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UNIFIED FIELD THEORIES OF BAGS AND STRINGS

Patrizio Vinciarelli

CERN -- Geneva

A B S T R A C T

Within our models of temporary quark confinement in field theoretical bags, we exhibit, in the tree approximation, string-like solutions. We argue that the bag-like and dual string-like pictures complement each other with the former describing the low-lying hadronic states and the latter the excited states of large angular momentum. When certain quantum fluctuation effects are included, we find Regge trajectories, in a certain dynamical regime, to always approach straight lines in a (J, E^2) plane.

INTRODUCTION

Localized excitations of the vacuum, in field theories with spontaneously broken symmetries, provide a self-consistent potential capable of binding bare constituents with large (though finite) masses into low mass hadrons. This approach¹⁾ offers an elegant resolution to the puzzle of unseen quarks and incorporates naturally many desirable properties of the static quark model and of parton dynamics, without abandoning the conventional framework of quantum field theory.

In a recent paper²⁾, we exhibited, at the level of the tree approximation, in theories of interacting spinor and scalar fields, explicit implementations of such a suggestion leading to spherically symmetric quark bound state solutions in three-space-one-time dimensions. These states are phenomenologically approximated in some limits by states constructed within the so-called MIT bag model³⁾.

Here we present, within the context of the same field theories and dimensionality of space-time, cylindrically symmetric solutions. These states are phenomenologically approximated in some limits by states constructed within the dual string models of hadrons⁴⁾.

It is our intention in this paper to argue in favour of a complementary, bag-like and string-like, picture of the hadronic spectrum. The transition from bag-like to string-like structures would be associated with the excitation of large angular momenta, as depicted in Fig. 1. This connection determines the character of those excited states which are relevant to the behaviour of the leading Regge trajectories in our models.

Field theoretical models with semi-classical solutions which approximate dual strings have been known to exist for some time^{5,6)}. They have been based on the Higgs-Kibble Lagrangian for interacting vector and scalar fields, which provides a relativistic generalization for the Ginzburg-Landau Lagrangian: the latter accounts for the vortex line solutions manifested by type II superconductors. An important feature of these solutions is that vortices always close on themselves as the flux of magnetic field along the vortex line cannot be interrupted without the introduction of additional sources, i.e., magnetic monopoles. Thus, the corresponding dual strings are closed strings with vacuum quantum numbers (Pomerons), corresponding to the Shapiro-Virasoro dual model.

Our string-like solutions, on the other hand, which derive from a theory of interacting spinor and scalar fields (and, possibly, non-abelian vector fields), are not dependent upon vortex lines and may lead to open as well as closed strings. They also naturally carry fermion as well as other quantum numbers.

The plan of the paper is the following. We consider two implementations of our general field theoretical approach to quark confinement differing in the character of the interaction. One of them yields by a limiting procedure⁷⁾ the MIT

model, about which certain conclusions will therefore be drawn. For both of these field theories we exhibit in the tree approximation cylindrically symmetric solutions. These are static, straight, infinitely long "strings" with a finite transverse size. Their properties are related to the Hamiltonian parameters of our models and certain strong coupling limits are briefly discussed.

We then attack the problem of terminating such "strings" at a finite length. We compensate for the instabilities that arise by putting the string in rapid rotation. Making extensive use of energetic considerations, we discuss both open and closed strings and their associated leading Regge trajectories. When certain quantum fluctuation effects are included, in the presence of appropriate colour interactions, these trajectories are seen in all cases to asymptotically approach straight lines in a (J, E^2) plane.

The approach presented here may be interpreted to suggest an approximate dual picture in a dynamical regime where hadronic states of large angular momentum are described by constituent quarks concentrated at the periphery of a rotating string.

