CM-P00060598

Archives

Ref.TH.1950-CERN

RESTRICTIONS FOR ASYMMETRY AND POLARIZATIONS
OF RECOIL IN MUON CAPTURE

J. Bernabeu
CERN -- Geneva

A B S T R A C T

Using the helicity formalism, we discuss muon capture by targets of spin-zero. Owing to the definite neutrino helicity, three independent observables define a complete experiment. The precise relation between asymmetry α and longitudinal polarization P_L of recoil, $\alpha = 1 + 2jP_L$, comes only from rotational invariance. When time-reversal invariance is inserted, there is an additional restriction between the average polarization P_{av} and the longitudinal polarization P_L . On the basis of the experimental result $P_{av} = 0.43 \pm 0.10$ for ^{12}C , we predict

$$P_L = -(0.99_{+0.01}^{-0.04}).$$

It seems that the measurement of angular distributions and polarizations of recoils, in nuclear muon capture, is already a new field to explore. This can settle old questions, as universality of β decay and μ capture form factors and, in particular, if the G-T relation for the pseudoscalar coupling constant is satisfied. A measurement of the average polarization (see below) of the recoil nucleus in the process $\mu^- + {}^{12}\text{C} \rightarrow {}^{12}\text{B} + \nu$ has been made by the Louvain-Saclay group ¹⁾ and the result compared with the theory of Devanathan et al. ²⁾⁻³⁾. A value of $g_p = (12 \pm 5)g_A$ is obtained, when all other form factors are fixed to their canonical values. The same group plans to measure other observables in the same process.

The purpose of this note is to show, without any explicit form of the nuclear muon capture interaction, what are the relations between the different observables which are accessible experimentally. We only use rotational invariance and the defined helicity of the neutrino. For the time being, we restrict ourselves to targets of spin zero in order to avoid the problems of the hyperfine interaction in the initial state. Apart from that, the results are completely general, in particular, they are valid for any spin of the final nuclear state.

Although stated in another language, the kind of restrictions in which we are interested are well known ⁴⁾ for hyperon decays, such as the asymmetry parameters in $\Lambda \rightarrow p\pi^-$. In some sense, our process is similar to this case, at least for the purpose of counting different amplitudes. The initial state (μN_i) in muon capture is bound and has a definite angular momentum. In the final state, both particles N_f and ν have spin, but the helicity of the neutrino is $-\frac{1}{2}$. If time-reversal invariance is imposed, the absence of final state interactions in muon capture gives the reality condition without any problems.

We consider the reaction

$$\mu^- + N_i (J_i = 0) \rightarrow N_f (J_f = \frac{1}{2}) + \nu \quad (1)$$

in the rest frame of the initial state. This corresponds, in the final state, to the c.m. system, with the recoil momentum $\vec{p} = -\vec{\nu}$. We characterize the physical states in the helicity basis, and we expand them in terms of basis states of the irreducible representations of the rotation group. The moving reference

system, associated with the direction $\hat{p}$, is obtained from the rest system through Euler angles $(\phi, \theta, 0)$ (see Fig. 1). The convention about phases, normalizations, ..., follows the book of Martin-Spearman ⁵⁾.

The final state is constructed as the direct product of the two single-particle states in the c.m. system, characterized by the helicity values λ_f , $-\frac{1}{2}$, for the recoil nucleus and neutrino, respectively. The expansion in terms of definite angular momentum is written

$$|\theta \phi; \lambda_f, -\frac{1}{2}\rangle = \sum_{s, m} \left(\frac{2s+1}{4\pi}\right)^{1/2} \mathcal{D}_{m\lambda}^{(s)}(\phi, \theta, 0) |s, m; \lambda_f, -\frac{1}{2}\rangle \quad (2)$$

where $\lambda = \lambda_f + \frac{1}{2}$, and $\mathcal{D}^{(s)}$ is the matrix representation of the rotation operator. As the initial state is characterized by angular momentum $s = \frac{1}{2}$, the transition amplitude is given by

$$f_{\lambda m} \equiv \langle \theta \phi; \lambda | M | \frac{1}{2} m \rangle = \left(\frac{1}{2\pi}\right)^{1/2} \mathcal{D}_{m\lambda}^{(1/2)*}(\phi, \theta, 0) M_{\lambda} \quad (3)$$

where M_{λ} is the partial wave helicity amplitude, corresponding to total angular momentum $\frac{1}{2}$.

From Eq. (3) it is clear that two amplitudes $M_{\frac{1}{2}}$ and $M_{-\frac{1}{2}}$ determine everything that can be learnt from the process (1) (this is due to have a fixed helicity for the neutrino). With the two absolute values and the relative phase at our disposal, three independent observables exhaust all the information we can obtain from (1).

