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THE $\Delta I = \frac{1}{2}$ RULE IN NON-LEPTONIC WEAK DECAYS

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We investigate the $\Delta I = \frac{1}{2}$ rule for non-leptonic decays in unified theories of weak and electromagnetic interactions. In such theories, weak interactions are mediated not only by gauge vector bosons, but also through the exchange of scalar Higgs bosons. Also in such theories, the electromagnetic and weak interaction parts of the Lagrangian and the mass term are completely specified. Hence the divergence of the weak current $\partial_\mu J_\mu$ is completely known. For a non-leptonic decay $\alpha(p) \rightarrow \beta(p'+k)$, one has $\langle \beta | i\partial_\mu J_\mu | \alpha \rangle = 0$ because of energy-momentum conservation for particles on the mass shell. We examine the restrictions due to the above condition on non-leptonic decays. In particular, we discuss whether through such restrictions one can understand the suppression of $\Delta I = \frac{3}{2}$ in non-leptonic decays. Our conclusions are that for the eight-quark version of the Georgi-Glashow model, we get the $\Delta I = \frac{1}{2}$ rule for parity-violating decays only in the $SU(3)$ limit. For the Salam-Weinberg model, we find that parity-violating non-leptonic decays arise in first order of $SU(3)$ symmetry breaking, while the parity conserving part can arise in the $SU(3)$ limit. The violation of the $\Delta I = \frac{3}{2}$ rule for parity-violating decays is of second order in $SU(3)$ breaking, whereas it is of first order in $SU(3)$ breaking for parity-conserving decays.

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1. INTRODUCTION

In at least some of the models based on the notion of spontaneously broken gauge symmetry for unified theories of weak and electromagnetic interactions, weak interactions are mediated not only by gauge vector bosons but also through the exchange of scalar Higgs bosons. The former are coupled to the vector and axial vector currents, while the latter are coupled to the scalar and pseudo-scalar densities. It has been observed by Lee and Treiman¹⁾ that in the eight-quark version of the Georgi-Glashow model²⁾ (we are concerned here with the hadronic sector of the model), the scalar boson exchanges in the lowest order can be arranged so as to have a pure $\Delta I = \frac{1}{2}$ or octet rule for the uncharged $|\Delta S| = 1$ non-leptonic decays. Since it is difficult to assess the relative effective strengths of the contributions from the scalar and vector boson exchanges to the uncharged non-leptonic decays, one cannot say whether the amplitudes for such decays exhibit octet dominance.

For a non-leptonic decay of the form $\alpha(p) \rightarrow \beta(p' + k)$, one has

$$\langle \beta | i \partial_\mu J_\mu^7 | \alpha \rangle = 0,$$

because of energy-momentum conservation for particles on the mass shell. Here J_μ^7 denotes the uncharged neutral current which has the structure

$$- \frac{i}{2} \left[i \bar{N} \gamma_\mu (1 + \gamma_5) \lambda - i \bar{\lambda} \gamma_\mu (1 + \gamma_5) N \right].$$

Now in spontaneously broken gauge theories, the electromagnetic and weak interaction parts of the Lagrangian are completely specified and so is the mass term. Hence $\partial_\mu J_\mu^7$ is completely known. The purpose of this paper is to examine the restrictions due to the above condition on non-leptonic decays. In particular, we want to discuss whether through such restrictions one can understand suppression of $\Delta I = 3/2$ in non-leptonic decays.

One consequence of the above condition is that the matrix elements for scalar and vector boson exchanges in a non-leptonic decay become related. As will be argued below, such relations lead to the following consequences.

- i) For the eight-quark version of the Georgi-Glashow model, we get octet dominance or the $\Delta I = \frac{1}{2}$ rule for the parity-violating part of the uncharged observed $|\Delta S| = 1$ non-leptonic decays only in the SU(3) limit. For the parity-conserving part of non-leptonic decays we do not, in general, get the $\Delta I = \frac{1}{2}$ rule. We can get the $\Delta I = \frac{1}{2}$ or octet rule in this case if the term $\left[(m_P - m_\pi)/2 \bar{P}P + (m_\lambda + m_N)/2 \sin^2 \beta (\bar{N}N + \bar{\lambda}\lambda) \right] [\bar{N} \gamma_5 \lambda - \bar{\lambda} \gamma_5 N]$

is suppressed compared to $m_q \sin^2 \beta (\bar{q}q) (\bar{N}\gamma_5 \lambda - \bar{\lambda}\gamma_5 N)$, where q is the uncharmed SU(3) singlet quark with hypercharge zero.

