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RARE DECAY MODES OF THE K MESONS AND $K_L - K_S$ MASS DIFFERENCE
IN AN ASYMPTOTICALLY FREE GAUGE THEORY

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ABSTRACT

Rare decay modes of the K mesons are calculated for the Weinberg-Salam model of weak and electromagnetic interactions, assuming that the strong interactions are described by an asymptotically free gauge theory.

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Higher order weak and electromagnetic interactions can induce rare decays of the kaons such as $K \rightarrow \mu \bar{\mu}$, $K \rightarrow \pi \nu \bar{\nu}$, $K \rightarrow \gamma \gamma$ and $K \rightarrow \pi e \bar{e}$. In a unified gauge theory of the weak and electromagnetic interactions, predictions for these processes can, in general, be made provided some assumption is made about the effect of the strong interactions.

Gaillard and Lee ¹⁾ have computed these decays in a free quark model in which the weak and electromagnetic interactions are described by the Weinberg-Salam ²⁾ $SU(2) \times U(1)$ model using the Glashow-Iliopoulos-Maiani ³⁾ mechanism to suppress $\Delta S = 1$ direct neutral currents. To explain the small $K_L - K_S$ mass difference and the non-suppression of $K_L \rightarrow 2\gamma$, they find $m_{\rho'} < 5 \text{ GeV}$ and $m_{\rho}/m_{\rho'} \ll 1$ where m_{ρ} is the mass of the ρ quark and $m_{\rho'}$ the mass of the charmed quark. Using this mass constraint, they are then able to predict certain other rare kaon decay modes.

Of course, their calculation assumes that the strong interactions effectively "switch off" for the purposes of these calculations. As shown by Wilson ⁴⁾, the second order contributions to rare kaon decays come principally from the short distance part of the integration range. Thus the free quark model assumption of Gaillard and Lee corresponds to assuming canonical dimensions at short distances for the operators contributing to the decays. Recently it has been shown ⁵⁾ that in a model in which the strong interactions are described by a non-Abelian gauge theory, the anomalous dimensions do indeed vanish. However, in such an asymptotically free gauge theory (AFGT), the free quark model results are modified by terms involving logarithms of momenta.

In this paper we consider in detail the effects of such modifications for rare kaon decays. We work with the "standard" picture in which the strong interactions are described by an $SU(n)$ gauge group coupled to the colour of the quarks and commuting with the weak and electromagnetic gauge group. For the Weinberg-Salam model with three colours, we find that the logarithmic deviations from the free quark model give a suppression of $\sim 10^{-1}$ to the K_L, K_S mass difference calculation of Ref. 1). Similar effects are found for $K \rightarrow 2\mu$ and $K \rightarrow \pi \nu \bar{\nu}$. As a result, the bound on the charmed mass is relaxed to give $m_{\rho'} < 15 \text{ GeV}$ and a favoured value of $m_{\rho'} = 5 \text{ GeV}$. With this choice, definite predictions are made for $K \rightarrow \pi e \bar{e}$, $K_L \rightarrow \gamma \gamma$, $K_S \rightarrow \pi \gamma \gamma$, and $K^+ \rightarrow \pi^+ \gamma \gamma$.

The Weinberg-Salam model with the GIM mechanism has charged currents J_{μ}^+ given by

$$J_{\mu}^+ = \bar{p} \gamma_{\mu} (1 + \gamma_5) (n \cos \theta + \lambda \sin \theta) + \bar{p}' \gamma_{\mu} (1 + \gamma_5) (\lambda \cos \theta - n \sin \theta) \quad (1)$$

together with neutral currents mediated by the Z intermediate vector boson and the photon γ . Together these currents satisfy the algebra of $SU(2) \times U(1)$.

Second order contributions to $K \rightarrow \mu\bar{\mu}$, $K \rightarrow \pi\nu\bar{\nu}$, $K \rightarrow \gamma\gamma$, $K \rightarrow \pi\gamma\gamma$ and $K \rightarrow \pi e\bar{e}$ involve loops with W boson propagators, and the leading diagrams are shown in Fig. 1 and Fig. 2 [cf. Ref. 1)]. The dominant diagram for $K^0 \rightarrow \bar{K}^0$ is shown in Fig. 3 where, following Ref. 1), all calculations are to be made in the 't Hooft-Feynman gauge.