PRELIMINARIES

Postponing until later the introduction of "colour", let us consider, for the moment, just a singlet Fermi field ψ (the "quark") in interaction with a neutral scalar field ϕ , a system characterized by the Hamiltonian density:

$$\mathcal{H} = \psi^\dagger \frac{\alpha \cdot \nabla}{i} \psi + \frac{1}{2} (|\nabla \phi|^2 + \dot{\phi}^2) + \mathcal{I}(\phi, \psi) + \mathcal{V}(\phi). \quad (1)$$

We shall assume that the ("effective") self-interaction of ϕ , $\mathcal{V}(\phi)$, to be such that spontaneous symmetry breakdown and generation of a fermion mass term occurs. The absolute minimum of $\mathcal{V}(\phi)$ should then correspond to a value λ of the classical field different from zero

$$\mathcal{V}(0) - \mathcal{V}(\lambda) = \mathcal{B} \geq 0, \quad (2)$$

so that in the ground state or vacuum, $|0\rangle$, the expectation value of ϕ is different from zero:

$$\langle 0 | \phi | 0 \rangle = \lambda \neq 0. \quad (3)$$

Thus we take $\mathcal{V}(\phi)$ to be of the type represented in Fig. 2 and parametrize it in the form:

$$\mathcal{V}(\phi) = \mathcal{B} + \mathcal{B} \phi^2 (9\phi^2 - 16\lambda\phi + 6\lambda^2) / \lambda^4 + \mathcal{H} (\phi^2 - \lambda\phi)^2 / \lambda^4 \quad (4)$$

Here, B , λ , and H are all positive constants: B is the potential difference defined in Eq. (2); λ is the expectation value of ϕ in the vacuum, defined in Eq. (4); and H characterizes, in the limit $H \gg B$, the height of the barrier between the local minimum of the potential occurring at $\phi = 0$ and the ground state.

We will explicitly consider two different possible choices for the "interaction" term", $I(\phi, \psi)$:

$$I(\phi, \psi) = \begin{cases} \frac{\mu}{\lambda} \bar{\psi} \psi \phi \\ \frac{\mu}{\lambda^2} \bar{\psi} \psi \phi^2 \end{cases} \quad (5)$$

Notice that the second choice would lead to a non-renormalizable interaction. However, this is of no concern to us here, since, as argued earlier²⁾, we only regard $\mathcal{H}(x)$ in Eq. (1) as an effective Hamiltonian, and we restrict our calculations for the moment, to the level of the tree approximation. Physically, our justification for this choice is that ϕ may effectively be taken to describe composite (rather than elementary) excitations of the system, such as bound quark pairs. In the case of spherically symmetric configurations of the system (at the first-quantized level) previously discussed²⁾, the more singular choice for I in Eq. (5) led to solutions which appeared to be more desirable from a phenomenological viewpoint. Indeed, the quark gas is then confined in such a way that its effective mass and its interactions within the "prison" may be negligible. This would lead to a simple, intuitive explanation for Bjorken scaling while blaming the absence of partons in final states on the strong coupling regime prevailing at the boundaries of the prison, which "temporarily" confine the parton-quarks. These features may not be easily carried over to systems with the "renormalizable" $\bar{\psi} \psi \phi$ interaction. However, such systems are nevertheless interesting and will deserve our attention in our present discussion of string-type configurations.

Following a logical pattern already discussed²⁾, involving the construction of quantum trial states and the use of a variational principle^{8,9)}, from the Hamiltonian (1) we may deduce, in the tree approximation, the following system of equations for our "classical" fields ($f \leftrightarrow \phi$, $u \leftrightarrow \psi$):

$$\begin{aligned} \nabla^2 f - \frac{dV}{df} &= \frac{\partial I}{\partial f}, \\ (-i\alpha \cdot \nabla + \frac{\partial I}{\partial u}) u &= \epsilon u, \end{aligned}$$

where ϵ is the spinon energy eigenvalue.