- ii) In the Salam-Weinberg model³⁾, the parity-conserving and parity-violating parts of observed non-leptonic decays arise in different orders of SU(3) breaking; the latter arising in the first order of SU(3) breaking, while the former can arise in the SU(3) limit. The violation of octet rule for the parity-violating part is of second order in SU(3) breaking, whereas it is of first order in SU(3) breaking for the parity-conserving part.

2. GEORGI-GLASHOW MODEL

An eight-quark version (we are concerned here only with the hadron sector of the model) is based on spin $\frac{1}{2}$ fermions grouped into two SO(3) triplets and the singlets

$$\Psi_N = \begin{bmatrix} P \\ (N_c \sin \beta + q \cos \beta)_L + q_R \\ z \end{bmatrix}$$

$$\Psi_\lambda = \begin{bmatrix} P' \\ (\lambda_c \sin \beta + q' \cos \beta)_L + q'_R \\ z' \end{bmatrix}$$

$$S_N = (N_c \cos \beta - q \sin \beta)_L, N_{cR}$$

$$S_\lambda = (\lambda_c \cos \beta - q' \sin \beta)_L, \lambda_{cR},$$

where

$$N_c = N \cos \theta_c + \lambda \sin \theta_c$$

$$\lambda_c = -N \sin \theta_c + \lambda \cos \theta_c.$$

Here β is a parameter of the model and θ_c is the Cabibbo angle. P , N , and λ are uncharged quarks and form an $SU(3)$ triplet; q is uncharged and has zero hypercharge; other quarks carry charm quantum numbers. The integral quark charges are:

$$Q = \begin{bmatrix} 1, & P, P' \\ 0, & N, \lambda, q, q' \\ -1, & \bar{z}, \bar{z}' \end{bmatrix}.$$

The mass Lagrangian is given by

$$\begin{aligned} \mathcal{L}_m^q = & - (m_P \bar{P} P + m_N \bar{N} N + m_\lambda \bar{\lambda} \lambda) \\ & - m_q \bar{q} q - m_z \bar{z} z - m_{P'} \bar{P}' P' \\ & - m_{q'} \bar{q}' q' - m_{z'} \bar{z}' z'. \end{aligned} \quad (1)$$

The interaction Lagrangian with the Higgs scalar ϕ is

$$\mathcal{L}_\phi^q = - \frac{e}{m_W} J_\phi \phi, \quad (2a)$$

where

$$J_\phi = J_\phi^0 + J_\phi^1 \quad (2b)$$

with

$$\begin{aligned} J_\phi^0 = & \frac{m_P - m_z}{2} (\bar{P} P - \bar{z} z) \\ & + \sin^2 \beta \left[\frac{m_\lambda + m_N}{2} (\bar{N} N + \bar{\lambda} \lambda) \right. \\ & \quad \left. + m_q \bar{q} q + m_{q'} \bar{q}' q' \right] \\ & + \frac{m_{P'} - m_{z'}}{2} (\bar{P}' P' - \bar{z}' z') \\ & - \frac{m_\lambda - m_N}{2} \sin^2 \beta [\bar{N} N - \bar{\lambda} \lambda]. \end{aligned} \quad (2c)$$

$$J_\phi^1 = \sin \beta \cos \beta \left\{ \frac{m_\lambda + m_N}{2} \left[\bar{N}_c \frac{1 + \gamma_5}{2} q + h.c. \right] - m_q \left[\bar{N}_c \frac{1 - \gamma_5}{2} q + h.c. \right] + \frac{m_\lambda + m_N}{2} \left[\bar{\lambda}_c \frac{1 + \gamma_5}{2} q' + h.c. \right] - m_{q'} \left[\bar{\lambda}_c \frac{1 - \gamma_5}{2} q' + h.c. \right] + \frac{m_\lambda - m_N}{2} \left[\bar{N}_c \frac{1 + \gamma_5}{2} q + \bar{\lambda}_c \frac{1 + \gamma_5}{2} q' + h.c. \right] \right\}, \quad (2d)$$