The evaluation of these diagrams involves the (time ordered) product of two or more currents, and as shown by Wilson⁴⁾ and Roy⁶⁾ the dominant contribution to the integral comes from the short distance part where all integration variables are of order $1/M_W$.

In order to compute the short distance behaviour in an AFGT, we first write the Wilson expansion in the form

$$T[J_\mu(x) J_\nu(y) \dots J_\lambda(0)] \underset{x, y \rightarrow 0}{\sim} \sum_n G_{\mu\nu\dots\lambda}^n(x, y, \dots) O_n(0)$$

where the sum n runs over all operators O_n contributing in the Wilson expansion. The functions G^n satisfy the renormalization group equations⁷⁾

$$\left[\mu \frac{\partial}{\partial \mu} - \beta(g) \frac{\partial}{\partial g} + (1 + \gamma_{\oplus}(g)) m \frac{\partial}{\partial m} - D_{A_0} - \gamma_0 \right] G(\mu^{-1}x, \mu^{-1}y, \dots, g, m) \underset{= 0}{=} \quad (2)$$

where g is the renormalized strong coupling constant, and m stands for any renormalized masses present. However, these will be evaluated at unphysical momenta and their relation to physical masses is unclear. $\gamma_{\oplus}$ and γ_0 are anomalous dimensions associated with the mass m and the operator O ⁷⁾.

We have included the mass terms in the renormalization group equations, since in the model considered the rare decays are forbidden by P spin in the absence of mass splitting between the p and p' quarks¹⁾. Thus the first non zero contributions will involve the mass and it is essential to include the effects of the anomalous dimensions $\gamma_{\oplus}$.

The solution to these equations takes the form

$$G^n(\mu^{-1}x, \mu^{-1}y, \dots, g, m) = \mu^{D_{A_0}} \exp \left\{ \int_1^\mu \gamma_0(g(x')) \frac{dx'}{x'} \right\} \times G^n(x, y, \dots, g(\mu), m(\mu)) \quad (3)$$

where D_0 is the dimension of the operator O and with $g(\mu)$, $n(\mu)$ given by:

$$\begin{aligned} \mu \frac{\partial}{\partial \mu} g(\mu) &= \beta(g(\mu)) \quad ; \quad g(1) = g \\ \mu \frac{\partial}{\partial \mu} m(\mu) &= -[1 + \gamma_{\oplus}(g(\mu))] m(\mu) \quad ; \quad m(1) = m \end{aligned} \quad (4)$$

For an asymptotically free gauge theory

$$g(\mu) \xrightarrow{\mu \rightarrow \infty} 0$$

and

$$m(\mu) \approx \mu^{-1} m \exp \left\{ - \int_1^{\mu} \gamma_{\oplus}(g(x')) \frac{dx'}{x'} \right\}$$

* power series in $(\ln \mu)^{-1}$

If the leading non-vanishing term in G^n then has a factor m^s , say $m^s G^{n,s}$

$$\begin{aligned} G^n(\mu^{-1}x, \mu^{-1}y \dots, g, m) &\approx (\mu)^{D_0 - s} G^{n,s}(x, y \dots, 0, 0) \\ &\times \exp \left\{ \int_1^{\mu} [\gamma_0(g(x')) - s \gamma_{\oplus}(g(x'))] \frac{dx'}{x'} \right\} \end{aligned} \quad (5)$$

provided, of course, $G^{n,s}$ exists.

For a free quark model, the term

$$\exp \left\{ \int_1^{\mu} [\gamma_0(g(x')) - s \gamma_{\oplus}(g(x'))] \frac{dx'}{x'} \right\}$$

is absent.

If we had used the Callan-Symanzik equations ⁸⁾ to discuss the short distance behaviour, we could have proceeded by making an expansion in powers of mass about the P spin symmetric theory ⁷⁾. However, because of

the P spin symmetry, the leading operators contributing to the operator product expansion would have involved the mass operator and would have introduced anomalous dimensions $\gamma_{\odot}$ as in Eq. (5).

