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VACUUM POLARIZATION PHENOMENOLOGY :THE μ MESIC ATOM X-RAY DISCREPANCYAND THE $2P_{\frac{3}{2}}-2S_{\frac{1}{2}}$ SEPARATION IN THE $(\mu^4\text{He})^+$ SYSTEM

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A B S T R A C T

The possible existence of non-perturbative vacuum polarization modifications and the possible existence of a weakly-coupled light scalar boson have been recently proposed to explain the apparent μ -mesic atom X-ray discrepancy. Their effects are shown to introduce in the separation of the $2P_{\frac{3}{2}}$ and $2S_{\frac{1}{2}}$ levels of the $(\mu^4\text{He})^+$ system a discrepancy between recent experiment and theory - otherwise in agreement - of the same level as the original μ -mesic atom X-ray discrepancy.

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Among all the low-energy experiments in Quantum Electrodynamics to check with high accuracy the vacuum polarization effects, the recent measurement made by Bertin et al.¹⁾ of the separation $\Delta E_{\text{He}}(2P_{3/2}-2S_{1/2})$ between the $2P_{3/2}$ and the $2S_{1/2}$ levels of the helium muonic ion $(\mu \text{ } ^4\text{He})^+$ is certainly one of the most significant. In the same region of mean momentum transfer ($\sim \alpha m_\mu$) and relatively to the vacuum polarization contribution, the precision of this experiment ($\sim 1\%$) has to be compared with the relative precision of the electron (g_e-2) experiment ($\sim 3\%$) and of the positronium hyperfine structure ($\sim 4\%$). Even considering the other experiments testing the vacuum polarization in different regions of mean momentum transfer, this figure (1%) remains the best one (see Table 1).

Among these experiments, of particular interest are now the measurements made by two different groups²⁾ of the transition energies between large orbits in high-Z μ -mesic atoms, since recent studies³⁾ of them have shown persistent discrepancies between theory and experiment. It seems therefore important -- as suggested by Adler⁴⁾, who has recently undertaken a phenomenological analysis of this discrepancy -- to try to compare the results of the muonic atom X-ray and of the helium muonic ion experiments.

Let us summarize the Adler analysis as follows. A phenomenological potential

$$\delta V^{(Z,A)} = - (Z,A) c \frac{e^{-\mu r}}{r} \quad (1)$$

is considered which could possibly modify the μ -nucleus interaction. This potential is proportional either to the charge Ze of the nucleus if, at a more fundamental level, it has a purely electromagnetic source, or to the atomic number A of the nucleus if it describes an interaction mediated by a scalar, isoscalar boson ϕ , of undetermined mass μ , as perhaps called for in unified gauge theories of the weak and electromagnetic interactions. The constant c is a phenomenological coupling, which is either of order α^2 in the pure electromagnetic case, or of order 10^{-6} - 10^{-7} in the ϕ -boson case³⁾. Instead of Eq. (1), in the case of an electromagnetic source for the potential $\delta V(Z)$, one should more properly consider a superposition of Yukawa potentials. However, for the present discussion -- as we shall see -- it is enough to consider the potential of Eq. (1) with some mean value for the mass μ , typically greater than $(5-6)m_e$ in order not to introduce discrepancies into the very low momentum transfer experiments, like the hydrogenic Lamb shift (see Table 1). In the case of the ϕ -boson its mass μ can also be smaller, because a ratio m_e/m_μ is expected in all gauge models between the ϕ -electron and the ϕ -muon coupling, so that even a very light ϕ -meson will have negligible effects on the electron experiments. We will take $\mu > 0.6 \text{ MeV}$ because -- according to a discussion by Adler, Dashen and Treiman⁵⁾ -- a lighter mass for the ϕ -boson would not be compatible with the neutron-electron and electron-deuteron scattering data presently available.