where

$$\bar{N}_c = -N \cos \theta_c + \lambda \sin \theta_c \quad (2e)$$

$$\bar{\lambda}_c = N \sin \theta_c + \lambda \cos \theta_c. \quad (2f)$$

The weak interaction Lagrangian is given by

$$\mathcal{L}_W^q = g_W \left[J_\mu^W \bar{W}_\mu + \bar{J}_\mu^W W_\mu \right], \quad (3a)$$

where

$$g_W = -\frac{e}{2} \sin \beta, \quad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{m_W^2} = \frac{e^2}{4m_W^2} \sin^2 \beta \quad (3b)$$

$$J_\mu^W = J_\mu^{(8)} + J_\mu^{(3)} + J_\mu^{(3^*)} + J_\mu^{(\text{singlet})} \quad (3c)$$

$$\bar{J}_\mu^W = \bar{J}_\mu^{(8)} + \bar{J}_\mu^{(3^*)} + \bar{J}_\mu^{(3)} + \bar{J}_\mu^{(\text{singlet})} \quad (3d)$$

with

$$\begin{aligned} J_\mu^{(8)} &= i \bar{P} \gamma_\mu (1 + \gamma_5) N_c \\ &= \left[\cos \theta_c i \bar{P} \gamma_\mu (1 + \gamma_5) N + \sin \theta_c i \bar{P} \gamma_\mu (1 + \gamma_5) \lambda \right] \\ &= \cos \theta_c J_\mu^0 + \sin \theta_c J_\mu^1 \end{aligned} \quad (4a)$$

$$J_{\mu}^{(3)} = -i \bar{N}_c \gamma_{\mu} (1 + \gamma_5) \chi - i \bar{\lambda}_c \gamma_{\mu} (1 + \gamma_5) \chi' + \cot \beta i \bar{P} \gamma_{\mu} (1 + \gamma_5) \psi + \frac{1}{\sin \beta} i \bar{P} \gamma_{\mu} (1 - \gamma_5) \psi \quad (4b)$$

$$J_{\mu}^{(3^*)} = i \bar{P}' \gamma_{\mu} (1 + \gamma_5) \lambda_c \quad (4c)$$

$$J_{\mu}^{(\text{singlet})} = \cot \beta [i \bar{P}' \gamma_{\mu} (1 + \gamma_5) \psi' - i \bar{\psi} \gamma_{\mu} (1 + \gamma_5) \chi - i \bar{\psi}' \gamma_{\mu} (1 + \gamma_5) \chi'] + \frac{1}{\sin \beta} [i \bar{P}' \gamma_{\mu} (1 - \gamma_5) \psi' - i \bar{\psi} \gamma_{\mu} (1 - \gamma_5) \chi - i \bar{\psi}' \gamma_{\mu} (1 - \gamma_5) \chi'] \quad (4d)$$

$$\bar{J}_{\mu}^{(i)} = J_{\mu}^{\dagger (i^*)} (1 - 2 \delta_{i4}), \quad i = 8, 3, 3^*, \text{singlet}. \quad (5)$$

In the above $J_{\mu}^{(8)}$ denotes the octet [with respect to the SU(3)] Cabibbo current. J_{μ}^0 and J_{μ}^1 are the usual uncharged $\Delta S = 0$, $\Delta S = 1$ currents. Superscripts 3, 3^* , singlet, signify the currents which belong to 3, 3^* , singlet representations of SU(3), respectively.

Consider now the $|\Delta S| = 1$ uncharged neutral currents

$$J_{\mu}^{6+i7} = i \bar{N} \gamma_{\mu} (1 + \gamma_5) \lambda$$

$$J_{\mu}^{6-i7} = i \bar{\lambda} \gamma_{\mu} (1 + \gamma_5) N \quad (6)$$

Then, writing

$$i J_{\mu}^7 = \frac{1}{2} (J_{\mu}^{6+i7} - J_{\mu}^{6-i7}) \quad (7)$$

we see that for a non-leptonic decay of the form $\alpha(p) \rightarrow \beta(p') + k$ [for example $\alpha = \Lambda(p)$, $\beta = p(p') + \pi^-(k)$],

$$\langle \beta | i \partial_\mu J_\mu^7 | \alpha \rangle = 0 \quad (8)$$

This is because of the energy-momentum conservation $p = p' + k$ for the particles on the mass shell. Now

$$i \partial_\mu J_\mu^7 = i [i Q^7, \mathcal{L}], \quad (9a)$$

where

$$Q^7 = -i \int J_4^7(x, 0) d^3x \quad (9b)$$

$$\begin{aligned} \mathcal{L} = & e J_\mu^{em} A_\mu + g_W (J_\mu^W \bar{W}_\mu + \bar{J}_\mu^W W_\mu) \\ & - \frac{e}{m_W} J_\phi \phi + \mathcal{L}_m^q \end{aligned} \quad (10)$$

