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CONSTRAINTS IMPOSED BY THE DATA ABOVE 1 GeV
ON THE LOW ENERGY $\pi\pi$ AMPLITUDES

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A B S T R A C T

We study the influence of the region above 1 GeV on low energy $\pi\pi$ amplitudes by using minimum subtracted dispersion relations, in addition to the twice subtracted ones. When our results are compared with the recent phase shift analysis below 1 GeV, we find that the S wave isospin zero scattering length has to lie between 0.5 and 1.1.

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1. INTRODUCTION

It has recently been shown ^{1),2)} that one can construct many $\pi\pi$ amplitudes which fulfill theoretical constraints such as fixed t dispersion relations, crossing symmetry, unitarity (including some inelastic effects like the $K\bar{K}$ production) and which in addition accommodate various phenomenological requirements. Needless to say that all these constraints are approximately fulfilled, and that the "solutions" have to be selected according to the criteria given in Ref. 2).

In this framework, most of the recent sets of published data below 1 GeV ($\pi\pi$ centre-of-mass energy) are found equally compatible with general principles. One can then simply predict, for each of these data sets, the range of variation of some very low ^{2),3)} or very high ^{4),5)} energy parameters.

Between these two extreme regions, accurate experimental data ⁶⁾ are now available in the range 1 GeV $\sim$ 2 GeV. Their possible consequences on the low energy behaviour of the first partial waves can be explored in at least two ways. The first is to use improved $\pi\pi$ equations, like those recently given by Mahoux, Roy and Wanders ⁷⁾ which allow to enlarge the domain of analysis from 1 GeV to 1.5 GeV. Another possibility is to keep the usual Roy's equations ⁸⁾ but to add to them some supplementary conditions which enhance the role of the 1 GeV $\sim$ 2 GeV data at very low energy.

In this work we have chosen to explore this second method because it uses as a starting point the $\pi\pi$ spectrum already found in previous analyses. The implications of any new constraint are then easy to distinguish. Our supplementary constraints come from fixed t dispersion relations with the minimal number of subtractions allowed by Regge asymptotic behaviour. They are described in Section 3, after a brief recall in Section 2 of the analogous equations in case of two subtractions. The results can be found in Section 4.

2. IMPLICATIONS OF TWICE SUBTRACTED DISPERSION RELATIONS

As previously done ^{1),2),9),10)}, we use the Roy equations in the following "phenomenological form" :

$$\begin{aligned} \operatorname{Re} f_I(s) &= L_I(s) + f_I^\infty + \frac{1}{\pi} \int_4^{s_M} R_I^J(s;s') \operatorname{Im} f_J(s') ds' \\ 4 \leq s < s_M \quad ; \quad m_\pi &= 1 \end{aligned} \quad (2.1)$$

where $f_I(s)$ are the S waves ($I=0,2$) and the reduced P wave ($I=1$). Below 1 GeV the phase shifts $\delta_I(s)$ are real and given by

$$\begin{aligned} f_I(s) &= \tau(s) e^{i\delta_I} \sin \delta_I \quad I=0,2 \quad \tau(s) = \sqrt{s/(s-4)} \\ f_1(s) &= 4 \tau(s) e^{i\delta_1} \sin \delta_1 / (s-4) \end{aligned} \quad (2.2)$$

The scattering lengths a_I are defined by $a_I = f_I(4)$, $I=0,1,2$ [they contribute to the linear terms $L_I(s)$ given in Appendix B].

We recall that Eq. (2.1) comes from the projections on the S and P waves of twice subtracted dispersion relations of the isospin amplitudes $F^I(s,t,u)$ after the following approximations have been done :

- for $4 \leq s' \leq s_M$ the absorptive parts $\text{Im} F^I(s',t)$ are replaced by their S and P wave contributions. This gives the integral term in (2.1). (The kernels are listed in Appendix B.) We choose $s_M = (E_M/0.14)^2$, $E_M = 0.95$ GeV.
- the dispersive contributions of the absorptive parts for $E' > 0.95$ GeV give the "driving terms" $f_I^\infty(s)$. We distinguish two regions. (This is particularly important in the next section.) For $0.95 \text{ GeV} \leq E' \leq 1.9 \text{ GeV}$ we reconstruct the absorptive parts from the data of Ref. 6). In the remaining part $E' > 1.9 \text{ GeV}$ we use standard Regge forms ¹¹⁾ :

$$\begin{aligned} \text{Im} F^{I_t=0}(s,t) &= a(t) s^{\alpha_P(t)} \\ a(t) &= 0.05 \exp(0.016 t) ; \quad \alpha_P(t) = 1 + 0.0057 t \\ \text{Im} F^{I_t=1}(s,t) &= 0.53 \sin(\pi \alpha_S(t)) \Gamma(1 - \alpha_S(t)) \left(\frac{s}{50}\right)^{\alpha_S(t)} \\ \alpha_S(t) &= 0.5 + 0.017 t \\ \text{Im} F^{I_t=2}(s,t) &= 0 \end{aligned} \quad (2.3)$$