CYLINDRICAL SOLUTIONS

We now adopt cylindrical variables (r, θ, z) and search for z -independent solutions such that¹⁰⁾:

$$f = f(r), \quad u = \begin{pmatrix} u_1(r) \chi \\ i u_2(r) \sigma_z \chi \end{pmatrix}, \quad (6)$$

where χ is a two-component spinor and $\sigma_r = \sigma_x \cos \theta + \sigma_y \sin \theta$. Thus we obtain the following system of equations:

$$\begin{aligned} \left(\frac{d}{dr}\right)^2 f + \frac{1}{2} \frac{d}{dr} f - \frac{dV}{df} = \frac{\partial I}{\partial f} &= \begin{cases} \frac{\mu}{\lambda} (u_1^2 - u_2^2) \\ \frac{2\mu}{\lambda} (u_1^2 - u_2^2) f \end{cases}, \\ \left(\frac{d}{dr}\right)^2 u_1 + \left[\frac{1}{2} + (\mu_e + \varepsilon)^{-1} \frac{d\mu_e}{dr} \right] \frac{d}{dr} u_1 + (\varepsilon^2 - \mu_e^2) u_1 &= 0, \\ u_2 &= -(\mu_e + \varepsilon)^{-1} \frac{d}{dr} u_1, \end{aligned} \quad (7)$$

where the "effective" quark mass μ_e is defined by:

$$\mu_e = \frac{\partial^2 I}{\partial \bar{\psi} \partial \psi} = \begin{cases} \mu f / \lambda \\ \mu f^2 / \lambda^2 \end{cases} + \text{"bare mass"} \quad (8)$$

These equations must, of course, be solved with the boundary conditions that are derived from the requirements of regularity at the origin and asymptotic fall-off at infinity:

$$\begin{aligned} (r \rightarrow 0) \quad & \begin{cases} \frac{d}{dr} f(r) \rightarrow 0 \\ \frac{d}{dr} u_1(r) \rightarrow 0 \\ u_2(r) \rightarrow 0 \end{cases} \\ (r \rightarrow \infty) \quad & \begin{cases} f(r) \rightarrow \lambda \\ u_1(r) \rightarrow 0 \\ u_2(r) \rightarrow 0 \end{cases} \end{aligned} \quad (9)$$

Note, in particular, that asymptotic fall-off for $f(r)$ means approaching λ sufficiently fast, i.e. the vacuum expectation value of ϕ . These conditions will insure that the energy density (per unit length) of our solutions will be finite. We shall also require our solutions to satisfy the following conditions of normalization per unit length:

$$2\pi \int dr (u_1^2 + u_2^2) = n \quad (10)$$

where n is, for the moment, an arbitrary parameter.

Finally, our bound state picture²⁾ demands that:

$$f(r) \approx 0, \quad (r \rightarrow 0) \quad (11)$$

Using standard techniques, asymptotic solutions to the system of Eqs. (7) with the appropriate behaviour at the origin and at infinity, as dictated by Eq. (9), are easily found:

$$\begin{aligned} u_1(r) &\underset{r \rightarrow 0}{\sim} A J_0[(\epsilon^2 - \mu_e^2)^{1/2} r] \\ u_2(r) &\underset{r \rightarrow 0}{\sim} -A \frac{(\epsilon^2 - \mu_e^2)^{1/2}}{\epsilon + \mu_e} J_1[(\epsilon^2 - \mu_e^2)^{1/2} r] \end{aligned} \quad (12)$$

$$\begin{aligned} f(r) &\underset{r \rightarrow 0}{\sim} \begin{cases} \eta + \frac{1}{4} \left(\frac{\mu}{\lambda} A^2 + m_0^2 \eta \right) r^2 & (\bar{\psi} \psi \phi) \\ \eta I_0 \left[\left(\frac{2\mu}{\lambda^2} A^2 + m_0^2 \right) r \right] & (\bar{\psi} \psi \phi^2) \end{cases} \end{aligned}$$

$$\begin{aligned} u_1(r) &\underset{r \rightarrow \infty}{\sim} A' \exp[-(\mu^2 - \epsilon^2)^{1/2} r] / r \\ u_2(r) &\underset{r \rightarrow \infty}{\sim} -\frac{(\mu^2 - \epsilon^2)^{1/2}}{\epsilon + \mu} A' \exp[-(\mu^2 - \epsilon^2)^{1/2} r] / r \\ f(r) &\underset{r \rightarrow \infty}{\sim} \lambda - \eta' \exp[-m r] / r \end{aligned} \quad (13)$$

