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THE VARIATION OF THE VACUUM POLARIZATION POTENTIAL
OF ORDER $(Z\alpha)^3$ ON NUCLEAR CHARGE DISTRIBUTIONS

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A B S T R A C T

Nuclear size effects are calculated using the expression of the vacuum polarization potential of order $(Z\alpha)^3$ previously derived by Owen. The variation of this potential on nuclear charge distributions is shown for point charge, Yukawa and exponential distributions for ^{208}Pb .

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1. INTRODUCTION

Muonic x-ray data are not only a means for better test of quantum electrodynamics, but give information on nuclear sizes and shapes as well ¹⁾. The increasingly more accurate data provide further challenge for those engaged in calculating more precise nuclear charge distributions ²⁾. Not only must previous work be done more carefully and to a higher degree of accuracy, but more subtle finite size effects must be taken into account as those occurring in recoil corrections ³⁾ as well as in radiative corrections.

In the present paper we shall try to determine the importance of nuclear size effects in modifying the third order vacuum polarization potential. This potential which results from the diagram in Fig. 1 is the most important radiative correction in which the finite size effects are non-linear. The method used is based on the formulation of the vacuum polarization by Owen ⁴⁾ which provides an alternative to several recent works ⁵⁾ which are based on the work of Wichmann and Kroll ⁶⁾ and to a numerical study by Gyulassy ⁷⁾. The calculations based on the Wichmann-Kroll method tend to increase the existing discrepancy between theory and experiment ¹⁾ thus making an alternate calculation desirable.

2. THE THIRD ORDER VACUUM POLARIZATION POTENTIAL

The vacuum polarization potential arising from the Feynman-like graph in Fig. 1 can be written in a compact form ⁴⁾. In momentum space it is given by

$$V_{VP}(q) = -i \frac{4\pi\alpha}{q^2} \int \frac{d^4p}{(2\pi)^4} \left\{ 4 (p - eA)_\mu (g^{\mu\nu} + ie \Delta_c F^{\mu\nu}) \Delta_c \right. \\ \left. \times \frac{1}{1 - \frac{e^2}{2} F^{\sigma\tau} \Delta_c F_{\sigma\tau} \Delta_c} \right\} \quad (1)$$

where notations of Ref. 4) are used. For the graph (Fig. 2) giving the third order contribution the quantity between brackets is calculated to be

$$4 e A_0(\vec{p} - \vec{w}) e A_0(\vec{w} - \vec{v}) e A_0(\vec{v} - \vec{k}) \mathcal{R}(q) \quad (2)$$

where

$$\begin{aligned} \mathcal{G}(q) = & \left[p_0^4 + p_0^2 (6m^2 + \vec{w} \cdot \vec{p} + \vec{p} \cdot \vec{v} + \vec{p} \cdot \vec{k} + \vec{w} \cdot \vec{v} \right. \\ & + \vec{v} \cdot \vec{k} + \vec{w} \cdot \vec{k}) + m^4 + m^2 (\vec{p} \cdot \vec{w} + \vec{w} \cdot \vec{v} \\ & - \vec{w} \cdot \vec{k} + \vec{v} \cdot \vec{k} - \vec{p} \cdot \vec{v} + \vec{p} \cdot \vec{k}) + (\vec{p} \cdot \vec{w}) (\vec{w} \cdot \vec{v} \\ & - \vec{w} \cdot \vec{k} + \vec{v} \cdot \vec{k}) + w^2 (\vec{p} \cdot \vec{k} - \vec{p} \cdot \vec{v}) \left. \right] \left\{ [p_0^2 - E^2(p)] \right. \\ & \left. [p_0^2 - E^2(w)] [p_0^2 - E^2(v)] [p_0^2 - E^2(k)] \right\}^{-1} \end{aligned} \quad (3)$$

with $k = p - q$. The renormalized expression for (2) is

$$4e A_0(\vec{p} - \vec{w}) e A_0(\vec{w} - \vec{v}) e A_0(\vec{v} - \vec{k}) [\mathcal{G}(q) - \mathcal{G}(0)] \quad (4)$$

Expression (1) was evaluated using (4) as the integrand for different values of the external potential A_0 . Point charge and Yukawa ^{*)} and exponential charge distributions were considered. The form of the distributions have been chosen as to make the calculation manageable. $V_{vp}^{(3)}(q)$ can be written as follows

$$V_{vp}^{(3)}(q) = \frac{3}{2\pi q^2} \alpha (Z\alpha)^3 V_D(t) \quad (5)$$

where $t = q^2/m^2$.

*) We wish to thank S.J. Brodsky for pointing out the calculational advantages of the Yukawa charge distribution.

$V_D(t)$ depends on the charge distribution. It is expressed by a sum of seven six-fold integrals which were evaluated numerically for the example of ^{208}Pb . The integration routine used is basically a Rieman sum with an automatic adjustment of the intervals on each axis ⁸⁾. An accuracy of 5% was requested and obtained. The computed values are plotted in Fig. 3. From this figure one can assert that finite size effects are important for this diagram.

In particular, we note that in the region of large t , that is, the region in which the wave function (for s states) is large, the finite size strongly dampens the strength of the potential. This suggests that the finite size effect lowers the energy correction of the s states (as compared to the point-charge value) for the $(Z\alpha)^3$ vacuum polarization diagram. On the other hand, since for high t the point charge distribution and the finite size distributions merge, we expect for non s states that the finite size effects will be negligible. Hence, qualitatively, we agree with the results of Rinker and Wilets ⁵⁾. Furthermore, it is self-evident from our method, that as we let our finite size parameter go to zero, we recover our point charge result. It is also interesting to note that the widely differing charge distributions, i.e., the Yukawa and the exponential lead to almost identical potentials which are markedly different, at large t , to the point charge potential.

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FIGURE CAPTIONS

- Fig. 1 : Vacuum-polarization diagram. The double line means a particle propagating in an external field.
- Fig. 2 : Third order vacuum-polarization diagram. X indicates the action of the external field.
- Fig. 3 : Variation of $V_D(t)$ on nuclear charge distributions. P, Y and E mean point charge and Yukawa and exponential charge distribution. $t = q^2/m^2$.

FIG. 1

FIG. 2

FIG. 3