In the region $4 \leq s \leq s_M$ the so-constructed driving terms are well approximated by

$$f_I^\infty(s) = \mathcal{D}_I^0 + \mathcal{D}_I^1 (s-4) + \mathcal{D}_I^2 (s-4)^2 + \mathcal{D}_I^3 \ln \left[\frac{s_M - s}{s_M - 4} \right] \quad (2.4)$$

The coefficients D_I^k are listed in Table 1a and in Fig. 1a we have drawn $f_0^\infty(s)$ as an example : it can be seen that the pure Regge contribution is small compared to the total one. [This allows us to consider the parameters entering Eq. (2.3) as fixed.]

Now, to impose Eq. (2.1), we first construct a unitary parametrization $f_I^M(s)$ of the S and P waves valid for $4 \leq s < s_M$. (This parametrization is given in Appendix A.) Inserting $\text{Im} f_I(s') = \text{Im} f_I^M(s')$ in the right-hand side of (2.1), this expression gives $\text{Re} f_I(s)$, which has to be equal to $\text{Re} f_I^M(s)$. This is approximately fulfilled by requiring

$$X_2 = \sum_{I=0}^2 \int_4^{s_M} \left[\text{Re} f_I(s) - \text{Re} f_I^M(s) \right]^2 ds \simeq 0 \quad (2.5)$$

When the parameters describing the S and P waves in our model are fixed through constraint (2.5) alone, we obtain a spectrum of $\pi\pi$ amplitudes whose main features have already been described in Refs. 1) and 2). Thus we just point out, since it is important for future comparisons, that among the parameters a_0 and m_0 remain uncorrelated. [m_0 is defined by $\delta_0(m_0) = \pi/2$.] The other parameters can vary around some mean value, as explained in Appendix A. For example the "universal band" can be described by $a_2 = a_2^c(a_0) \pm 0.015$, where the "central value" $a_2^c(a_0)$ is given in Table 2 for all a_0 ,

$$0.0 \lesssim a_0 \lesssim 1.5 \quad (2.6)$$

The range of variation (2.6) of a_0 is large and m_0 can vary appreciably, thus leading, even for a_0 fixed, to a very poor determination of $\delta_0(s)$ above 450 MeV (see, e.g., Fig. 2).

3. IMPLICATIONS OF MINIMALLY SUBTRACTED DISPERSION RELATIONS

We notice that in the analysis of Ref. 2), the cut-off is chosen equal to the "dual" value $s_M \simeq 115$. This implies small low energy driving terms (see Fig. 1a), not containing the $K\bar{K}$ cusp for example. (This effect is then put into the S^0 wave through the S^* .) In our case all these inputs are taken into account through the driving terms, which then play an important role which we want to emphasize here by using minimally subtracted dispersion relations.

We thus introduce the amplitudes $G^I(s, t, u)$ by

$$G^I(s, t, u) = M_J^I F^J(s, t, u) ; \quad M = \begin{pmatrix} 2 & -9 & -5 \\ 2 & 3 & -5 \\ 6 & -9 & 3 \end{pmatrix} \quad (3.1)$$

which obey the following relations

$$\begin{aligned} G^I(u, t, s) &= (-)^I G^I(s, t, u) \\ G^0(s, t, u) &= -4 F^0(t, s, u) + 10 F^2(t, s, u) \\ G^1(s, t, u) &= 6 F^1(t, s, u) \\ G^2(s, t, u) &= 18 F^2(t, s, u) \end{aligned} \quad (3.2)$$

Assuming the asymptotic behaviour (2.3), one can then write unsubtracted fixed t dispersion relations for $G^1(s, t, u)/(s-u)$ and $G^2(s, t, u)$:

$$G^I(s, t, u) = \frac{1}{\pi} \int_4^\infty \text{Im} G^I(s', t) \left\{ \frac{1}{s' - s} + \frac{(-)^I}{s' - u} \right\} ds' ; \quad I=1, 2 \quad (3.3)$$