The calculation of $\gamma_{\odot}$ to first order in g^2 is straightforward and involves the graphs of Fig. 4. It yields

$$\gamma_{\odot}(g) = \frac{3g^2}{16\pi^2} (n - \frac{1}{n}) \quad (6)$$

The calculation of γ_0 depends on which is the leading operator contributing to the operator product expansion.

For the $K^0 \rightarrow \bar{K}^0$ transition in the free quark model, the leading operator is the 27 plet operator $\bar{n}_{\mu} (1 + \gamma_5) \lambda \bar{\lambda} \gamma^{\mu} (1 + \gamma_5) n$. The anomalous dimensions of this have been computed⁹⁾

$$\gamma_0(g) = - \frac{g^2}{8\pi^2} 3(n-1)/n \quad (7)$$

Using (6) and (7) in Eq. (5) together with $g(x)$ given by Eq. (4) gives, for the calculation of the $K^0 \rightarrow \bar{K}^0$ transition, a suppression of the free quark model result by the amount

$$\left[1 - \left(\frac{g^2}{8\pi^2} \right) b \ln(\sqrt{-x^2} m_0) \right]^{-\left\{ 3(n - \frac{1}{n}) + 3(n-1)/n \right\} / b} \quad (8)$$

where we have written

$$\beta(g) = - \frac{g^3}{16\pi^2} b + O(g^2) \quad ; \quad b = (11n - 8)/3$$

and m_0 is the initial value of $(-x^2)^{-\frac{1}{2}}$.

Here a factor $2\gamma_m$ was necessary since the first non-vanishing term in Eq. (5) had $s=2$.

As discussed by Gaillard and Lee in the 't Hooft-Feynman gauge, the contributions of unphysical Higgs scalars are negligible compared to those of the vector bosons.

The effect of this suppression on the $K_L - K_S$ mass difference may be found by performing the space integral. If we take the Gaillard-Lee estimate for the operator matrix elements, then we find

$$\frac{m_L - m_S}{m_K} \approx \frac{G_F}{\sqrt{2}} f_K^2 \frac{\alpha}{4\pi} \epsilon'_0 \sin^2 \theta_c \cos^2 \theta_c \left(\frac{2}{3} \right) \quad (9)$$

where ϵ'_0 is related to ϵ_0 defined in Ref. 1) by

$$\epsilon'_0 = \epsilon_0 \left[1 + \left(\frac{g^2}{8\pi^2} \right) b \ln \frac{M_W}{m_0} \right]^{-\{3(n-\frac{1}{n}) + 3(n-1)/n\}/b}$$

For the free quark model, the result was Eq. (9) with ϵ'_0 replaced by ϵ_0 .

We recall that

$$\epsilon_0 = \frac{1}{M_W^2 \sin^2 \theta_W} \begin{cases} m_{\rho'}^2 & \text{if } M_W^2 \gg m_{\rho'}^2 \gg m_\rho^2 \\ 2(m_{\rho'}^2 - m_\rho^2) & \text{if } M_W^2 \gg m_{\rho'}^2 \approx m_\rho^2 \end{cases}$$

Note that these masses are evaluated at $q^2 = \mu^2$.

To estimate the magnitude of this change, we choose m_0 to be the onset of scaling of the form (8) and take $m_0 \simeq 1$ GeV with $g^2/4\pi \simeq 1$ at that point. Then with $m_W \simeq 100$ GeV and three colours, we have

$$1 + \frac{g^2}{8\pi^2} b \ln \left(\frac{M_W}{m_0} \right)^2 \approx 7$$

so that

$$\epsilon'_0 = \epsilon_0 (7)^{-1.2} \approx \epsilon_0 / 10$$

Thus the free quark calculation of Gaillard and Lee is suppressed by a factor of 10 in this asymptotically free gauge theory, most of the suppression coming from the anomalous dimension associated with the mass.

Experimentally, the left-hand side of Eq. (9) is about 0.7×10^{-14} , so we have

$$\epsilon_0 \approx 2.1 \times 10^{-2}$$

A value of ϵ_0 this large is only compatible with $m_{\rho'} \gg m_\rho$ and $m_{\rho'}^2 \simeq 30 \text{ GeV}^2$.

