ (apart from an arbitrary constant).
- iii) Concerning the energy behaviour of f_2 , let us first stress that the interesting question to be discussed is that of deviations from linearity in Y . The simple limiting case $L = \infty$ (no diffraction) yields $f_2 = \text{const.}$, but any non-diffractive spectrum different from a simple $(1-x)^{\lambda-1}$ behaviour would generate a linear f_2 [see Eq. (11) and Ref. 5]. In two-component models, a quadratic term in Y is present in f_2 ¹⁰⁾. Here, diffractive and non-diffractive events are treated on the same footing, and only one component is present.

*) The asymptotic properties of B_1 and B_2 can be obtained analytically by a local study near $z = 0$ of their Laplace transforms $B_1(z)$, $B_2(z)$ which verify first-order differential equations. Some of this information is displayed below [see points (ii) and (iii) of this section].

Moreover, asymptotically in Y , the singular part $1/(1-x)$ of the kernel overcomes its regular part L , so that f_2 tends to a constant, the $L = 0$ solution. The resulting behaviour, as exhibited most clearly by the curve for $L = 10$ in Fig. 2, is then easy to understand: at low energy, diffraction is unimportant and f_2 is flat (here normalized to 0 at $Y = 0$); at intermediate energies both parts of the kernel contribute and positive correlations are generated, and f_2 finally flattens out to a constant when the singular part of the kernel dominates. One can show that the next term after the constant is $-2L/Y$ and this explains why the approach to constancy is not seen in our scale for $L = 15$.

There are two main points to be remembered from the study of this simple example. First, in this NBM, diffraction asymptotically dominates in the integral equation for the multiplicity distribution, due to the singularity of the kernel at $x = 1$. As a consequence, the average multiplicity flattens out at some energy depending on the relative strengths of the diffractive and non-diffractive parts of the kernel. From the orders of magnitude given above, this flattening may well occur in the ISR range. Secondly, the behaviour of the f_2 coefficient is characteristic neither of short-range nor of long-range correlations. The curvature observed between low and asymptotic energies rather corresponds to a transition regime where the diffractive and non-diffractive parts of the kernel are in competition. In this respect, the NBM is very different from a two-component model.

Though we believe that the features of the NBM which we have discussed above in a simple example including diffraction are quite general, the phenomenological implications of such models at presently accessible energies have to be investigated with the help of a more realistic spectrum σ_L . As already mentioned, an important property of σ_L is its behaviour near $x = 0$. If it is large there, high multiplicities will be produced and small deviations from exact scaling of the leading-particle spectrum in this region will induce appreciable energy dependence in the multiplicity distribution. This question will be discussed in the next section.

3. QUANTITATIVE FEATURES OF THE MODEL AT PRESENTLY ACCESSIBLE ENERGIES

3.1 On the behaviour of $\sigma_L(x,s)$ near $x = 0$

In deriving a more realistic model $\sigma_L(x,s)$, the leading-particle spectrum, we consider the proton spectrum in a pp collision as a guide. More precisely, since baryon-antibaryon pairs are produced in the central region, we consider the difference σ_{pp}^- of the p and $\bar{p}$ spectra as a representative of the memory the system has of the incoming particles. Experimentally, the corresponding invariant spectrum $x_0 \sigma_{pp}^-(x,s)$ falls down rather slowly with energy near $x = 0$ ¹¹⁾, and is still clearly different from zero at ISR energies. It has been pointed out by Ern¹²⁾ that

this phenomenon, together with the assumption of exact scaling of the spectrum away from $x = 0$, is able to account for about a half of the inelastic cross-section rise in the ISR range, via the baryon number conservation sum rule. We invoke this experimental observation to suggest that the leading-particle spectrum we are looking for could well have similar non-scaling properties near $x = 0$. On the other hand, Finkelstein and Peccei in their work about pionization⁴⁾, have studied the connection between the asymptotic behaviour of $\sigma_L(x)$ near $x = 0$ and the approach to scaling of the inclusive cross-section $f(x,s)$ at $x = 0$. They showed that an approach to scaling of $f(0,s)$ proportional to $1/(\sqrt{s})^{1-\alpha_0}$, like in a double-Regge Mueller diagram, is induced by a behaviour of $x_0\sigma_L(x,s)$ proportional to $|x|^{1-\alpha_0}$ near $x = 0$, as s goes to infinity. In the NBM, the approach to scaling is from below.