Here, the J 's denote conventionally defined, ordinary Bessel functions; $I_0(z) = J_0(iz)$ is a modified Bessel function of imaginary argument; μ_e is the effective quark mass near $r = 0$ [this will actually be zero unless a bare quark mass term is added to the Hamiltonian, Eq. (1)]; m_0^2 is the second derivative of the effective potential $V(\phi)$ [Eq. (4)] at the origin

$$m_0^2 = \frac{d^2 V}{d\phi^2} \Big|_{\phi=0} = 2(H + 6B); \quad (14)$$

and m is the physical mass of the scalar field ϕ :

$$m^2 = \frac{d^2 V}{d\phi^2} \Big|_{\phi=\lambda} = 2(H + 12B). \quad (15)$$

The constant coefficients A , A' , η , η' , and the spinor energy eigenvalue ϵ , are, of course, left unspecified by the "linearized" equations from which the asymptotic solutions (12) and (13) are derived. They will have to be determined from the complete solution and the normalization condition Eq. (10). Note, however, that we expect

$$\eta \ll 1 \quad (16)$$

by virtue of Eq. (11).

The asymptotic solutions at the origin, Eq. (12), and at infinity, Eq. (13), can be joined numerically to obtain complete solutions to Eq. (7) with any degree of accuracy. A procedure similar to the one summarized in the Appendix of Ref. 2 can be adopted. As we are here, however, only interested in the general features of the solutions, the approximate arguments based on the use of mechanical analogies and energetic considerations, also described in Ref. 2, will suffice.

The typical shape of the radial dependence for the bound state solutions under consideration, corresponding to the two interaction terms of Eq. (5), is exhibited in Fig. 3a and 3b. As indicated by the asymptotic solutions near the origin Eq. (12), the main effect of going from a Yukawa-type interaction ($\bar{\psi}\psi\phi$) to the more singular interaction $\bar{\psi}\psi\phi^2$ is to change the way in which the field amplitude $f(r)$ approaches zero "inside the bound state". Namely, in the latter case the approach is very drastic, of exponential type, as was the case for the spherically symmetric solutions (corresponding to the same choice of interaction) studied in detail in Ref. 2. In the former case of a Yukawa-type interaction, on the other hand, the approach to zero is much slower, only power behaved. Thus, the quark gas could then only be effectively free and massless along an essentially one-dimensional domain in the z -direction (... in the present instance of cylindrical solutions; and at one point, the origin, in the case of spherically symmetric solutions).

STRONG COUPLING LIMITS

We may estimate the energy density per unit length, $\mathcal{E}$, of such strings by approximately integrating the Hamiltonian density, Eq. (1), over the radial field configurations exhibited in Fig. 3a and 3b:

$$\mathcal{E} = \int dr \, 2\pi r \, \mathcal{H}(f(r), u(r)). \quad (17)$$

A very rough estimate, ignoring numerical coefficients, yields in the two cases, respectively (in analogy with Ref. 2):

$$\mathcal{E} \sim \begin{cases} \frac{m}{R} + \lambda^2 + (B+H)R^2 + I(m, \mu, \lambda, R), & (18) \\ \frac{m}{R} + \lambda^2 \frac{R}{D} + B R^2 + H R D, & (19) \end{cases}$$

where R parametrizes the transverse size of the strings and D the depth of the region in which the field amplitude $f(r)$ makes its transition from the condition of vacuum excitation ($f \sim 0$) to the condition of normal vacuum ($f \sim \lambda$). Note that, in the case of Fig. 3a, $D \sim R$ and it is complicated to estimate for general values of the Hamiltonian parameters the interaction energy I :

$$I = 2\pi \int dz \approx \frac{\mu}{\lambda} \bar{\psi}\psi \phi. \quad (20)$$

The analogous quantity is negligible in the case of Fig. 3b.