Then

$$\begin{aligned} i \partial_\mu J_\mu^7 = & i e [i Q^7, J_\mu^{em}] + i g_W [i Q^7, \bar{J}_\mu^W] W_\mu \\ & + i g_W [i Q^7, J_\mu^W] \bar{W}_\mu - \frac{i e}{m_W} [i Q^7, J_\phi] \phi \\ & + i [i Q^7, \mathcal{L}_m^q] \end{aligned} \quad (11)$$

It is easy to see that

$$[i Q^7, J_\mu^{em}] = 0 \quad (12a)$$

$$[i Q^7, J_\mu^W] = h_\mu^W, \quad [i Q^7, \bar{J}_\mu^W] = \bar{g}_\mu^W \quad (12b)$$

$$[i Q^7, J_\phi] = j_\phi, \quad (12c)$$

where

$$h_{\mu}^W = h_{\mu}^{(8)} + h_{\mu}^{(3^*)} + h_{\mu}^{(3)} \quad (13a)$$

$$\bar{g}_{\mu}^W = \bar{g}_{\mu}^{(8)} + \bar{g}_{\mu}^{(3)} + g_{\mu}^{(3^*)} \quad (13b)$$

$$j_{\phi} = j_{\phi}^0 + j_{\phi}^1 \quad (14)$$

with

$$\begin{aligned} h_{\mu}^{(8)} &= -\cos \theta_c \, i \, \bar{P} \gamma_{\mu} (1 + \gamma_5) \lambda \\ &\quad + i \, \bar{P} \gamma_{\mu} (1 + \gamma_5) N \\ &= -\cos \theta_c \, J_{\mu}^1 + \sin \theta_c \, J_{\mu}^0 \end{aligned} \quad (15a)$$

$$h_{\mu}^{(3^*)} = i \, \bar{P}' \gamma_{\mu} (1 + \gamma_5) N_c \quad (15b)$$

$$h_{\mu}^{(3)} = i \, \bar{\lambda}_c \gamma_{\mu} (1 + \gamma_5) \lambda - i \, \bar{N}_c \gamma_{\mu} (1 + \gamma_5) \lambda' \quad (15c)$$

$$\begin{aligned} \bar{g}_{\mu}^{(8)} &= \sin \theta_c \, i \, \bar{N} \gamma_{\mu} (1 + \gamma_5) P \\ &\quad - \cos \theta_c \, i \, \bar{\lambda} \gamma_{\mu} (1 + \gamma_5) P \\ &= \sin \theta_c \, \bar{J}_{\mu}^0 - \cos \theta_c \, \bar{J}_{\mu}^1 \end{aligned} \quad (16a)$$

$$\bar{g}_{\mu}^{(3)} = i \, \bar{N}_c \gamma_{\mu} (1 + \gamma_5) P' \quad (16b)$$

$$\begin{aligned} \bar{g}_{\mu}^{(3^*)} &= i \, \bar{\lambda} \gamma_{\mu} (1 + \gamma_5) \lambda_c \\ &\quad - i \, \bar{\lambda}' \gamma_{\mu} (1 + \gamma_5) N_c \end{aligned} \quad (16c)$$

$$j_\phi^0 = \sin^2 \beta \left\{ -\frac{m_\lambda + m_N}{2} [\bar{N} \gamma_5 \lambda - \bar{\lambda} \gamma_5 N] \right. \\ \left. + \frac{m_\lambda - m_N}{2} [\bar{N} \lambda + \bar{\lambda} N] \right\} \quad (17a)$$

$$j_\phi^1 = \sin \beta \cos \beta \left\{ \left[m_q \bar{\lambda}_c \frac{1 - \gamma_5}{2} q + h.c. \right] \right. \\ \left. - \left[m_q' \bar{N}_c \frac{1 + \gamma_5}{2} q + h.c. \right] \right\}. \quad (17b)$$

Thus

$$i \partial_\mu J_\mu^7 = i g_W [\bar{g}_\mu^W W_\mu + h_\mu^W \bar{W}_\mu] \\ - \frac{ie}{m_W} j_\phi \phi + i [i Q^7 + \mathcal{L}_m^q]. \quad (17c)$$