The shifts produced by the potential of Eq. (1) to the levels of a hydrogen-like muonic atom are immediately computed in the non-relativistic approximation. For the transition energies of interest in the μ -mesic atoms (from states of principal quantum number n and angular momentum $n-1$, to states of principal quantum number $n-1$ and angular momentum $n-2$)

$$\delta E_{\gamma}^{(Z,A)}(\mu) = \delta E_{n,n-1}^{(Z,A)}(\mu) - \delta E_{n-1,n-2}^{(Z,A)}(\mu),$$

we have the following reduced contribution

$$\delta \bar{E}_{\gamma}^{(Z,A)}(2m_e e^{\omega/2}) = \left(\frac{1}{Z}, \frac{1}{A}\right) \frac{\delta E_{\gamma}^{(Z,A)}(2m_e e^{\omega/2})}{\frac{\alpha^2}{3\pi} Z \alpha m_{\mu} \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)} = \frac{c}{\alpha^2} f_{\gamma}(\omega), \quad (2)$$

$$f_{\gamma}(\omega) = \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)^{-1} \left\{ \frac{1}{(n-1)^2} \left[1 + \frac{m_e}{m_{\mu}} e^{\omega/2} \frac{n-1}{Z\alpha}\right]^{-2(n-1)} - \frac{1}{n^2} \left[1 + \frac{m_e}{m_{\mu}} e^{\omega/2} \frac{n}{Z\alpha}\right]^{-2n} \right\} \quad (3)$$

where, for numerical convenience, we have put $\mu = 2m_e e^{\omega/2}$. Analogously, for the contribution $\delta E_{\text{He}}^{(Z,A)}(\mu)$ coming from the potential of Eq. (1) to the separation $\Delta E_{\text{He}}(2P_{3/2} - 2S_{1/2})$ of the $(\mu \text{ } ^4\text{He})^+$ system, we have*)

$$\begin{aligned} \delta E_{\text{He}}^{(Z,A)}(2m_e e^{\omega/2}) &= 0.13(Z,A) c \alpha m_{\mu} \left(\frac{m_e}{\alpha m_{\mu}}\right)^2 f_{\text{He}}(\omega) = \\ &= (Z,A) \frac{c}{\alpha^2} f_{\text{He}}(\omega) 248 \times 10^{-3} \text{ eV} \end{aligned} \quad (4)$$

$$f_{\text{He}}(\omega) = \frac{e^{\omega}}{0.13} \left[1 + \frac{m_e}{\alpha m_{\mu}} e^{\omega/2}\right]^{-4} \quad (5)$$

Note that both here and in the previous formulae the spin (or the total angular momentum) does not play any role, since the potential in Eq. (1) is spin-independent and we are working in the non-relativistic approximation. In particular, the same contribution as in Eq. (4) would arise from the potential of Eq. (1) to the Lamb shift $\Delta E_{\text{He}}(2P_{1/2} - 2S_{1/2})$ of the helium muonic ion system⁴⁾.

*) In the definition of $f_{\text{He}}(\omega)$ the numerical factor $1/0.13$ is introduced -- as Adler does -- to have $f_{\text{He}}(0) \approx 1$.

To make now a prediction for δE_{He} , starting from the high-Z muonic atom experiments, we first take -- following Adler -- an average $\bar{f}_Y(\omega)$ of the functions $f_Y(\omega)$ defined in Eq. (3) between six of the measured transitions ($4f \rightarrow 3d$ in $_{47}\text{Ag}$, $_{48}\text{Cd}$, $_{50}\text{Sn}$ and $5g \rightarrow 4f$ in $_{80}\text{Hg}$, $_{81}\text{Tl}$, $_{82}\text{Pb}$) which all have nearly identical $f_Y(\omega)$, and we plot it against $f_{\text{He}}(\omega)$. This is done in Fig. 1. One sees that from $\omega \approx -1$ (corresponding to $\mu \approx 0.6$ MeV), up to $\omega \approx 5-6$ ($\mu \approx 15-20$ MeV) -- where both $\bar{f}_Y(\omega)$ and $f_{\text{He}}(\omega)$ start being vanishingly small*) -- the two functions coincide within 20%.

Therefore, independently of the value of the mass $\mu \gtrsim 0.6$ MeV, one can write

$$0.2(Z, A) \text{ eV} < \delta E_{\text{He}}^{(Z, A)} / \delta \bar{E}_Y^{(Z, A)} < 0.3(Z, A) \text{ eV} \quad (6)$$

Now, if the potential of Eq. (1) has to explain the μ -mesic X-ray discrepancy, we can replace in Eq. (6) its average reduced contribution $\delta \bar{E}_Y^{(Z, A)}$ to the above-mentioned transition energies with the reduced average discrepancies⁴⁾

$$\begin{aligned} \delta \bar{E}_Y^{\text{Exp}}(Z) &= -(54.5 \pm 10) \times 10^{-3} \\ \delta \bar{E}_Y^{\text{Exp}}(A) &= -(21.4 \pm 5) \times 10^{-3} \end{aligned}$$

obtained by putting in Eq. (2), in place of the theoretical contribution $\delta E_Y^{(Z, A)}(2m_e^{\omega/2})$, the discrepancies themselves between present theory and experiments.