In the latter case, the equilibrium values of R and D are simply obtained by minimization of $\mathcal{E}$ (in analogy with Ref. 2):

$$D \sim \frac{\lambda}{\sqrt{H}}, \quad \frac{n}{R^3} \sim \frac{\lambda \sqrt{H}}{R} + B. \quad (\bar{\psi}\psi \phi^2) \quad (21)$$

Substituting back the quantities calculated in this way into the expression for the energy density $\mathcal{E}$ Eq. (19), it is easy to convince oneself that there exists a limit of the Hamiltonian parameters in which $D \rightarrow 0$ while $\mathcal{E}$ remains finite. This limit involves taking $\lambda \rightarrow 0$, $H \rightarrow \infty$, $\mu \rightarrow \infty$ and yields, in the case of spherically symmetric solutions, states that are better and better approximated by the MIT bags^{7,2)}. In the present case of cylindrical solutions, the same limit yields strings with infinitely sharp and deep (in the sense of being unpenetrable to the quark gas) walls. Notice however that these strings have a finite transverse size, R , (whose magnitude is mainly controlled by B) which cannot be made to vanish without causing a simultaneous, unphysical blow up of $\mathcal{E}$. In fact, the quark kinetic energy term, n/R , diverges as $k \rightarrow 0$ and this divergence cannot be cancelled by any other term on the r.h.s. of Eq. (19), as all of these terms are positive definite.

This conclusion is quite general. For instance, with the ordinary Yukawa interaction, Eq. (18), one can see that, for certain ranges of values of the parameters, the string's transverse size, R , is controlled by the bare quark mass μ so that, as $\mu \rightarrow \infty$, $R \sim 0$. However, in the same limit one finds $\mathcal{E} \rightarrow \infty$.

Thus, although strong coupling limits of our field theoretical models in the tree approximation are capable of generating "almost perfect bags", they are unable to generate "ideal strings", i.e. strings without a non-negligible transverse size. Needless to say, the reason is that while a "perfect bag" confines hadronic fields to a region of bounded support but with the same dimensionality, an "ideal string" confines fields to a manifold of lower dimensionality, which requires an infinite amount of kinetic energy.

ROTATING STRINGS

Our cylindrical string-like solutions are, as they stand, imperfect in another respect: they are infinitely long. What happens if we arbitrarily cut them off at two points along the z -axis is depicted in Fig. 4a; they become unstable against

contraction in the z-direction. Such a conclusion is obvious from the energetic point of view, for it is then energetically convenient for the system to shape itself in a spherically symmetric configuration.

One may attempt to cure the above mentioned instability by closing the string on itself, thereby changing its topological structure in favour of a toroidal configuration (Fig. 4b). However, it is apparent that for any finite value of the major radius L of the toroid another instability develops, this time against contraction in the radial direction L.

Both kinds of instability are effectively cured through the appearance of balancing centrifugal forces by imparting angular momenta to the systems. Such states, of non-vanishing angular momenta, may be studied within a variational approach, in the tree approximation, by solving the Euler-Lagrange equations derived from a variational principle with a Lagrange multiplier enforcing the angular momentum constraint. More modestly, one can simply solve the stability problem within certain types of configurations and attempt to abstract from these studies general conclusions.

Cylindrical configurations

We shall first consider our truncated cylindrical solutions. In the expression for the energy density per unit length (at rest) $\mathcal{E}$, Eq. (19), let us take, for simplicity, $D \sim R$ and write:

$$\mathcal{E} \sim \frac{m}{R} + \lambda^2 + K R^2, \quad K = B + H. \quad (22)$$

Setting such a truncated cylindrical solution of length L (this is, for the time being, a free parameter) in motion around its centre of mass, we obtain a system of energy $\mathcal{E}$ given roughly by (up to numerical coefficients):

$$\begin{aligned} \mathcal{E} &\sim \left(\frac{m}{R} + \lambda^2 + K R^2 \right) \int_0^L dL (1 - v_\perp^2)^{-1}, \\ &\sim \left(\frac{m}{R} + \lambda^2 + K R^2 \right) \frac{1}{\omega} \int_0^{\omega L} dv_\perp (1 - v_\perp^2)^{-1}, \end{aligned} \quad (23)$$

in obvious notations. Ignoring the spins of the constituents, the angular momentum of the system J is

$$\begin{aligned} J &\sim \left(\frac{m}{R} + \lambda^2 + K R^2 \right) \int_0^L dL L (1 - v_\perp^2)^{-1} v_\perp, \\ &\sim \left(\frac{m}{R} + \lambda^2 + K R^2 \right) \frac{1}{\omega^2} \int_0^{\omega L} dv_\perp (1 - v_\perp^2)^{-1} v_\perp^2. \end{aligned} \quad (24)$$