As the amplitude $G^0(s, t, u)$ needs two subtractions, we write

$$\begin{aligned} G^0(s, t, u) &= G^0(4-2t, t, t) + I^0 \\ I^0 &= \frac{1}{\pi} \int_4^\infty \text{Im} G^0(s', t) \left\{ \frac{1}{s' - s} + \frac{1}{s' - u} - \frac{1}{s' + 2t - 4} - \frac{1}{s' - t} \right\} ds' \end{aligned} \quad (3.4)$$

and we notice that

$$G^0(4-2t, t, t) \equiv G^1(4-2t, t, t) \quad (3.5)$$

since $F^1(4-2t, t, t) \equiv 0$.

Thus, replacing in (3.4) the subtraction function by $G^1(4-2t, t, t)$ given by (3.3), we get the representation

$$G^I(s, t, u) = \frac{1}{\pi} \int_4^\infty K_J^I(s, s', t, u) \text{Im} G^J(s', t) ds' \quad I=0, 1, 2 \quad (3.6)$$

on which we perform the same approximations and projections as those already explained in Section 2. Coming back to isospin amplitudes, we obtain for the S and P waves

$$\text{Re } g_I(s) = g_I^\infty(s) + \frac{1}{\pi} \int_4^{s_M} H_I^J(s';s) \text{Im } g_J(s') ds' \quad (3.7)$$

where the kernels $H_I^J(s',s)$ are given, in Appendix C.

The driving terms $g_I^\infty(s)$ (which do not vanish at threshold now) can be well approximated by ($4 \leq s < s_M$) :

$$g_I^\infty(s) = G_I^0 + G_I^1(s-4) + G_I^2(s-4)^2 + G_I^3 \ln \left\{ \frac{s_M - s}{s_M - 4} \right\} \quad (3.8)$$

(The values of the coefficients G_I^k are listed in Table 1b.) As can be seen on the examples given in Fig. 1b, the Regge contribution remains small compared to the total one : as wanted, these driving terms reflect at low energy the experimental features of $\pi\pi$ scattering between 1 and 2 GeV.

We fulfil Eq. (3.7) exactly like Eq. (2.1), i.e., by inserting $\text{Im } g_I(s') = \text{Im } f_I^M(s')$ and imposing

$$X_0 \equiv \sum_{I=0}^2 \int_4^{s'_M} \left[\text{Re } g_I(s) - \text{Re } f_I^M(s) \right]^2 ds \simeq 0 \quad (3.9)$$

[We take $s'_M \simeq 30$ to eliminate uncertainties linked to high values of $|t|$ in (3.6).]

Before reporting our results which come from the minimization of $X = \text{Sup}(X_0, X_2)$ we add a remark. The representation (3.6) does not display $t \rightleftharpoons u$ symmetry, and one has in general to supply Eq. (3.7) with conditions implementing these crossing constraints on the partial waves. However, we do not need them here because the S and P waves which satisfy (3.7) fulfil at the same time the usual Roy representation (2.1), where these supplementary conditions begin with the D waves.

4. RESULTS AND CONCLUSIONS

We denote by R the spectrum of S and P phase shifts fulfilling Eq. (2.1) alone and by R' the spectrum of solutions to Eqs. (2.1) and (3.7) simultaneously.

Our first result is that R and R' coincide below 400 MeV. In this region the spectrum is essentially a one-parameter family whose parameter (a_0) can vary in the range given by (2.6). In other words, this shows that without any other assumptions, Regge sum rules cannot restrict the values of the scattering lengths. However, as the driving terms associated to R' are important ($g_0^{\infty}(4) \simeq 0.88$), this implies strong correlations between the partial waves at higher energies.

We find that these correlations have to be done with the remaining degrees of freedom and it turns out to give strong constraints on m_0 . In Fig. 2, for example, we give the range of δ_0 in R for a fixed value of a_0 ($a_0 = 0.6$). As can be seen m_0 may vary between 550 MeV and 950 MeV. In R' its range is constrained to be between 750 MeV and 900 MeV (ex., Fig. 3, band 2). The overlap between families of δ_0 associated to different a_0 remain important above 650 MeV, [see, e.g., the overlap between bands 2) and 3) of Fig. 3 which correspond to $a_0 = 0.6$ and $a_0 = 0.2$, respectively] but is considerably smaller in R' than in R , where it begins to be important at 500 MeV.