We propose to include these properties in s and x by choosing for $\sigma_L(x,s)$ itself at small x and large s , a double Mueller-Regge diagram contribution which reads, up to a normalization constant

$$\sigma_L^0(x,s) = \left(\frac{\sqrt{s}}{m_\perp}\right)^{\alpha_0} \frac{C h (1-\alpha_0) y}{C h y} \quad (17)$$

with, as usual,

$$x = \frac{2 m_\perp}{\sqrt{s}} \ln y$$

$m_\perp$ being an average transverse mass. At fixed x the asymptotic behaviour of $x_0\sigma_L^0$ is $|x|^{1-\alpha_0}$, in agreement with the proposal made by Finkelstein and Peccei. At $x = 0$, σ_L^0 behaves like $(\sqrt{s}/m_\perp)^{\alpha_0}$, similarly to the σ_{pp}^- spectrum studied by Ern. As an illustration of the effects on the multiplicity distribution of such a term in σ_L , we show the solution of Eq. (5) for $\langle n \rangle$ (Fig. 3) and f_2 (Fig. 4), when $\sigma_L = \sigma_L^0$ with $\alpha_0 = 0.5$, $m_\perp = 1$ GeV. For comparison, these figures also show the asymptotic limits $\langle n \rangle_{as} = 1.63 \log s$ and $f_{2as} = 0.9 \langle n \rangle_{as}$ worked out in Ref. 4 for $\sigma_L = 1/\sqrt{x}$. A curvature in the upward direction is clearly seen, especially for f_2 : this is a reflection of the rise with energy of σ_L at $x = 0$, which is partly balanced in $\langle n \rangle$ (connected to the one-particle density at $x = 0$) by the simultaneous rise of the inelastic cross-section. A simple mathematical example illustrates this phenomenon: suppose that the invariant leading-particle spectrum is a constant in x (limit $\alpha_0 = 1$). Then the inelastic cross-section behaves like $\log s$, and it is straightforward to show that $\langle n \rangle$ and f_2 are polynomials of degree 2 and 3, respectively, in $\log s$, which explains the curvatures observed at finite energy for $\alpha_0 < 1$.

In actual applications of the model we then propose the following leading-particle spectrum

$$\sigma_L(x, s) = A \sigma_L^0(x, s) (1-x)^{1-2\alpha_R} + \sigma_L^P(x) + R(x, s) \quad (18)$$

The first term, where A is a normalization constant, behaves near $x = 0$ as described above. Near $x = 1$ it is of the RRP triple-Regge type, with secondary-Regge intercept α_R . It represents the pionization part of the kernel, which scales asymptotically. $\sigma_L^P(x)$ is the diffractive part of the kernel. The parametrization used in what follows is

$$\sigma_L^P(x) = B \frac{\lambda}{(1-x)(\lambda - \log(1-x))} \quad (19)$$

a PPP term, where B is a normalization constant and $\lambda = a/2\alpha'_P$, the ratio of the triple-Pomeron residue slope to twice the Pomeron trajectory slope. Finally $R(x, s)$ stands for possible non-scaling terms of type RRR which we write

$$R(x, s) = \frac{G}{(1-x)^{\alpha_R}} \left(\frac{s}{M_0^2} \right)^{\alpha_R - 1} \quad (20)$$

We want now to apply Eq. (5) with the kernel (18) to the π^- multiplicity distribution. We choose to examine the π^- case because they are the lightest particles, the least affected by kinematical effects, and because they are not correlated by charge conservation. Their correlation coefficient f_2^{--} is then expected to be dynamically significant.