Correspondingly, the predictions for the contribution to the separation $\Delta E_{\text{He}}(2P_{3/2} - 2S_{1/2})$ in the $(\mu \text{ } ^4\text{He})^+$ system are**)

$$-(33 \pm 6) < \delta E_{\text{He}}^{(Z)} / (10^{-3} \text{ eV}) < -(22 \pm 4)$$

in the case of the potential of electromagnetic origin proportional to Z , and

$$-(26 \pm 6) < \delta E_{\text{He}}^{(A)} / (10^{-3} \text{ eV}) < -(17 \pm 4)$$

when a scalar ϕ -boson of mass $\mu \gtrsim 0.6$ MeV is exchanged.

Let us then turn to the present experimental and theoretical knowledge of $\Delta E_{\text{He}}(2P_{3/2} - 2S_{1/2})$. The expected theoretical value

$$\Delta E_{\text{He}}^{\text{TR}}(2P_{3/2} - 2S_{1/2}) = \Delta E_{\text{He}}^{\text{QED}} + \Delta E_{\text{He}}^{\text{NFS}} + \Delta E_{\text{He}}^{\text{NP}}$$

*) An anomalously big weight of this function -- for example if one starts with a superposition of Yukawa potentials -- above the region $-1 \lesssim \omega \lesssim 5-6$ is not allowed in order not to violate the current limits on deviations in g_{μ}^{-2} ⁴⁾.

**) Here and in the following the +, - signs are correlated.

receives a contribution from QED for a point-like nucleus (electron vacuum polarization, fine structure, muonic Lamb shift), computed by Campani and Di Giacomo⁶⁾,

$$\Delta E_{He}^{QED} = (1808.8 \pm 1.3) \times 10^{-3} \text{ eV}$$

a contribution from the nuclear polarizability, recently evaluated by Bernabeu and Jarlskog⁷⁾

$$\Delta E_{He}^{NP} = 3.1 \times 10^{-3} \text{ eV}$$

and finally a contribution from the nuclear finite-size⁶⁾

$$\Delta E_{He}^{NFS} = -105 \times 10^{-3} \langle r^2 \rangle \text{ eV}$$

with the squared charge radius $\langle r^2 \rangle$ of the ${}^4\text{He}$ nucleus expressed in fermi². The weighted average $\langle \overline{r^2} \rangle^{1/2}$ of the presently available experimental values⁸⁾ gives

$$\langle \overline{r^2} \rangle^{1/2} = 1.65 \pm 0.025 \text{ fermi},$$

so that $\Delta E_{He}^{NFS} = -(286 \pm 8.7) \times 10^{-3} \text{ eV}$.

Summing up all the partial contributions, one gets^{*)}

$$\Delta E_{He}^{TR} (2P_{3/2} - 2S_{1/2}) = (1525 \pm 9) \times 10^{-3} \text{ eV} \quad (7)$$

It is perhaps interesting to note that the hadronic vacuum polarization gives a contribution to this separation

$$\Delta E_{He}^{HVP} \simeq \frac{\alpha^2}{3\pi} 8\alpha^3 m_\mu \left(\frac{m_\mu}{m_{had}} \right)^2 \simeq \left(\frac{m_\mu}{m_{had}} \right)^2 2 \times 10^{-3} \text{ eV},$$

where

$$m_{had}^{-2} = \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} R(s)$$

is the effective inverse hadronic mass squared connected with the usual ratio $R(s)$ defined as

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons}; s)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-; s)}$$

A numerical evaluation, using the presently existing data for $R(s)$, gives

$m_{had} \simeq 2.3 m_\pi$, so that ΔE_{He}^{HVP} is roughly an order of magnitude smaller than the present experimental precision.

*) Here and in the following we take for the total error $\sigma = (\sum_i \sigma_i^2)^{1/2}$.