As a result, the phenomenological implications of R' can be different from those of R when the data below 600 MeV are taken into account. This does not hold for K_{e4} results as R and R' coincide below 400 MeV. [Anyway the present K_{e4} data lead to inconclusive results¹²⁾.] The set of data that we have considered are shown in Fig. 4a. Also shown in this figure is the variation allowed in R' of δ_0 when $a_0 = 1.1$. (This value of a_0 appears to be the maximal one which allows a global fit of the data.) Similarly we get a lower limit for a_0 . If we exclude the data of Baton et al., this limit is $a_0 \gtrsim 0.5$ (best fit $a_0 \simeq 0.7$). If, however, this set of data is included, we get $a_0 \geq 0.1$. The associated values for a_2 are small and positive, δ_2 quickly getting negative and being small (Fig. 4b). (The detailed values of all the parameters, for different values of a_0 between 0 and 1.5, are given in Table 2.)

Our main result is then to favour a strong S^0 wave at very low energy, stronger than generally expected due to some PCAC arguments. However, it has been already pointed out¹³⁾ that small scattering lengths are implied by

current algebra and PCAC only through specific models which generally do not allow strong unitarity effects (like those associated to the S waves of R or R' for $a_0 \gtrsim 1$). The compatibility of PCAC with strong low energy $\pi\pi$ scattering is then an open question which is now under study since we know ¹⁴⁾ how to construct in a model independent way the off-mass shell $\pi\pi$ spectrum associated to R'.

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APPENDIX A - S AND P WAVE PARAMETRIZATION

We use K matrix forms expressed in a variable z which takes into account the left-hand cut ($s \leq 0$) and the inelastic cut ($E \geq E_1$, $E_1 = 1$ GeV) :

$$z = z(s) = \frac{\sqrt{s(s_1-4)} - \sqrt{4(s_1-s)}}{\sqrt{s(s_1-4)} + \sqrt{4(s_1-s)}} \quad s_1 = (E_1 / 0.14)^2 \quad (\text{A.1})$$

(The s plane cut for $s \leq 0$ and $s \geq s_1$ is mapped onto the unit disc.)
The S waves $f_0(s)$ and $f_2(s)$ are given by

$$f_I(s) = N_I(z) / (1 + h(s) N_I(z)) \quad (\text{A.2})$$

where $h(s)$ is the Chew-Mandelstam function

$$h(s) = \frac{2}{\pi} g(s) \ln \left\{ \frac{\sqrt{s} + \sqrt{s-4}}{2} \right\} - i g(s) \quad (\text{A.3})$$

$$g(s) = \sqrt{(s-4)/s}$$

and

$$N_0(z) = (a_0 + x_2 z)(1-z) / (1 - x_3 z)(1 + x_4 z + x_5 z^2) \quad (\text{A.4})$$

$$N_2(z) = (a_2 + y_2 z) / (1 + y_3 z + y_4 z^2) \quad (\text{A.5})$$

As suggested by the data ⁶⁾ we impose the following "boundary conditions" :

$$f_0(s_1) = 0 \quad ; \quad -40^\circ \leq \delta_2(s_1) \leq -20^\circ \quad (\text{A.6})$$

The reduced P wave is given by

$$f_1(s) = N_1(z) / (s_p - s - i \left(\frac{s-4}{4} \right) g(s) N_1(z))$$

$$N_1(z) = \frac{4z}{(s-4)} \left[\frac{z_1 + z z_2}{1 + z z_3} \right] \quad (\text{A.7})$$

with the boundary condition $\delta_1(s_1) \simeq 155^\circ$.

The P wave is then function of three parameters which can be linked to the ρ mass and width (m_ρ, Γ_ρ) and to a_1 . We impose

$$755 \text{ MeV} \leq m_\rho \leq 775 \text{ MeV}$$

$$135 \text{ MeV} \leq \Gamma_\rho \leq 155 \text{ MeV}$$

(A.8)

Then the S and P waves are described by 13 parameters x_i, y_i, z_i .