3.2 Application to the π^- multiplicity distribution in a pp collision

In principle, a multichannel calculation of the type developed in a paper by Kronenfeld and Peccei¹³⁾ should be performed, taking properly into account quantum-number conservation. However, our aim in this section is not to present a complete NBM for hadron production, but rather to show that a NBM including multiply diffractive events, though predicting a non-conventional behaviour of the multiplicity distribution, can nevertheless be successfully confronted by the data. So we shall now present a simplified calculation of $\langle n^- \rangle$ and f_2^{--} in a pp collision. We assume that in fact clusters rather than stable particles are to be considered as the leading object. By that we mean that, while we are looking for π^- 's, other particles (π 's of different charges, or resonances) can be leading. Let $Q(t)$ be the generating function for the π^- probability distribution inside a leading cluster, $G^{PP}(t, s)$ the generating function we are looking for. Given a leading-cluster spectrum $\sigma_L(x, s)$, the integral Eq. (5) reads

$$G^{PP}(t, s) = \frac{Q(t)}{\sigma_{in}(s)} \int_0^{1 - M_0^2/s} dx \sigma_L(x, s) G^{PP}(t, s(1-x)) \quad (21)$$

with [Eq. (3)]

$$\sigma_{in}(s) = \int_0^{1 - M_0^2/s} dx \sigma_L(x, s) \quad (22)$$

We are aware of the crudeness of this treatment of charges, but we believe that the approximations made do not alter the significance of our results.

Finally, since our arguments for constructing a leading spectrum are likely to be valid at sufficiently high energies only, we assume that $\sigma_L(x, s)$ for $s \geq s_0$ is given by Eq. (18). For $s \leq s_0$, we take the multiplicity distribution from the data. Let $G_0^{PP}(t, s)$ be the given generating function for $s \leq s_0$. Taking $u = s(1 - x)$ as a new integration variable and separating out the integral in u from M_0^2 to s_0 , we obtain

$$G^{PP}(t, s) = \frac{Q(t)}{s \sigma_{in}(s)} \left\{ \int_{M_0^2}^{s_0} du \sigma_L(1 - u/s, s) G_0^{PP}(t, u) + \int_{s_0}^s du \sigma_L(1 - u/s, s) G^{PP}(t, u) \right\} \quad (23)$$

This is the integral equation we have to solve. It is no longer homogeneous, due to our choice of replacing the lack of knowledge of σ_L at low energies by experimental information on G^{PP} in that region, which fixes the over-all normalization of G^{PP} for any t . Recall that this normalization was left free by the general NBM equation (5). Writing Eq. (23) for $s = s_0$, where G must verify the matching condition $G^{PP}(t, s_0) = G_0^{PP}(t, s_0)$ for all t 's, allows us to eliminate $Q(t)$. The resulting integral equation so depends only on the leading-particle spectrum at $s \geq s_0$ and on observables at $s \leq s_0$. It reads

$$G^{PP}(t, s) = \frac{s_0}{s} \frac{\sigma_{in}(s_0)}{\sigma_{in}(s)} \frac{G_0^{PP}(t, s_0)}{\int_{M_0^2}^{s_0} du \sigma_L(1-u/s_0, s_0) G_0^{PP}(t, u)} \times$$

$$\times \left\{ \int_{M_0^2}^{s_0} du \sigma_L(1-u/s, s) G_0^{PP}(t, u) + \int_{s_0}^s du \sigma_L(1-u/s, s) G^{PP}(t, u) \right\} \quad (24)$$