If the initial muon, when it stays at the 1s orbit of the atom, has a residual polarization $\vec{P}_{\mu}$, its density matrix is given by

$$\rho_i = \frac{1}{2} \left\{ 1 + 2 \vec{P}_{\mu} \cdot \vec{S} \right\} \quad (4)$$

Their matrix elements are taken in the initial basis, corresponding to a reference in which the polarization $\vec{P}_\mu$ is pointing along the z axis. The final state density matrix is expressed in the basis of states (2) and it is obtained from (4) in the form

$$(\rho_f)_{\lambda\lambda'} = \frac{1}{2} \sum_m f_{\lambda m} \left\{ 1 + 2m |\vec{P}_\mu| \right\} f_{\lambda' m}^* \quad (5)$$

This equation (5) is our starting point to calculate all the observables we are interested in. From Eq. (3) it is a simple task to show

$$\begin{aligned} \frac{1}{2} \sum_m f_{\lambda' m} f_{\lambda m}^* &= \frac{1}{4\pi} \delta_{\lambda, \lambda'} |M_\lambda|^2 \\ \frac{1}{2} \sum_m 2m f_{\lambda' m} f_{\lambda m}^* &= \frac{1}{4\pi} \left\{ \delta_{\lambda, \lambda'} 2\lambda |M_\lambda|^2 \cos \theta \right. \\ &\quad \left. - [\delta_{\lambda-\lambda', 1} M_{-\frac{1}{2}} M_{\frac{1}{2}}^* + \delta_{\lambda-\lambda', -1} M_{\frac{1}{2}} M_{-\frac{1}{2}}^*] \sin \theta \right\} \end{aligned} \quad (6)$$

Equations (6) show explicitly the axial symmetry of our problem. It is obvious physically that all observables must be independent of ϕ .

The angular distribution of recoils is obtained from the trace of ρ_f . Using (6), with $\lambda = \lambda'$, in Eq. (5) we obtain for

$$I(\theta, \phi) = \frac{\Gamma}{4\pi} \Lambda(\theta)$$

the following result

$$\begin{aligned} \Gamma &= \sum_\lambda |M_\lambda|^2 \\ \Lambda(\theta) &= 1 + 2 \frac{\sum_\lambda \lambda |M_\lambda|^2}{\sum_\lambda |M_\lambda|^2} |\vec{P}_\mu| \cos \theta \end{aligned} \quad (7)$$

The value of Γ is the capture rate, whereas the asymmetry parameter α in the angular distribution

$$\Lambda(\theta) = 1 + \alpha (\vec{P}_\mu \cdot \hat{p})$$

is given by

$$\alpha = 2 \frac{\sum_{\lambda} \lambda |M_{\lambda}|^2}{\sum_{\lambda} |M_{\lambda}|^2} \quad (8)$$

For the longitudinal polarization of the recoil nucleus we write

$$(\vec{P} \cdot \hat{p}) I(\theta, \phi) = \text{Tr} \left[\frac{(\vec{J} \cdot \hat{p})}{J} \rho_f \right] \quad (9)$$

and we use $(\vec{J} \cdot \hat{p})_{\lambda' \lambda} = (\lambda - \frac{1}{2}) \delta_{\lambda \lambda'}$. A simple glance at Eqs. (6) gives the answer

$$(\vec{P} \cdot \hat{p}) \Lambda(\theta) = P_L \left\{ 1 - (\vec{P}_\mu \cdot \hat{p}) \right\} \quad (10)$$

with the following expression for the longitudinal polarization P_L in the case of unpolarized muons

$$P_L = \frac{\sum_{\lambda} (\lambda - \frac{1}{2}) |M_{\lambda}|^2}{\sum_{\lambda} |M_{\lambda}|^2} \quad (11)$$

It is worth noticing the relation between the asymmetry in the angular distribution (8) and the longitudinal polarization of the residual nucleus (11). The precise connection is given by

$$\alpha = 1 + 2j P_L \quad (12)$$

and it is, therefore, a consequence of rotational invariance. This relation had to appear due to the lack of interference between the two helicity amplitudes in these observables. Recently ⁶⁾, Eq. (12) has been shown to hold for the Fujii-Primakoff effective Hamiltonian responsible for the nuclear muon capture process, after an explicit calculation involving nucleon coupling constants and nuclear matrix elements. We have seen that this connection is not only independent of the particular values of these last ingredients, but instead, it is completely independent of the dynamics. The form of the weak interaction Lagrangian and its extension to nuclei only affect the explicit values of the two amplitudes $M_{\frac{1}{2}}$ and $M_{-\frac{1}{2}}$. Equation (12) is independent of all that. We remember that, apart from the characterization of states in terms of basis states of the representations of the rotation group, inherent to the formalism, we have used the fact that the neutrino helicity is definite.