In Table 2 the given values of all these parameters can be used to reconstruct the "central solutions" in R' associated to different values of a_0 between 0 and 1.5. These selected solutions are "central" in the sense that the parameters uncorrelated to a_0 , like m_ρ , or weakly correlated, like Γ_ρ , are fixed to their mean value, i.e.,

$$m_\rho = 850 \text{ MeV}$$

$$m_\rho = 756 \text{ MeV}$$

$$\Gamma_\rho = 154 \text{ MeV}$$

$$\delta_1(s_1) = 152^\circ$$

$$\delta_2(s_1) = -23^\circ$$

(A.9)

APPENDIX B - S AND P WAVE KERNELS FROM THE TWICE SUBTRACTED
DISPERSION RELATIONS

$$R_0^0 = 1/(s'-s) + (4-s-s')/s'(s'-4) + R_2^0 \quad (B.1)$$

$$R_0^1 = 3(2s+s'-4)Q_0(z)/(s-4) - 3(3s+2s'-4)/4s' \quad (B.2)$$

$$R_0^2 = 5 R_2^0 \quad (B.3)$$

$$R_1^0 = 16 Q_1(z)/3(s-4)^2 - 4/9 s'(s'-4) \quad (B.4)$$

$$R_1^1 = 1/(s'-s) + 6(2s+s'-4)Q_1(z)/(s-4)^2 - 3/2s' \quad (B.5)$$

$$R_1^2 = 10/9 s'(s'-4) - 40 Q_1(z)/3(s-4)^2 \quad (B.6)$$

$$R_2^0 = 4 Q_0(z)/3(s-4) + (s+4-2s')/3 s'(s'-4) \quad (B.7)$$

$$R_2^1 = - R_0^1/2 \quad (B.8)$$

$$R_2^2 = 1/(s'-s) + R_2^0/2 + (4-s-s')/s'(s'-4) \quad (B.9)$$

where

$$z = 1 + 2s'/(s-4)$$

$$Q_0(z) = \frac{1}{2} \ln \left\{ \frac{1+z}{-1+z} \right\} \quad Q_1(z) = z Q_0(z) - 1 \quad (B.10)$$

For the reduced p.w., the linear subtraction terms $L_I(s)$ are :

$$\begin{bmatrix} L_0(s) \\ L_1(s) \\ L_2(s) \end{bmatrix} = \begin{bmatrix} a_0 \\ 0 \\ a_2 \end{bmatrix} + \frac{2a_0 - 5a_2}{12} \begin{bmatrix} s-4 \\ 2/3 \\ (4-s)/2 \end{bmatrix} \quad (B.11)$$

APPENDIX C - S AND P WAVE KERNELS FROM THE UNSUBTRACTED DISPERSION
RELATIONS

$$H_0^0 = 1/(s'-s) + H_2^0 \quad (C.1)$$

$$H_0^1 = 3(4s-s'-4)Q_0(z)/4(s-4) - 3/2 \quad (C.2)$$

$$H_0^2 = 5 H_2^0 / 4 \quad (C.3)$$

$$H_1^0 = 16 Q_1(z)/3(s-4)^2 \quad (C.4)$$

$$H_1^1 = 1/(s'-s) + 6(s-s') Q_1(z)/(s-4)^2 \quad (C.5)$$

$$H_1^2 = -5 H_1^0 / 2 \quad (C.6)$$

$$H_2^0 = 4 Q_0(z)/3(s-4) \quad (C.7)$$

$$H_2^1 = 3/4 - 3(2s'+s-4) Q_0(z)/2(s-4) \quad (C.8)$$

$$H_2^2 = 1/(s'-s) - H_2^0 \quad (C.9)$$

where $Q_0(z)$ and $Q_1(z)$ have the same definition as in Eq. (B.10).

I	D_I^0	D_I^1	D_I^2	D_I^3
0	0	-0.0006740	0.000110	-0.060681
1	0.000385	0.0000144	-0.0000034	-0.003557
2	0	0.0002793	0.0000852	-0.008941

TABLE 1a - Values of the coefficients used in formulae (2.4) to parametrize the driving terms $f_I^{\omega}(s)$ on the range $4 \leq s < s_M$.