The recent measurement¹⁾ gives, in fact, for the wavelength corresponding to the energy difference $\Delta E_{\text{He}}(2P_{3/2} - 2S_{1/2})$

$$\lambda = (8120 \pm 7) \text{ \AA}$$

and therefore

$$\Delta E_{\text{He}}^{\text{Exp}}(2P_{3/2} - 2S_{1/2}) = \frac{hc}{\lambda} = (1526.9 \pm 1.3) \times 10^{-3} \text{ eV}$$

in good agreement with the expected theoretical value, Eq. (7).

If, on the contrary, one assumes the existence of the potential of Eq. (1), modifying the μ -nucleus interaction as discussed, the theoretical value $\Delta E_{\text{He}}^{\text{Th}}(Z, A) \equiv \Delta E_{\text{He}}^{\text{Th}} + \delta E_{\text{He}}^{\text{Th}}(Z, A)$ is lowered either to the value

$$(1492 \pm 11) < \delta E_{\text{He}}^{\text{TR}(Z)} / (10^{-3} \text{ eV}) < (1503 \pm 10)$$

in the case of the potential proportional to Z , or to the value

$$(1499 \pm 11) < \delta E_{\text{He}}^{\text{TR}(A)} / (10^{-3} \text{ eV}) < (1508 \pm 10)$$

if the potential is coupled to the atomic number A (ϕ -boson exchange).

In both cases one introduces a discrepancy $\delta E(Z, A) \equiv \Delta E_{\text{He}}^{\text{Th}}(Z, A) - \Delta E_{\text{He}}^{\text{Exp}}$ between theory and experiment

$$- (35 \pm 11) < \delta E^{(Z)} / (10^{-3} \text{ eV}) < - (24 \pm 10)$$

$$- (28 \pm 11) < \delta E^{(A)} / (10^{-3} \text{ eV}) < - (19 \pm 10)$$

of magnitude comparable with the high- Z μ -mesic atom discrepancy invoked to introduce the potential $\delta V^{(Z, A)}$. This seems to rule out the possibility of its existence.

A conclusive test on the potential $\delta V^{(Z)}$ of pure electromagnetic origin will be the next $g_{\mu} - 2$ experiment, whose precision is anticipated⁹⁾ to be around 15 ppm. It has, in fact, been shown by Adler⁴⁾ that a superposition of Yukawa potentials $\delta V^{(Z)}$, with a monotonically increasing weight function, which can resolve the high- Z μ -mesic atom discrepancy, produces also on the muon anomaly a_{μ} an effect

$$\delta a_{\mu} / a_{\mu} \leq - 84 \pm 16 \text{ ppm}$$

which is larger than the likely remaining uncertain contributions to a_{μ} .

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Table 1

Tests of the vacuum polarization effects S_{vp}

Experiment	Exp. precision $\frac{\Delta S}{S}$ (ppm)	$\frac{S_{vp}}{S}$ (Th.)	$\frac{\Delta S}{S_{vp}}$ (1‰)	Mean momentum transfer
Lamb shift in hydrogen	60	2.5×10^{-2}	2-3	αm_e
Hyperfine structures in hydrogen and muonium	4 a)	α^2	100	αm_e
$(\mu \text{ } ^4\text{He})^+ \Delta E(2P_{3/2} - 2S_{1/2}) \left\{ \right.$	850	1.2	0.7	αm_μ
	6000 a)	1.2	5	αm_μ
$g_e - 2$	3	10^{-4}	30	m_e
Hyperfine structure in positronium	25	$\frac{1}{3} \left(\frac{\alpha}{\pi} \right)^{b)}$	40	$2m_e$
High-Z μ -mesic atoms	50	5×10^{-3}	10	$\frac{Z\alpha m_\mu}{n}$
$g_\mu - 2 \left\{ \begin{array}{l} \text{present} \\ \text{[future c)} \end{array} \right.$	270	$2 \left(\frac{\alpha}{\pi} \right)$	70	m_μ
	15	$2 \left(\frac{\alpha}{\pi} \right)$	3-4	m_μ]

a) This figure represents the theoretical uncertainty.

b) Contribution of the vacuum polarization in the annihilation channel.

c) See Ref. 9.

Figure caption

Fig. 1 : Plots of the functions $\bar{f}_\gamma(\omega)$ and $f_{\text{He}}(\omega)$.

Fig. 1