We shall make the identification

$$L \sim \frac{c}{\omega} = \frac{1}{\omega}, \quad (c=1) \quad (25)$$

as this clearly leads to the highest angular momentum state for a given energy. Observing that

$$n \sim \frac{1}{L} \sim \omega, \quad (26)$$

we find the stability condition for the (rest frame) transverse size of the rotating "string":

$$\left\{ \frac{\partial \mathcal{E}}{\partial R} \sim -\frac{\omega}{R^2} + KR = 0 \right\} \Rightarrow R \sim \left(\frac{\omega}{K} \right)^{1/3}. \quad (27)$$

Substitution into Eq. (23) and (24) yields the following expressions:

$$\mathcal{E} \sim (\lambda^2 + K^{1/3} \omega^{2/3}) \frac{1}{\omega}, \quad (28)$$

$$J \sim (\lambda^2 + K^{1/3} \omega^{2/3}) \frac{1}{\omega^2}. \quad (29)$$

For large J these relations imply approximately straight Regge trajectories

$$J \sim \alpha' \mathcal{E}^2 \quad (30)$$

of slope

$$\alpha' \sim \lambda^{-2}. \quad (31)$$

Our cylindrical solutions become correspondingly longer

$$L \sim \frac{1}{\omega} \sim \frac{\mathcal{E}}{\lambda^2}, \quad (32)$$

and thinner

$$R \sim \left(\frac{\omega}{K} \right)^{1/3} \sim \left(\frac{\lambda^2}{\mathcal{E} K} \right)^{1/3}, \quad (33)$$

as the angular momentum (energy) increases.

It is worth pointing out what the parallel result in the case of the MIT model is. As this model does not provide for surface kinetic energy and tension, Eq. (22) is replaced by

$$\mathcal{E} \sim \frac{M}{R} + BR^2 \quad (\text{MIT}) \quad (34)$$

where B is now the MIT "bag constant". Equation (30) is replaced by:

$$J \sim B^{-1} \mathcal{E}^4 \quad (35)$$

The corresponding "strings" become longer

$$L \sim \frac{1}{\omega} \sim \epsilon^3/B, \quad (36)$$

and thinner

$$R \sim \left(\frac{\omega}{B}\right)^{1/3} \sim \epsilon^{-1}, \quad (37)$$

much more rapidly than in the previous case. These conclusions will, however, be crucially modified by the inclusion of colour exchange and quantum fluctuation effects.

Toroidal configurations

Next, we wish to briefly discuss toroidal configurations¹¹⁾. Denoting by L and R the major and minor radii of the toroid, we obtain:

$$\epsilon \sim \left[(2\pi L)^2 \left(\frac{\gamma}{R} + \lambda^2 + K R^2 \right)^2 + J^2/L^2 \right]^{1/2}. \quad (38)$$

The stability condition for the minor radius R is identical to Eq. (27) and leads, with the help of Eq. (25) to

$$\epsilon \sim \left[(2\pi L)^2 \left(\lambda^2 + K^{1/3} L^{-2/3} \right)^2 + J^2/L^2 \right]^{1/2}. \quad (39)$$

Minimization of expression (39) yields, for large J , the following stability condition for the major radius L :

$$L \sim J^{1/2} / (\lambda \sqrt{2\pi}) \quad (40)$$

Thus, in the limit of large J , we obtain again straight Regge trajectories as in Eq. (30), with a slope

$$\alpha' \sim (2\pi \lambda^2)^{-1} \quad (41)$$

Notice that we have kept some numerical factors in this case, for this enables us to make the amusing observation of topological origin that Pomeron (closed strings) may naturally have a slope which is $1/2\pi$ smaller than ordinary trajectories.