We have numerically solved the two first equations obtained by deriving Eq. (24) once and twice with respect to t . These equations yield $\langle n \rangle$ and $\langle n(n-1) \rangle$. We have chosen $s_0 = 100 \text{ GeV}^2$, corresponding to $p_{lab} \approx 50 \text{ GeV/c}$, an energy at which the π^- distribution is known to be Poisson-like. Data below s_0 have been taken from a fit given in Rossi et al.¹⁴⁾ for $\langle n^- \rangle$, and $f_2^{--15)}$ has been represented by the formula $f_2^{--} = \log s (\log s - 4.6)/18$. The threshold value M_0^2 has been fixed at 2 GeV^2 . In Figs. 5 and 6, the results for $\langle n^- \rangle$ and f_2^{--} are compared with the experimental data¹⁴⁻¹⁶⁾. The inelastic cross-section is given in Fig. 7 (the data are from Refs. 16 and 17) and the leading-cluster invariant spectrum at three energies ($s = 100, 400$ and 2800 GeV^2) in Fig. 8. These figures correspond to the following set of parameters occurring in Eqs. (17) to (20), $m_\perp = 1 \text{ GeV}$, $\alpha_0 = 0.7$, $\alpha_R = 0.25$, $A = 12 \text{ mb}$, $B = 1 \text{ mb}$, $\lambda = 10$, $C = 36 \text{ mb}$. No over-all fit to the data has been performed, so that better solutions, or equally good sets of parameters could be found. Owing to the assumptions made, we believe this would be unuseful, if not misleading. Our procedure was the following. First we fix $m_\perp$ and λ . 1 GeV is a reasonable order of magnitude for the average transverse mass of a cluster containing a few particles. The constant α_R is the intercept of an effective secondary trajectory. The value $\lambda = a/(2\alpha'_P) = 10$ is obtained for $\alpha'_P = 0.25 \text{ GeV}^{-2}$ and $a = 5 \text{ GeV}^{-2}$. For a given α_0 , A , B and C are essentially determined by the size and behaviour of $\sigma_{in}(s)$. It thus appears that the only really important parameter once $\sigma_{in}(s)$ is given is α_0 . It has been adjusted to obtain the fit to $\langle n^- \rangle$ from $s = 100$ to $s = 2800 \text{ GeV}^2$. In this fit, the precise values of α_R and C are unimportant.

We would like to make a few comments on the realization presented here of the NBM:

- i) Half of the 3 mb rise of $\sigma_{in}(s)$ in the ISR range is due to diffraction, the other half to non-scaling effects in the central region [similarly to the Erné results¹²⁾].
- ii) Our curve for $\langle n^- \rangle$ has a curvature downwards in the highest energy range.

The data points have presently errors too big to draw any conclusion, but they are certainly compatible with this tendency.

iii) In the present model, the behaviour of $\langle n \rangle$ and f_2 is highly non-asymptotic. It is worth noting, however, that this feature is not related to any high-energy scale. The only scale of interest here is $m_\perp$, set equal to 1 GeV. As illustrated in Section 2, the important quantity, which actually sets the scale, is the relative size of the diffractive and non-diffractive parts of the kernel.

iv) As a by-product of this calculation, the composition in π^- of the leading cluster, represented by a function $Q(t)$, can be computed from Eq. (23) written at $s = s_0$. We find

$$\left. \frac{dQ(t)}{dt} \right|_{t=1} = 0.53 \quad \text{and} \quad \left. \frac{d^2Q(t)}{dt^2} \right|_{t=1} = 0.15$$

If, for example, a leading cluster contains only 0, 1 or 2 π^- 's with respective probabilities β_0 , β_1 and β_2 , one finds $\beta_0 = 0.54$, $\beta_1 = 0.38$ and $\beta_2 = 0.08$. We attach no importance to the precise values of these probabilities, but we believe they are of a very reasonable order of magnitude. In fact, β_2 could be brought down to zero, for example by choosing a slightly different $\sigma_L(x, s)$ near $x = 1$, including two different secondary trajectories, which would generate extra short-range correlations.