The longitudinal polarization jP_L is allowed to have values between -1 and 0, whereas the asymmetry α goes from -1 to +1 ^{*)}.

With present techniques, the detection of the direction of the recoil nucleus appears difficult to accomplish, whereas it is accessible to measure polarizations when averaged over certain regions of the polar angle θ [see the discussions in Refs. 1) and 3)]. We are going to show that, in the general framework of our study, it is possible to learn, from these average polarizations, the same information which is contained in detailed angular distributions of the polarization.

As we have commented, the formalism is well suited in order to discuss the polarization vector in terms of the reference system of the final state. The longitudinal polarization, along the z' axis, was obtained in Eqs. (10) and (11). In a similar way, we can get the transverse polarization, i.e., the

*) If values for these observables are obtained which are outside the limits, as in Ref. 3), this must be due to computational round-off. It cannot be blamed as a counter-indication for particular values of coupling constants.

component along the x' axis in Fig. 1. From Eq. (6) we see that, dynamically, this component is determined by the value of $\text{Re} (M_{\frac{1}{2}} M_{-\frac{1}{2}}^*)$, a new piece of information different from P_L or α . When we refer the polarization to the rest system, the component along the z axis is given by

$$P_z = P_{z'} \cos \theta - P_{x'} \sin \theta \quad (13)$$

If we average over all unphysical azimuthal angles ϕ , this component P_z is the only one that remains. We are interested in the polarization when averaged over certain directions of the recoil nucleus. In fact, if this direction is not observed we need to integrate over values of θ , in (13), between 0 and π . However, if the observed polarization is the one which has been maintained only within some hemisphere, one can define two different experimental quantities

$$\begin{aligned} \vec{P}_f &\equiv \frac{1}{2} \int_0^{\pi/2} d\theta \sin \theta \vec{P}(\theta) \Lambda(\theta) \\ \vec{P}_b &\equiv \frac{1}{2} \int_{\pi/2}^{\pi} d\theta \sin \theta \vec{P}(\theta) \Lambda(\theta) \end{aligned} \quad (14)$$

corresponding to the situations in which the final polarization remains in the forward region (and it is observed from the corresponding β decay) and the case in which it is preserved only in the backward hemisphere. The total average polarization is $\vec{P}_{av} = \vec{P}_f + \vec{P}_b$.

Using the matrix element of $J_{x'}$, for the final nucleus

$$(J_{x'})_{\lambda\lambda'} = \frac{1}{2} \sqrt{j(j+1)} [\delta_{\lambda-\lambda',1} + \delta_{\lambda-\lambda',-1}]$$

together with Eqs. (13) and (6) we are able to calculate the polarizations (14) with the result

$$\vec{P}_f = \frac{1}{2} \vec{P}_{av} + \frac{1}{4} P_L \hat{z} \quad ; \quad \vec{P}_b = \frac{1}{2} \vec{P}_{av} - \frac{1}{4} P_L \hat{z} \quad (15)$$

The value of P_L is given in (11) and the total average polarization is expressed as

$$\vec{P}_{av} = \vec{P}_\mu \frac{1}{3} \left\{ \sqrt{\frac{j+1}{j}} \frac{2 \operatorname{Re}[M_{\frac{1}{2}} M_{-\frac{1}{2}}^*]}{\sum_\lambda |M_\lambda|^2} - P_L \right\} \quad (16)$$

We see explicitly that the measurements of $\vec{P}_f$ and $\vec{P}_b$ determine the values of $\vec{P}_{av}$ and P_L . Of course, in order to extract from $\vec{P}_{av}$ in (16) the information we want, the value of the residual muon polarization $|\vec{P}_\mu|$ must be known independently. The average polarization depends on the relative phase between the two helicity amplitudes and thus gives the new independent observable we needed. This completes the information which is possible to obtain from (1). Whereas this statement gives a restriction to the number of independent combinations of weak form factors which can be "measured" from a given reaction, it is saying that the determination of $\vec{P}_f$ and $\vec{P}_b$ (together with the capture rate) defines a complete experiment, no matter whether the recoil direction cannot be detected. The experimental determination of the asymmetry parameter α , for example, should be considered, when compared with P_L through Eq. (12), as a test of the neutrino helicity, i.e., in that situation we are at the level of isolating the V-A character of the leptonic sector.

The next question we wish to consider is the following : what is the result of imposing time-reversal invariance for the present process ? If we allow for the well-accepted dynamical input that the transition operator M , in Eq. (3), is described by a first order weak interaction effective Hamiltonian, time-reversal invariance says that the helicity amplitudes M_λ must be real : $M_\lambda = M_\lambda^*$. With the absence of final state interactions in our problem, this condition will be satisfied in real life too.