We can now write

$$0 \equiv \langle \beta | i \partial_\mu J_\mu^7 | \alpha \rangle \\ = \mathcal{T}^W(\alpha \rightarrow \beta) + \mathcal{T}^\phi(\alpha \rightarrow \beta) + \mathcal{T}^M(\alpha \rightarrow \beta), \quad (18)$$

where

$$\mathcal{T}^W(\alpha \rightarrow \beta) = \frac{e^2 \sin^2 \beta}{8} \frac{1}{(2\pi)^4} \int d^4 q \frac{S_{\lambda\mu} + \frac{q_\lambda q_\mu}{m_W^2}}{q^2 + m_W^2} g_{\lambda\mu}^W \quad (19a)$$

$$\mathcal{T}^\phi(\alpha \rightarrow \beta) = \frac{e^2}{2m_W^2} \frac{1}{(2\pi)^4} \int d^4 q \frac{1}{q^2 + m_\phi^2} g^\phi \quad (19b)$$

$$\tau^M(\alpha \rightarrow \beta) = i \langle \beta | [i Q^7, d_m^q] | \alpha \rangle, \quad (19c)$$

with

$$g_{\lambda\mu}^W = i \int d^4x e^{-iq \cdot x} \left[\langle \beta | T(J_\lambda^W(x) \bar{g}_\mu^W(0)) | \alpha \rangle + \langle \beta | T(\bar{J}_\lambda^W(x) g_\mu^W(0)) | \alpha \rangle \right] \quad (20a)$$

$$g^\phi = i \int d^4x e^{-iq \cdot x} \langle \beta | T(J_\phi(x) j_\phi(0)) | \alpha \rangle. \quad (20b)$$

First we observe that spontaneously broken gauge theories are renormalizable theories. Thus any infinities in $\tau^W(\alpha \rightarrow \beta)$ and $\tau^\phi(\alpha \rightarrow \beta)$ must be matrix elements of scalar or pseudoscalar densities⁴⁾, which transform under SU(3), as a singlet or octet. Thus we should be able to absorb these infinities in $\tau^M(\alpha \rightarrow \beta)$ and the latter can be transformed away. Let us denote the finite parts of τ^W and τ^ϕ by τ_f^W and τ_f^ϕ . Then the condition (18) becomes

$$\tau_f^W(\alpha \rightarrow \beta) + \tau_f^\phi(\alpha \rightarrow \beta) = 0. \quad (21)$$

This is our basic result. Next we try to relate this result with the observed $|\Delta S| = 1$ non-leptonic decays.

Now the matrix elements for the non-leptonic decay $\alpha \rightarrow \beta$ are given by

$$F(\alpha \rightarrow \beta) = F^W(\alpha \rightarrow \beta) + F^\phi(\alpha \rightarrow \beta), \quad (22a)$$

where

$$F^W(\alpha \rightarrow \beta) = \frac{e^2 \sin^2 \beta}{8} \frac{1}{(2\pi)^4} \int d^4q \frac{\delta_{\lambda\mu} + \frac{q_\lambda q_\mu}{m_W^2}}{q^2 + m_W^2} \mathcal{F}_{\lambda\mu}^W \quad (22b)$$

$$F^\phi(\alpha \rightarrow \beta) = \frac{e^2}{2m_W^2} \frac{1}{(2\pi)^4} \int d^4q \frac{1}{q^2 + m_\phi^2} \mathcal{F}^\phi, \quad (22c)$$

with

$$\begin{aligned} \mathcal{F}_{\lambda\mu} = i \int d^4x e^{-i q \cdot x} & \left[\langle \beta | T(J_{\lambda}^W(x) \bar{J}_{\mu}^W(0)) | \alpha \rangle \right. \\ & \left. + \langle \beta | T(\bar{J}_{\lambda}^W(x) J_{\mu}^W(0)) | \alpha \rangle \right] \end{aligned} \quad (22d)$$

$$\mathcal{F}^{\phi} = i \int d^4x e^{-i q \cdot x} \langle \beta | T(J_{\phi}(x) \bar{J}_{\phi}(0)) | \alpha \rangle. \quad (22e)$$