COLOUR INTERACTIONS AND QUANTUM FLUCTUATIONS

Let us now attempt to see what modifications are implied by the introduction of coloured gluon interactions within our string-like solutions. We assume that the hadrons under considerations are colour-singlet states and, furthermore, that the expectation value of any coloured current j_c within such states vanishes identically:

$$\langle \text{had} | j_c(x) | \text{had} \rangle \equiv 0 . \quad (42)$$

Thus, at the first quantized level, colour interactions do not play any role and leave our previous conclusions unaffected.

The situation changes drastically when quantum fluctuations of the fields are included because of the non-vanishing of current correlations:

$$\langle \text{had} | j_c(x) j_c(0) | \text{had} \rangle \neq 0 . \quad (43)$$

To acquire a feeling for their effect, we may argue in the following terms.

The interior of our string-like solutions is in a state of permanent vacuum excitation, in the sense that $\langle \phi \rangle \sim 0$, and the effective mass of quarks and colour gauge fields vanishes. The exterior of our string-like solutions, on the other hand, resembles the spontaneously broken vacuum of the theory which, in turn, is reminiscent of the ground state of a superconductor. Thus $\langle \phi \rangle \neq 0$, quark and gauge fields have a non-vanishing (large) effective mass in it, and their amplitudes fall off within a "penetration depth". The complete system is simulated by a superconductor filling the entire space with a cylindrical hole in it.

Quark-anti-quark pairs (of vanishing effective mass) are copiously present (produced and destroyed) within the hole, as this costs little energy. The effect of a pair being separated at two points x_1, x_2 along the string (hole) is to temporarily generate a colour flux proportional to the colour charge g between x_1 and x_2 . Now, if the colour in question is of "magnetic" type, i.e. if it is a direct source to the dual of the "electromagnetic" tensor, the associated flux will tunnel completely within the hole (increased by a penetration depth) as the "magnetic" field is tangent to the superconductor's surface. Then, between x_1 and x_2 , at times, there will be a (space-) average "magnetic" field inversely proportional to the cross-section of the string (hole) ($\sim g/R^2$). This leads to an energy density per unit length:

$$\mathcal{E}' \sim \left(\frac{g}{R^2} \right)^2 R^2 \sim \frac{g^2}{R^2} . \quad (44)$$

If, instead, the colour in question is of ordinary, "electric" type, i.e. if it is a direct source to the "electromagnetic" tensor itself, "flux leakage" to the superconductor will occur, as the "electric" field is orthogonal to the superconductor's surface. We are then not any longer led to the energy density term given in Eq. (44). The latter is however present (by fiat) in the case of the MIT model in virtue of the magnetic type boundary conditions that are postulated for the gauge fields in that model.

The inclusion of the energy density term given in Eq. (44) into Eq. (22) yields:

$$\mathcal{E} \sim \frac{q^2}{R^2} + \frac{\omega}{R} + \lambda^2 + K R^2 \quad (45)$$

and modifies our previous conclusions. Following through our analysis, one sees that Eq. (27) is replaced by

$$\frac{\partial \mathcal{E}}{\partial R} \sim -\frac{q^2}{R^3} - \frac{\omega}{R^2} + K R = 0 \quad (46)$$

which implies the existence of an asymptotic value for the string's transverse size R

$$R_{\min} \sim \sqrt{q/K}^{1/4} \quad (47)$$

Eq. (30) is still valid asymptotically, but the associated slope α' is now given by:

$$\alpha' \sim (\lambda^2 + \sqrt{K} q)^{-1} \quad (48)$$

This reduces to

$$\alpha' \sim (\sqrt{B} q)^{-1} \quad (49)$$

in the case of the MIT model, for which the Regge trajectories are now also "straight" asymptotically. Similarly, the slope of the trajectories corresponding to toroidal configurations, Eq. (41), is replaced by:

$$\alpha' \sim [2\pi (\lambda^2 + \sqrt{K} q)]^{-1} \quad (50)$$

and for the particular case of the MIT model:

$$\alpha' \sim (2\pi \sqrt{B} q)^{-1} \quad (51)$$

DOG-BONE CONFIGURATIONS

To establish stronger links with the duality framework, it is interesting to consider configurations in which the quark amplitudes are mostly concentrated toward the ends of the strings. We shall refer to these as dog-bone configurations because of obvious analogies (Fig. 5). Corresponding to such configurations a net, non-vanishing colour charge [possibly magnetic (monopole) charge¹²⁾] could be present at one end, balanced by the opposite charge at the other end.