Although the estimates made in obtaining this number are dubious, we hope that they will not mislead us as to the order of magnitude. Also the relation between the "mass" of the p' quark and the mass of charmed hadrons is unclear. But to the extent that the calculation describes the degree to which the cancellation between charmed and uncharmed states must be effective, we expect that $m_{p'}$ gives an estimate of the mass difference between physical charmed and uncharmed states.

Thus we may conclude that charmed states in this model should have a mass < 15 GeV with preferred value $\simeq 5$ GeV.

We turn now to the calculation of $K \rightarrow 2\mu$ and other rare decay modes. For both the graphs of Fig. 1, the leading operator is found to be J_{σ}^{6+i7} , the strangeness changing neutral current, and is proportional to m^2 . Since this should be conserved on the light cone it will have zero anomalous dimension, therefore γ_0 is zero in Eq. (5). This means that both the diagrams of Fig. 1 are suppressed by the same factor

$$\left[1 - \left(\frac{g^2}{8\pi^2} \right) b \ln(\sqrt{x^2} m_0) \right]^{\{3(n-\frac{1}{2})\}/b} \quad (10)$$

wherever m^2 enters.

As a result, the cancellation of the axial vector part $\bar{\mu}\gamma_{\alpha}\gamma_5\mu$ between the W^+W^- and Z contributions found by Gaillard and Lee will not be affected and, again using their estimates for the operator matrix elements we find

$$\begin{aligned} T(K_L \rightarrow \mu\bar{\mu}) &= -\sqrt{2} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi} \epsilon' \cos\theta_c \sin\theta_c \\ &\times [if_K(p_K)^{\alpha}] \sin^2\theta_w \bar{\mu}\gamma_{\alpha}\mu = 0 \end{aligned} \quad (11)$$

with

$$\epsilon' = \epsilon (7)^{-0.96} = \epsilon / 6.5$$

and

$$\epsilon = \frac{m_{p'}^2 - m_p^2}{M_W^2 \sin^2\theta_w} \left\{ \ln\left(\frac{6.5 M_W^2}{m_{p'}^2}\right) - 1 \right\}$$

i.e.,

$$\epsilon' \simeq 2.1 \times 10^{-2}$$

Thus again we expect the dominant mechanism for $K_{L,S} \rightarrow \mu\bar{\mu}$ to be $K_{L,S} \rightarrow \gamma\gamma \rightarrow \mu\bar{\mu}$. Even without the complete cancellation of the axial vector part with the suppression factor ϵ' , the amplitude given by Eq. (11) would be within the bounds implied by the experimental data.

The calculation of $K \rightarrow \pi\nu\bar{\nu}$ again requires the evaluation of the graphs of Fig. 1. Proceeding as for $K \rightarrow \mu\bar{\mu}$, we obtain the results

$$\begin{aligned} T(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= O(G_F \alpha \epsilon'^2) \\ \frac{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{2 \Gamma(K^+ \rightarrow \pi^0 e \nu)} &= \frac{\Gamma(K_S \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K_L \rightarrow \pi^0 e \nu)} = 2 \left(\frac{3\alpha}{4\pi} \epsilon' \cos\theta_c \right)^2 \quad (12) \end{aligned}$$

With the value for ϵ' corresponding to $m_{\rho'}^2 = 30 \text{ GeV}^2$, we have

$$\begin{aligned} \frac{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\Gamma(K^+ \rightarrow \text{all})} &\approx 3 \times 10^{-10} \\ \frac{\Gamma(K_S \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K_S \rightarrow \text{all})} &\approx 2 \times 10^{-12} \\ \frac{\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K_S \rightarrow \pi^0 e \nu)} &\approx 4 \times 10^{-4} \end{aligned} \quad (13)$$