Again the infinities in F^W and F^{ϕ} are matrix elements of scalar and pseudo-scalar densities which transform under $SU(3)$ as a singlet or octet. One should be able to absorb them in $\langle \beta | \mathcal{L}_m^q | \alpha \rangle$. We are interested in the finite parts of F^W and F^{ϕ} which are of order G_F . Thus we write for the $|\Delta S| = 1$ non-leptonic decays

$$F_f(\alpha \rightarrow \beta) = F_f^W(\alpha \rightarrow \beta) + F_f^{\phi}(\alpha \rightarrow \beta). \quad (23)$$

Next we note that in calculating τ^W and F^W , we encounter terms of the form $J_{\lambda}^W(x) \bar{g}_{\mu}^W(0)$, $\bar{J}_{\lambda}^W(x) h_{\mu}^W(0)$, and $J_{\lambda}^W(x) \bar{J}_{\mu}^W(0)$, $\bar{J}_{\lambda}^W(x) J_{\mu}^W(0)$, respectively. We now observe that $3 \otimes 3$, $3 \otimes 8 (= 8 \otimes 3)$ and their conjugates involve respectively the representations $3 \oplus 6$, $3 \oplus 6 \oplus 15$ and their conjugates. Such representations cannot contribute to the uncharged $|\Delta S| = 1$ observed non-leptonic decays, since they are not contained in $8 \otimes 8 \otimes 8$. Moreover, $3 \otimes 3^*$, $3^* \otimes 3$ involve only octets and singlets, while $8 \otimes 1$ and $1 \otimes 8$ involve only octets. Thus it follows from Eqs. (3c,d), (4), (5), (13), (15) and (16) that for uncharged $|\Delta S| = 1$ observed non-leptonic decays (writing only those terms which contribute), we have

$$\begin{aligned} J_{\lambda}^W(x) \bar{g}_{\mu}^W(0) = -\cos^2 \theta_c J_{\lambda}^0(x) \bar{J}_{\mu}^1(0) + \sin^2 \theta_c J_{\lambda}^1(x) \bar{J}_{\mu}^0(0) \\ + \text{octet part} \end{aligned} \quad (24a)$$

$$\begin{aligned} \bar{J}_{\lambda}^W(x) h_{\mu}^W(0) = -\cos^2 \theta_c \bar{J}_{\lambda}^0(x) J_{\mu}^1(0) + \sin^2 \theta_c \bar{J}_{\lambda}^1(x) J_{\mu}^0(0) \\ + \text{octet part} \end{aligned} \quad (24b)$$

$$J_{\lambda}^W(x) \bar{J}_{\mu}^W(o) = \sin \theta_c \cos \theta_c (J_{\lambda}^0(x) \bar{J}_{\mu}^1(o) + J_{\lambda}^1(x) \bar{J}_{\mu}^0(o))$$

+ octet part

(24c)

$$\bar{J}_{\lambda}^W(x) J_{\mu}^W(o) = \sin \theta_c \cos \theta_c (\bar{J}_{\lambda}^0(x) J_{\mu}^1(o) + \bar{J}_{\lambda}^1(x) J_{\mu}^0(o))$$

+ octet part

(24d)

Since in the expression (19a) with (20a) for τ^W and in the expression (22b) with (22d), for F^W , one can interchange λ and μ , it is easy to see by using the translational invariance, and the relations (24) that

$$\hat{F}_{\lambda}^W(\alpha \rightarrow \beta) = \frac{2 \sin \theta_c \cos \theta_c}{\sin^2 \theta_c - \cos^2 \theta_c} \hat{\tau}_{\lambda}^W(\alpha \rightarrow \beta) + \eta^W(\alpha \rightarrow \beta),$$
(25)

where $\wedge$ signifies that we are confining ourselves to uncharged observed $|\Delta S| = 1$ non-leptonic decays and $\eta^W(\alpha \rightarrow \beta)$ denote some combination of the octet parts which appear in the relations (24). Now using the fundamental relation (21), we have

$$\hat{\tau}_f^W(\alpha \rightarrow \beta) = - \hat{\tau}_f^{\phi}(\alpha \rightarrow \beta).$$
(26)

Thus using Eqs. (25) and (26), Eq. (23) becomes

$$\hat{F}_f^W(\alpha \rightarrow \beta) = \eta_f^W(\alpha \rightarrow \beta) + \tan 2 \theta_c \hat{\tau}_f^{\phi}(\alpha \rightarrow \beta) + \hat{F}_f^{\phi}(\alpha \rightarrow \beta).$$
(27)