I	G_I^0	G_I^1	G_I^2	G_I^3
0	0.8852	-0.01521	0.0000088	-0.1166
1	0.01155	-0.0002815	0.00000275	-0.003366
2	0.3134	-0.01716	0.000294	0.00522

TABLE 1b - Values of the coefficients used in formulae (3.8) to parametrize the driving terms $g_I^{\omega}(s)$ on the range $4 \leq s < s_M$.

a_0	$a_1 \times 10^3$	$a_2 \times 10^3$	x_2	x_3	$-x_4$	x_5	$-y_2$	$-y_3$	y_4	z_1	z_2	z_3
0.0	25	- 82	0.513	1.758	1.825	1.653	0.335	0.078	0.563	2.319	26.68	1.395
0.1	28	- 60	0.424	1.872	1.994	1.693	0.317	0.249	0.594	2.596	25.34	1.308
0.2	31	- 35	0.268	1.974	2.062	1.604	0.330	0.300	0.601	2.875	24.01	1.220
0.3	34	- 12	0.068	2.089	2.167	1.570	0.352	0.374	0.671	3.153	22.67	1.133
0.4	36.5	8	-0.086	2.184	2.169	1.497	0.377	0.386	0.700	3.386	21.56	1.060
0.5	39	25	-0.319	2.288	2.303	1.563	0.406	0.390	0.751	3.617	20.45	0.988
0.6	41	40	-0.433	2.363	2.232	1.458	0.398	0.610	0.888	3.803	19.56	0.929
0.7	43.5	55	-0.770	2.485	2.381	1.510	0.432	0.636	0.982	4.035	18.45	0.857
0.8	46	68	-0.961	2.574	2.370	1.454	0.461	0.650	1.052	4.266	17.33	0.784
0.9	48	80	-1.109	2.644	2.329	1.385	0.494	0.575	1.051	4.452	16.44	0.726
1.0	50	90	-1.257	2.719	2.328	1.376	0.506	0.821	1.304	4.637	15.55	0.668
1.1	51.5	100	-1.385	2.826	2.310	1.351	0.499	0.974	1.397	4.776	14.89	0.624
1.2	53	110	-1.506	2.883	2.273	1.304	0.536	0.890	1.409	4.916	14.22	0.580
1.3	54	120	-1.637	2.941	2.266	1.295	0.545	0.961	1.475	5.010	13.77	0.551
1.4	55	130	-1.900	2.972	2.339	1.346	0.571	0.929	1.502	5.101	13.33	0.522
1.5	55	140	-2.139	3.003	2.416	1.415	0.574	1.025	1.571	5.101	13.33	0.522

TABLE 2 - Values of the S and P wave parameters for different choices of a_0 . These values correspond to the "central solutions" of R' , as defined in Appendix A.

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FIGURE CAPTIONS

Figure 1a Curve 1 represents the total driving term $f_0^{\omega}(s)$ given by Eq. (2.4). Its Regge part is shown by curve 2. Curve 3 indicates the values of $f_0^{\omega}(s)$ used in Ref. 1).

Figure 1b Curve 1 represents the total driving term $g_0^{\omega}(s)$ given by Eq. (3.8). Curve 2 gives its Regge part. Curve 3 is $g_2^{\omega}(s)$.

Figure 2 The range of δ_0 for $a_0 = 0.6$ allowed in R when m_0 varies between 550 MeV (curve 1) and 900 MeV (curve 2). The hatched area shows the corresponding variation in R' .

Figure 3 The range of δ_0 allowed in R' for

- 1) $a_0 = 1.3$ (upper limit ; $m_0 \simeq 700$ MeV) ;
- 2) $a_0 = 0.6$ (total spread with m_0 ; $750 \text{ MeV} \leq m_0 \leq 900 \text{ MeV}$) ;
- 3) $a_0 = 0.2$ (total spread with m_0 ; $750 \text{ MeV} \leq m_0 \leq 900 \text{ MeV}$) ;
- 4) $a_0 = 0.0$ (lower limit ; $m_0 \simeq 950$ MeV).

Figure 4a Recent experimental values for δ_0 below 1 GeV.

P. Estabrooks et al. - Proc.Int.Conf.on $\pi\pi$ Scattering and Associated Topics, Tallahassee (1973).
The data of Ref. 6).
G. Villet et al. - Preprint D.Ph.P.E. CEN Saclay (1973).
J.P. Baton et al. - Phys.Letters 25B, 419 (1967).

The solid curves correspond to the band allowed in R' for $a_0 = 1.1$.

Figure 4b Isospin 2 S wave phase shift δ_2 of R' when a_0 varies between 0.1 (curve 1) and 1.1 (curve 2). The data are

G. Villet et al. - Preprint D.Ph.P.E. CEN Saclay (1973).
J.P. Baton et al. - Phys.Letters 25B, 419 (1967).

FIG. 1a

FIG. 1b

FIG.2

FIG.3

FIG.4 a

FIG. 4 b