The calculation of $K \rightarrow \gamma\gamma$ requires evaluation of the diagrams of Fig. 2. Again the leading operator contributing is J_σ^{6+17} so $\gamma_0 = 0$ and the free quark model result will be suppressed by the factor Eq. (10). Performing the coordinate space integration gives

$$iT(K_L \rightarrow \gamma\gamma) = -2 F_{\mu\nu}(k_1) \tilde{F}^{\mu\nu}(k_2) \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi} \cos\theta_c \sin\theta_c (2\sqrt{2} f_K) A_\gamma \quad (14)$$

where

$$A_\gamma = Q^2 \left[F\left(\frac{m_\rho^2}{6.5 m_K^2}\right) - F\left(\frac{m_{\rho'}^2}{6.5 m_K^2}\right) \right]$$

where Q is the charge of the p quark and $F(\beta)$ is the function defined in Ref. 1). For $\beta > 1$, $F(\beta)$ converges very rapidly to zero. Moreover, for $m_\rho \sim 0.3$ GeV we have, due to the suppression factor of 6.5,

$$A_\gamma \approx Q^2 F(0) = \frac{1}{2} Q^2 \quad (15)$$

If we take $Q = 2/3$ this leads to the prediction

$$\frac{\Gamma(K_L \rightarrow \gamma\gamma)}{\Gamma(K_L \rightarrow \text{all})} = 3.3 \times 10^{-5} \quad (16)$$

In the free quark model, Eq. (16) was an upper limit for this ratio.

The experimental branching ratio is $(4.9 \pm 0.4)10^{-4}$, therefore the prediction of the fractionally charged quark model is rather small. However, for this particular decay, we have had to sum to all orders in mass, while ignoring other non-leading terms in the solution of the renormalization group equations. Accordingly, this result should be treated with caution.

With the same note of caution we will calculate other two-photon decay modes. For $K_S \rightarrow \gamma\gamma$ to lowest order in M_W^{-2} there is no contribution. We expect the dominant mechanism to be $K_S \rightarrow \pi\pi \rightarrow \gamma\gamma$ 1).

Other two-photon decay modes may be calculated and we find

$$\begin{aligned} T(K_L \rightarrow \pi^0 \gamma\gamma) &= 0 \\ i T(K^+ \rightarrow \pi^+ \gamma\gamma) &= -i T(K_S \rightarrow \pi^0 \gamma\gamma) \\ &= -i \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi} \cos\theta_c \sin\theta_c Q^2 \left[F\left(\frac{m_\rho^2}{6.5t}\right) - F\left(\frac{m_{\rho'}^2}{6.5t}\right) \right] \\ &\quad \times \epsilon^\rho \epsilon^\sigma \frac{2}{t} \Lambda_{\rho\sigma\mu} \left[(p+q)^\mu f_+(t) + (p-q)^\mu f_-(t) \right] \end{aligned} \quad (17)$$

where $\Lambda_{\rho\sigma\mu}$ is defined in Ref. 1).

Due to the suppression factor $1/6.5$ we expect the approximation

$$F\left(\frac{m_\rho^2}{6.5t}\right) \approx F(0) = \frac{1}{2} \quad (18)$$

to be good for a dominant part of the t range $0 \leq t \leq (m_K - m_\pi)^2$. Taking $f(t) \approx f(0)$ we have

$$\begin{aligned}
 \Gamma(K_S \rightarrow \pi^0 \gamma \gamma) &= \Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) \\
 &= \frac{1}{2m_K} m_K^4 \left[\frac{G_F}{\sqrt{2}} \frac{4\alpha}{\pi} Q^2 \cos\theta_c \sin\theta_c \frac{1}{2} f_+(0) \right]^2 \\
 &\quad \times \frac{m_K^2}{64(2\pi)^3} \int_0^{0.53} dx (1-x)
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 \frac{\Gamma(K_S \rightarrow \pi^0 \gamma \gamma)}{\Gamma(K_S \rightarrow \text{all})} &\approx 3 \times 10^{-10} \\
 \frac{\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma)}{\Gamma(K^+ \rightarrow \text{all})} &\approx 3 \times 10^{-8}
 \end{aligned} \tag{20}$$

