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CONSTRAINTS ON THE CHARGED MULTIPLICITY DISTRIBUTIONFROM NEUTRAL CHARGED PARTICLE CORRELATIONSV. Gupta ^{*)}

CERN -- Geneva

A B S T R A C T

It is pointed out that the knowledge of $\bar{N}_0(n_-)$, the average number of neutrals as a function of n_- gives constraints on the probability $P(n_-)$ for producing n_- negatively charged particles, given some assumptions about higher neutral and negative particle correlations. These constraints enable one to calculate $P(n_-)$ given some input from data. Fits to the data on $P(n_-)$, for pp collisions above 50 GeV/c, are given for the assumption that $\bar{N}_0(n_-)$ is linear in n_- .

*) On leave from the Tata Institute of Fundamental Research, Bombay 400005, India.

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At present the only data on neutrals which are available at various energies, for pp collisions, are on $\bar{N}_0(n_-)$, the average number of neutrals for a given number n_- of negatively charged particles. The expression for $\bar{N}_0(n_-)$ depends on the neutral negative particle correlations and on the charged multiplicity distribution $P(n_-)$, that is, the probability for producing n_- negatively charged particles. Many authors¹⁾ have studied the n_- dependence of $\bar{N}_0(n_-)$ using various models for $P(n_-)$. In this note we turn the problem around by taking the dependence of $\bar{N}_0(n_-)$ on n_- as known. This coupled with some simple assumption about the higher neutral negative correlations gives us constraints on $P(n_-)$ in the form of recurrence relations. These then enable us to calculate or "predict" the $P(n_-)$ given some input from the experimental data.

Consider the generating function $G(h_-, h_0)$ where h_- and h_0 are the generators associated with the negative and neutral particles, respectively. The general form of G , incorporating charge conservation constraints, is

$$G(h_-, h_0) = \exp \left[\sum_{b,c} \frac{f_{0bc}}{b!c!} h_-^b h_0^c \right] \quad (1)$$

where f_{0bc} is the correlation integral²⁾, for zero positive, "b" negative and "c" neutral particles. The above equation also serves to define the f_{0bc} . Note that $f_{010} = \langle n_- \rangle$ and $f_{001} = \langle n_0 \rangle$ are the average number of negative and neutral particles, while f_{0bc} with $b, c \geq 1$ will introduce correlations among them. Further, f_{0b0} with $b \geq 2$ give the correlation integrals for the negative particles only. For these, we will use the simpler notation $f_b^- \equiv f_{0b0}$ for $b \geq 1$.

By definition,

$$P(n_-) \bar{N}_0(n_-) = \frac{1}{n_-!} \left(\frac{\partial}{\partial h_-} \right)^{n_-} \left(\frac{\partial}{\partial h_0} G(h_-, h_0) \right) \Big|_{h_0=0, h_-=-1} \quad (2)$$

It is clear that the expression for $\bar{N}_0(n_-)$ will only involve the neutral negative correlation integrals f_{0bc} with $c \leq 1$, b arbitrary. Hopefully, the correlation integrals f_{0bc} with $b > 2$ would be small so that one may get by with keeping only f_{011} and f_{021} . We study here the implications of having at most non-zero f_{011} and f_{021} with $f_{0b1} = 0$ for $b \geq 3$. From Eq. (2), one then obtains

$$\bar{N}_0(n_-) = \langle n_0 \rangle - f_{011} + \frac{1}{2} f_{021} + (f_{011} - f_{021}) \frac{P(n_- - 1)}{P(n_-)} + \frac{1}{2} f_{021} \frac{P(n_- - 2)}{P(n_-)} \quad (3)$$

Note that the alternative notation f_2^{-0} for f_{011} and f_3^{-0} for f_{021} is used by some authors.

A basic feature of the $\bar{N}_0(n_-)$ data ³⁾ is that the points seem to lie on a straight line and for high energies $\bar{N}_0(n_-)$ increases with n_- . However, the errors on the data points are rather large at present and the n_- dependence may well be different from a simple linear dependence. For most of our work we will take it that a linear dependence of $\bar{N}_0(n_-)$ on n_- gives a satisfactory description of the data from 50 GeV/c upwards. For the moment, consider that $\bar{N}_0(n_-)$ has a quadratic dependence on n_- , given by

$$\bar{N}_0(n_-) = a + b(n_- - \langle n_- \rangle) + c(n_- - \langle n_- \rangle)^2 \quad (4)$$

Then using the definitions $\langle n_0 \rangle = \sum n_- P(n_-) \bar{N}_0(n_-)$, etc., one can directly calculate ⁴⁾, from Eq. (4), that

$$a = \langle n_0 \rangle - cD, \quad \Delta b = A f_{011} - B f_{021}, \quad \Delta c = -C f_{011} + D f_{021} \quad (5)$$

where

$$A = f_4^- + 5 f_3^- + 4 f_2^- + 2 (f_2^- + \langle n_- \rangle)^2; \quad (6a)$$

$$B = C + D, \quad \Delta = AD - BC; \quad (6b)$$

$$C = f_3^- + 2 f_2^-, \quad D = f_2^- + \langle n_- \rangle. \quad (6c)$$

If $\bar{N}_0(n_-)$ is linear, then $c=0$, that is

$$f_{021} = C/D f_{011} \quad (7)$$

and (5) reduces to

$$a = \langle n_0 \rangle, \quad b = \frac{1}{D} f_{011} \quad (8)$$

It is important to note that the expressions for a , b , etc., depend only on the ansatz (4) for $\bar{N}_0(n_-)$ even though only f_{011} and f_{021} enter in their expressions and that no assumption is made that the higher correlation integrals are zero unlike Eq. (3) above. However, since only f_{011} and f_{021} enter, the higher correlations must be determined in terms of them and the f_b^- 's. An example of this is given by (7) when one assumes $\bar{N}_0(n_-)$ is linear in n_- . By comparing Eqs. (3) and (4), one obtains the constraints on $P(n_-)$. We consider the following simple cases.