Now $\eta^W(\alpha \rightarrow \beta)$ is a pure octet. $\hat{F}_f^{\phi}$ is determined by $J_{\phi}^1(x) J_{\phi}^1(o)$ and as emphasized by Lee and Treiman is a pure octet. We see from Eqs. (19b), (20b), (2b,c,d) and (17a,b) that $\hat{\tau}_f^{\phi}$ is determined by

$$\begin{aligned}
 J_\phi^0 j_\phi^0 &\sim \sin^2 \beta \left\{ \frac{m_P - m_r}{2} (\bar{P} P - \bar{r} r) \right. \\
 &+ \sin^2 \beta \left[\frac{m_\lambda + m_N}{2} (\bar{N} N + \bar{\lambda} \lambda) + m_q \bar{q} q \right. \\
 &\quad \left. + m_{q'} \bar{q}' q' \right] + \frac{m_{P'} - m_{r'}}{2} (\bar{P}' P' - \bar{r}' r') \\
 &\quad \left. - \frac{m_\lambda - m_N}{2} \sin^2 \beta [\bar{N} N - \bar{\lambda} \lambda] \right\} \times \\
 &\times \left\{ - \frac{m_\lambda + m_N}{2} [\bar{N} \gamma_5 \lambda - \bar{\lambda} \gamma_5 N] \right. \\
 &\quad \left. + \frac{m_\lambda - m_N}{2} (\bar{N} \lambda + \bar{\lambda} N) \right\} \quad (28)
 \end{aligned}$$

and $J_\phi^1 j_\phi^1$ [which is a pure octet as is clear from Eqs. (2d) and (17b)]. It is clear from Eqs. (27) and (28) that, in general, we do not get octet dominance or the $\Delta I = \frac{1}{2}$ rule for non-leptonic decays. However, for parity-violating decays, the violation of $\Delta I = \frac{1}{2}$ rule is of first order in SU(3) breaking.

Now all the terms in the first curly bracket in Eq. (28) are SU(3) singlets except the term $[(m_P - m_r)/2 \bar{P} P + (m_\lambda + m_N)/2 \sin^2 \beta (\bar{N} N + \bar{\lambda} \lambda)]$. This term violates the octet dominance. We can get octet dominance if this term is suppressed compared to $\sin^2 \beta (m_q \bar{q} q + m_{q'} \bar{q}' q')$. This is possible if $(m_\lambda + m_N)/2 \ll m_q, m_{q'}$ and $(m_P - m_r)/2 \ll m_q \sin^2 \beta, m_{q'} \sin^2 \beta$.

We conclude that for parity-conserving non-leptonic decays, in general we do not get the $\Delta I = \frac{1}{2}$ rule, whereas for parity-violating decays we get the $\Delta I = \frac{1}{2}$ rule only in the SU(3) limit.

3. SALAM-WEINBERG MODEL

In the four-quark version⁵⁾ of the Salam-Weinberg model, the lagrangian can be written

$$\begin{aligned}
 \mathcal{L} = & e J_\mu^{em} A_\mu + \frac{1}{2} (g^2 + g'^2)^{1/2} J_\mu^Z Z_\mu \\
 & + g_W (J_\mu^W \bar{W}_\mu + \bar{J}_\mu^W W_\mu) - \frac{\sqrt{2} g_W}{m_W} J_\phi \phi + \mathcal{L}_m^V, \quad (29)
 \end{aligned}$$

where

$$J_{\mu}^{em} = i \bar{q} \gamma_{\mu} Q q \quad (30a)$$

$$J_{\mu}^W = i \bar{q} \gamma_{\mu} (1 + \gamma_5) W q \quad (30b)$$

$$J_{\mu}^Z = \frac{1}{2} J_{\mu}^{W^0} - 2 \sin^2 \theta_W J_{\mu}^{em} \quad (30c)$$

$$J_{\mu}^{W^0} = i \bar{q} \gamma_{\mu} (1 + \gamma_5) W^0 q. \quad (30d)$$

Here q is the column matrix

$$\begin{bmatrix} P' \\ P \\ N \\ \lambda \end{bmatrix},$$

Q is the charge matrix, W is the 4×4 matrix $\begin{bmatrix} 0 & U \\ 0 & 0 \end{bmatrix}$,
with

$$U = \begin{bmatrix} -\sin \theta_c & \cos \theta_c \\ \cos \theta_c & \sin \theta_c \end{bmatrix},$$

θ_c being the Cabibbo angle, and $W^0 = [W, W^{\dagger}]_-$.