Finally, we consider the decay mode $K \rightarrow \pi e \bar{e}$. The dominant term comes from the one-photon exchange of Fig. 1(b). As before γ_0 is zero, and we find

$$\begin{aligned}
 i T(K^+(p) \rightarrow \pi^+(q) e^+ e^-) &= i T(K_S(p) \rightarrow \pi^0(q) e^+ e^-) \\
 &= \frac{i G_F}{\sqrt{2}} \frac{2\alpha}{3\pi} \delta' \cos\theta_c \sin\theta_c (p+q)_\mu f_+(t) \bar{e} \gamma^\mu e
 \end{aligned} \tag{21}$$

where

$$\delta' = Q \ln \frac{m_{p'}^2}{6.5 m_K^2}$$

This gives for $m_{p'}^2 = 30 \text{ GeV}^2$

$$\begin{aligned}
 \frac{\Gamma(K^+ \rightarrow \pi^+ e \bar{e})}{\Gamma(K^+ \rightarrow \text{all})} &\approx 0.5 \times 10^{-6} \quad (\text{cf. experiment } 0.3 \times 10^{-6}) \\
 \frac{\Gamma(K_S \rightarrow \pi^0 e \bar{e})}{\Gamma(K_S \rightarrow \text{all})} &\approx 0.2 \times 10^{-8}
 \end{aligned} \tag{22}$$

In the Table we summarize the results for the rare decay modes of the kaon considered here.

Of course the estimation of charmed masses depends critically on the choice of parameters $g^2/4\pi$ and M_W , and also on the assumption that effective masses evaluated at Euclidean momenta $\simeq 1$ GeV have any relation to physical masses. For this reason our calculation should perhaps be viewed merely as an example of the suppressions that can occur in AFGT. More reliable estimations of ϵ'_0 may be possible from an analysis of mass dependent effects in e^+e^- annihilation ¹⁰⁾.

However, if we treat m'_ρ and m_ρ as "free" parameters, then the calculation of relative K meson rare decay rates is more reliable. Here the main differences from the free quark model predictions are the relative suppression of the $K^0 \rightarrow \bar{K}^0$ transition to the other decay modes, the prediction that $K_L \rightarrow \gamma\gamma$, $K^+ \rightarrow \pi^+ \gamma\gamma$, $K_S \rightarrow \pi^- \gamma\gamma$ should be the maximum allowed by the free quark model, and the suppression of $K^+ \rightarrow \pi^+ e^+ e^-$ relative to the free quark model predictions. The accidental cancellation leading to suppression of the $K \rightarrow 2\mu$ decay seems to persist in AFGT.

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Table

Comparison of AFGT predictions for
3 quark quartet model with experiment

Process	Branching ratio Prediction (AFGT) $m_{\rho'}^2 = 30 \text{ GeV}^2$	Experimental rate
$K_L \rightarrow \mu\mu^-$	$\sim 10^{-8}$	10^{-8}
$K^+ \rightarrow \pi^+ \nu\bar{\nu}$ $K_L^0 \rightarrow \pi^0 \nu\bar{\nu}$ $K_S^0 \rightarrow \pi^0 \nu\bar{\nu}$	3×10^{-10} Forbidden 2×10^{-12}	$< 5.6 \times 10^{-7}$ - -
$K_L \rightarrow \gamma\gamma$	3×10^{-5}	$(4.9 \pm 0.4) 10^{-4}$
$K^+ \rightarrow \pi^+ \gamma\gamma$ $K_L^0 \rightarrow \pi^0 \gamma\gamma$ $K_S^0 \rightarrow \pi^0 \gamma\gamma$	3×10^{-8} $\sim 10^{-9}$ 3×10^{-10}	$< (3.5) 10^{-5}$ $< (2.4) 10^{-4}$ -
$K^+ \rightarrow \pi^+ e\bar{e}$ $K_L^0 \rightarrow \pi^0 e\bar{e}$ $K_S^0 \rightarrow \pi^0 e\bar{e}$	5×10^{-7} Strongly suppressed 2×10^{-9}	3×10^{-7} - -

FIGURE CAPTIONS

Fig. 1 : Diagrams for the reaction $\alpha \rightarrow \beta + \ell + \ell$.

Fig. 2 : Dominant contribution to $\alpha \rightarrow \beta + \gamma + \gamma$.

Fig. 3 : Dominant contribution to $K^0 \rightarrow \bar{K}^0$ transition.

Fig. 4 : a) Fermion self-energy due to the exchange of a coloured gluon.
b) Contribution to mass renormalization. The cross is where the term $-m\bar{\psi}\psi$ acts.

FIG. 1

FIG. 2

FIG. 3

FIG. 4