With the notations of Fig. 5, the energy-angular momentum relation for this configuration, in a regime in which most of the angular momentum is contributed by the "spheres" at the ends, may approximately be written in the form:

$$\mathcal{E} \sim \left\{ \left[\frac{1}{R} + \frac{g^2}{a^2} L + \lambda^2 (R+L) + K(R^3 + a^2 L) \right]^2 + \frac{J^2}{L^2} \right\}^{1/2} \quad (52)$$

The equilibrium conditions for the various parameters are immediately obtained:

$$R \sim K^{-1/4}, \quad (53)$$

$$a \sim (g/\sqrt{K})^{1/2}, \quad (54)$$

$$L \sim [J/(g\sqrt{K} + \lambda^2)]^{1/2}, \quad (55)$$

where, in determining R , we have, for simplicity, neglected the surface as compared to the volume energies of the spheres. Substitution into Eq. (52) yields again approximately straight Regge trajectories for large J of slope:

$$\alpha' \sim (\lambda^2 + g\sqrt{K})^{-1} \quad (56)$$

Notice that this expression is (up to unspecified numerical coefficients) identical to expression (48) for the cylindrical string. We expect a similar expression to also hold for the true equilibrium string configuration, which, of course, does not coincide with any of the particular configurations discussed here.

CONCLUSION

The main motivation for this note was to suggest a unified picture of hadrons as field-theoretical bags and strings. In analogy with rather well known phenomena in nuclear physics, the transition from spherical, to ellipsoidal and on to string-like structures would be governed by the hadron's angular momentum or spin. We hope

that our calculations and estimates carried out within particular field configurations [cylindrical, toroidal¹¹⁾, "dog-bone"], mostly at the first-quantized level, have made this attractive possibility more plausible.

An obvious question to ask ourselves at this point is if and when we would expect such a picture to break down. Assuming that colour is of ordinary, non-magnetic type, a partial answer to this question is that we would then expect it to occur eventually and to be intimately related to a turnover of Regge trajectories and the appearance of non-zero triality final states. This would be the time when our "temporary confinement" would end and our quarks would be set free. We remind the reader that in our models (in contrast to the MIT model) quark confinement is a "low" energy phenomenon as much as the confinement of the gauge bosons of the weak interactions is (presumably) a low energy phenomenon. In fact, quarks and weak interaction gauge bosons would acquire their bare masses (for some mysterious reason large, but finite) by similar mechanisms based upon spontaneous symmetry breakdown. Colour gauge fields would also be, in a similar way, only temporarily trapped; the "electric-type" colour flux that they carry would leak through the skins of bags and strings. Thus the string concept would also eventually break down.

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- 10) Note that these restrictions, while simplifying the discussion, leave out interesting string-like structures.
- 11) These configurations and a possible connection with the dual string picture have independently been considered in Ref. 8.
- 12) Strings with magnetic charges at the ends have been suggested by: Y. Nambu Lecture at the John Hopkins Workshop on Current Problems in High Energy Particle Theory (1974); G. Parisi, unpublished; M. Creutz, BNL preprint 18983 (1974); P. Olesen, unpublished.

Figure captions

- Fig. 1 : Picturesque view of the "deformation" of hadrons due to the excitation of large angular momenta.
- Fig. 2 : Self-interaction potential of the scalar field ϕ .
- Fig. 3 : Shape of the radial dependence of the scalar and quark amplitudes corresponding to bound state solution in models with $\bar{\psi}\psi\phi$ (a) and $\bar{\psi}\psi\phi^2$ (b) couplings.
- Fig. 4 : a) Truncated cylindrical solution.
b) Toroidal configuration.
- Fig. 5 : "Dog-bone" configuration.

FIG.1

FIG. 2

3,a

3,b

4,a

4,b

FIG. 5