5. CONCLUDING REMARKS

We have shown how the nested box model is well suited to include multiply diffractive events in a quite natural way, without encountering the difficulties of the multiperipheral model. The occurrence of an increasing number of large gap events makes the average multiplicities level off with increasing energy. Though, as we have shown, this phenomenon can be partially compensated at finite energy by a non-scaling behaviour of the leading-particle spectrum in the central region, our calculations indicate that a flattening of the average π^- multiplicity is predicted by the NBM in the ISR energy range. The behaviour of the integrated correlation coefficient f_2 is highly non-asymptotic even at ISR energies, and in contrast with the case of the average multiplicity, no compensation occurs: both the non-scaling behaviour of the kernel and the mismatch between its diffractive and non-diffractive parts contribute to the increase and curvature of f_2 in the NAL-ISR energy range. Our work also shows that these qualitative features of the NBM are independent of the precise values of the parameters.

From an experimental point of view, good measurements of the π^- average multiplicity at ISR energies would certainly be of great interest. Of course, in principle, the total charged average multiplicity should finally behave the same way, but the flattening effect can be weakened in this case by a relatively fast increase of heavy-particle multiplicities, due to kinematical constraints being released as the energy increases¹⁸⁾. Since measuring the π^- multiplicity distribution in a 4π solid angle at the ISR is a difficult task, one may also look at the one- and two-particle densities in the central region, for example by studying the π^- yield in a finite solid angle, centred around 90° with respect to the beam axis. Though we have not yet been able to compute these densities in the present form of the NBM, we believe that very interesting features of the production mechanism can be revealed by good measurements of their energy dependence. In particular, according to our calculation of $\langle n^- \rangle$, we expect the one π^- inclusive cross-section at $x = 0$ to increase with energy, but less rapidly than the inelastic cross-section throughout the ISR energy range. This statement, though not proven, is in agreement with present experimental knowledge⁹⁾. Also, since even in the absence of diffraction, f_2 is found to increase faster than linearly, due to a non-scaling behaviour near $x = 0$ of the leading-particle spectrum, some energy dependence might be present in the two π^- rapidity correlation in the small y region.

On the theoretical side, other models incorporate multiply diffractive events, or multi-Pomeron exchange, for example, the models which make use of the Gribov Reggeon calculus (GRC)²⁰⁾ and absorption models (AM)²¹⁾. Using renormalization group techniques, Caneschi and Jengo²²⁾ have studied the first term in an asymptotic expansion of the multiplicity distribution in these models. According to their work, average multiplicities are likely to be higher than $\log s$ in GRC models, and smaller than $\log s$ in AM. From this point of view, the NBM result looks rather like the AM prediction. Using the NBM, we are able to obtain quantitative predictions for presently accessible energies.

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Figure captions

- Fig. 1 : The average multiplicity corresponding to the kernel of Eq. (12), for $L = 0, 10, 15$ and ∞ .
- Fig. 2 : The correlation coefficient f_2 corresponding to the kernel of Eq. (12), for $L = 10$ and 15 . $f_2 = 0$ for $L = 0$ and $L = \infty$.
- Fig. 3 : The average multiplicity corresponding to the kernel of Eq. (17) (full line) and to $\sigma_L = 1/(\sqrt{x})$ (dashed line).
- Fig. 4 : The correlation coefficient f_2 corresponding to the kernel of Eq. (17) (full line) and to $\sigma_L = 1/(\sqrt{x})$ (dashed line).
- Fig. 5 : The result for $\langle n^- \rangle$. The data are from Refs. 14 to 16.
- Fig. 6 : The result for f_2^{--} . The data are from Ref. (6).
- Fig. 7 : The result for the inelastic cross-section. The data are from Refs. 16 and 17.
- Fig. 8 : The invariant leading-particle spectrum at $s = 100, 400$ and 2800 GeV^2 .

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8