As the modulus of each amplitude is determined by the capture rate and the longitudinal polarization

$$|M_{\frac{1}{2}}|^2 = \Gamma(1 + j P_L) \quad ; \quad |M_{-\frac{1}{2}}|^2 = -j P_L \Gamma \quad (17)$$

the value of P_{av}/P_μ in Eq. (16) can be predicted, allowing both possibilities for the relative sign of these two amplitudes in the first term within brackets.

It is possible to write a constraint between $P \equiv P_{av}/P_{\mu}$ and P_L in the following form

$$[1 + 4j(j+1)] P_L^2 + 2[3P + 2(j+1)] P_L + 9P^2 = 0 \quad (18)$$

For a value of the angular momentum of the final nucleus $j = 1$, the limiting values of P are given by

$$-\frac{1}{3} \leq P \leq \frac{2}{3} \quad (19)$$

and the correlation between the average polarization P and the longitudinal polarization P_L , Eq. (18), is plotted in Fig. 2. The study of the solutions of (18) shows that the full line, in the Figure, corresponds to equal signs for both amplitudes, whereas the dotted-dashed line gives the solution for opposite relative sign.

On the basis of the present experimental result ¹⁾ for the average polarization in $^{12}\text{C} \rightarrow ^{12}\text{B}$

$$P_{av}/P_{\mu} = 0.43 \pm 0.10 \quad (20)$$

we obtain the solutions for the longitudinal polarization

$$P_L = - (0.99 \pm 0.04) \quad (21)$$

or

$$P_L = - (0.19 \pm 0.08) \quad (22)$$

These results are indicated in Fig. 2 by projecting the value (20) on the restriction curve. We would like to stress that any experimental deviation from these predictions is a clear-cut evidence for time-reversal violation, without any detailed dynamical input.

It may be mentioned, at this point, that a simple glance at the theoretical calculations of Refs. 2) and 3) shows that the nuclear amplitudes of this particular process are such that, in our language, we can safely say that the helicity amplitude $|M_{-\frac{1}{2}}|$ is expected to be larger than $|M_{\frac{1}{2}}|$, unless pathological values of nucleon coupling constants (and/or, to a lesser extent, nuclear matrix elements) are considered. With this information, the value (22) can be ruled out. It is surprising that, from (20), one can predict the longitudinal polarization (21) with so small an error.

As we have emphasized before, once P_L is experimentally known, the separate measurement of α is a test of the definite neutrino helicity. From (21) and (12), the value

$$\alpha = - \left(0.98 - \frac{0.08}{+ 0.02} \right) \quad (23)$$

is expected. Actually this asymmetry parameter is large. That could help in seeing the effect from the angular distribution.

We close this discussion with the remark that, from a theoretical point of view, it follows from our study : all physically relevant observables, in the case of ^{12}C , have already been considered in the literature. The dynamical determination of P_{av} is undertaken in Ref. 2), whereas we refer to 7), for example, for the capture rate Γ . The lesson we have learnt from these studies is that, whereas Γ depends crucially on the nuclear physics part of the problem, P_{av} can give a quite accurate determination of the nucleon form factors.

ACKNOWLEDGEMENTS

The author would like to acknowledge critical remarks by T. Ericson, M. Jacob and N. Mukhopadhyay. N.M. brought Ref. 6) to his attention.

REFERENCES

- 1) A. Possoz, D. Favart, L. Grenacs, J. Lehmann, P. Macq, D. Meda,
L. Palfy, J. Julien and C. Samour, Phys.Letters 50B, 438 (1974).
- 2) V. Devanathan, R. Parthasarathy and P.R. Subramanian, Ann.Phys. 73,
291 (1972).
- 3) V. Devanathan and P.R. Subramanian, Madras preprint N.Ph./29/1974.
- 4) T.D. Lee and C.N. Yang, Phys.Rev. 108, 1615 (1957).
- 5) A.D. Martin and T.D. Spearman, "Elementary Particle Theory", North-
Holland, Amsterdam (1970).
- 6) P.R. Subramanian and V. Devanathan, Madras preprint N.Ph./30/1974.
- 7) N.C. Mukhopadhyay and M.H. Macfarlane, Phys.Rev.Letters 27, 1823 (1971) ;
Phys.Rev.Letters 28, 1545E (1972).

- . . -

FIGURE CAPTIONS

Fig. 1 Rest frame, and conventions for the final state reference system.
The z axis is pointing along the residual muon polarization.

Fig. 2 Restriction line between the average polarization P_{av}/P_{μ} and the
longitudinal polarization P_L , due to time-reversal invariance.
The full line corresponds to equal signs for the helicity amplitudes,
the dashed-dotted line to opposite signs. The experimental result ¹⁾
for P_{av} is projected on the curve to obtain the allowed values
for P_L .

FIG.1

FIG. 2