I $\bar{N}_0(n_-)$ is linear and $f_{021} = 0$, one then obtains

$$P(n_-) \left[1 + \frac{n_- - \langle n_- \rangle}{D} \right] = P(n_- - 1) \quad (9)$$

Thus, with $P(0)$, $\langle n_- \rangle$ and f_2^- as inputs from the experimental data, one can calculate the $P(n_-)$ for $n_- = 1, 2, \dots$

II $\bar{N}_0(n_-)$ is linear but $f_{021} \neq 0$ and would, in fact, be given by Eq. (7). In this case, one obtains

$$P(n_-) \left(n_- - \frac{1}{2} f_3^- \right) = \frac{C}{2} P(n_- - 2) + (D - C) P(n_- - 1) \quad (10)$$

The inputs required from experiment are $P(0)$, $P(1)$, $\langle n_- \rangle$, f_2^- and f_3^- to enable one to predict $P(n_-)$ for $n_- = 2, 3, \dots$

III $\bar{N}_0(n_-)$ is quadratic but $f_{021} = 0$. Then

$$P(n_-) \left\{ 1 + \frac{1}{\Delta} [CD - A \langle n_- \rangle - C \langle n_- \rangle^2 + (A + 2C \langle n_- \rangle) n_- - C n_-^2] \right\} = P(n_- - 1) \quad (11)$$

Five inputs $P(0)$, $\langle n_- \rangle$, f_2^- , f_3^- and f_4^- are required to be able to calculate $P(1)$, etc.

IV $\bar{N}_0(n_-)$ is quadratic and $f_{021} \neq 0$. Comparing (3) and (4) and treating f_{011} and f_{021} as independent gives us two constraints on $P(n_-)$. Since they have to be consistent for all n_- one finds that they reduce to Eq. (9).

Note that both Eqs. (10) and (11) reduce to (9) if one takes $C=0$, i.e., $f_3^- = -2f_2^-$. Also to the extent C is small both cases I and III should give similar results except for the different inputs. In fact, the data show that from 50 GeV/c to 405 GeV/c, C increases from a small negative value and becomes increasingly positive. This is due to the fact that f_2^- becomes positive and increases with energy. However, it is interesting to note that C is small from 50 to 205 GeV/c and consistent with zero within one standard deviation. We have numerically investigated both cases I and III and find the over-all fit to the $P(n_-)$ is better for case I. Consequently, we present the numerical results given by Eq. (9) only, which also includes case IV mentioned above. We also give the results for case II, namely Eq. (10). This case is interesting as it predicts that f_{021} is large compared to f_{011} asymptotically. This follows since the data on $P(n_-)$ seem to require that ⁵⁾ $f_b^- \sim \langle n_- \rangle^b$ so that from (7) $f_{021} \sim \langle n_- \rangle f_{011}$ asymptotically.

We have calculated the cross-section $\sigma(n_-)$ for n_- negative particles according to Eqs. (9) and (10) from 50 to 405 GeV/c for pp collisions. As a sample, the results of the calculation together with the experimental data ⁶⁾ are shown for the three energies 50, 205 and 405 GeV/c in Figs. 1, 2 and 3. The curves labelled I and II refer to the results from Eqs. (9) and (10), respectively. It should be noted that since the input quantities are taken from experiment, they have errors. Consequently, the calculated $\sigma(n_-)$ in the two cases will also have errors. These errors have not been shown in the figures for the sake of clarity and only the errors of the data points have been shown. As can be seen from Fig. 1, at 50 GeV/c the calculated values (without their error) agree within one standard deviation with the experimental values for both cases. As the energy increases one finds that Eq. (9) gives higher values for $\sigma(n_-)$ for small n_- than Eq. (10). However, for larger n_- the calculated values (without their error) again agree with experiment within the experimental error, though the value of n_- after which this happens is smaller for (10) than for (9). This can be seen in Fig. 2 at 205 GeV/c. But it should be noted that if one takes into account the errors in the calculated values, then they may overlap within one standard deviation with the experimental

values for all n_- in both the cases. This is true, for example, at 205 GeV/c primarily because $\sigma(0)$ has a large error. At 405 GeV/c, one can see from Fig. 3 that the values obtained from (10) give a much better fit than those from (9). Further, despite the inclusion of the error in the calculated values, the values obtained from Eq. (9) for small n_- remain too large. To summarize, what one finds is that in the range 50 to about 205 GeV/c both the formulas give a reasonable fit though one would prefer the simpler formula of Eq. (9), with fewer inputs. However, at higher energies, formula (10) gives a better fit and it would be interesting to check it at ISR energies.

It is clear that it is important to have precise measurements of $\bar{N}_0(n_-)$ since its n_- dependence can help us learn something about the $P(n_-)$ as well as about higher neutral negative correlations.

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FIGURE CAPTIONS

- Figure 1 Fit to $\sigma(n_-)$ at 50 GeV/c. Curves labelled I and II refer to formulas (9) and (10), respectively.
- Figure 2 Fit to $\sigma(n_-)$ at 205 GeV/c. Curves labelled I and II refer to formulas (9) and (10), respectively.
- Figure 3 Fit to $\sigma(n_-)$ at 405 GeV/c. Curves labelled I and II refer to formulas (9) and (10), respectively.

Fig. 1

Fig. 2