In this model

$$g_W^2 = g^2/8, \quad g_W^2 = \frac{e^2}{8 \sin^2 \theta_W}, \quad \frac{g^2 + g'^2}{4} = \frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W} \quad (31a)$$

$$\tan \theta_W = g'/g, \quad \frac{g^2 g'^2}{g^2 + g'^2} = e^2, \quad \frac{g_W^2}{m_W^2} = G_F/\sqrt{2}. \quad (31b)$$

The mass Lagrangian $\mathcal{L}_m^q$ and scalar current J_{ϕ} can be written

$$\mathcal{L}_m^q = -m_4 S^c - \epsilon_0 S_0 - \epsilon_8 S_8 - \epsilon_3 S_3 \quad (32a)$$

$$J_\phi = m_4 S^c + \epsilon_0 S_0 + \epsilon_8 S_8 + \epsilon_3 S_3, \quad (32b)$$

where

$$S^c = \bar{P}' P', \quad S_i = \bar{\Psi} \frac{\lambda_i}{2} \Psi. \quad (32c)$$

Here

$$\Psi = \begin{bmatrix} P \\ N \\ \lambda \end{bmatrix}, \quad \epsilon_0 = (2/3)^{1/2} (2\bar{m} + m_3),$$

$$\epsilon_8 = \frac{2}{\sqrt{3}} (\bar{m} - m_3), \quad \epsilon_3 = (m_1 - m_2)$$

$$\bar{m} = \frac{m_1 + m_2}{2} \quad \text{and} \quad c \equiv \epsilon_8 / \epsilon_0.$$

m_4, m_1, m_2 and m_3 are masses of quarks P', P, N and λ , respectively. We can write

$$J_\mu^W = J_\mu^{(8)} + J_\mu^{(3^*)}, \quad (33a)$$

$$\bar{J}_\mu^W = \bar{J}_\mu^{(8)} + \bar{J}_\mu^{(3)}, \quad (33b)$$

with

$$J_\mu^{(8)} = \cos \theta_c J_\mu^0 + \sin \theta_c J_\mu^1 \quad (34a)$$

$$J_\mu^{(3^*)} = i \bar{P}' \gamma_\mu (1 + \gamma_5) \lambda_c \quad (34b)$$

$$\bar{J}_\mu^{(i)} = J_\mu^{\dagger(i^*)} (1 - 2\delta_{i4}), \quad i = 8, 3^*. \quad (34c)$$

For this model

$$i \partial_\mu J_\mu^7 = i g_W \left[\bar{q}_\mu^W W_\mu + h_\mu^W \bar{W}_\mu \right] - i \frac{\sqrt{2} g_W}{m_W} j_\phi \phi \\ + i [i Q^7, d_n^V], \quad (35)$$

where

$$h_{\mu}^W = h_{\mu}^{(8)} + h_{\mu}^{(3^*)} \quad (36a)$$

$$\bar{g}_{\mu}^W = \bar{g}_{\mu}^{(8)} + \bar{g}_{\mu}^{(3)} \quad (36b)$$

with

$$h_{\mu}^{(8)} = -\cos \theta_c J_{\mu}^1 + \sin \theta_c J_{\mu}^0 \quad (37a)$$

$$h_{\mu}^{(3^*)} = i \bar{P}' \gamma_{\mu} (1 + \gamma_5) N_c \quad (37b)$$

$$\bar{g}_{\mu}^{(8)} = \sin \theta_c \bar{J}_{\mu}^0 - \cos \theta_c \bar{J}_{\mu}^1 \quad (37c)$$

$$\bar{g}_{\mu}^{(3)} = i \bar{N}_c \gamma_{\mu} (1 + \gamma_5) P' \quad (37d)$$

Here j_{ϕ} is given by

$$j_{\phi} = -\left(\frac{2}{3}\right)^{1/2} e_0 \left[\left(1 - \frac{c}{2\sqrt{2}}\right) P_7 + \frac{3c}{2\sqrt{2}} S_6 \right], \quad (38a)$$

where

$$P_7 = i \bar{\Psi} \gamma_5 \frac{\lambda_7}{2} \Psi \quad (38b)$$

In this model the matrix elements for non-leptonic decay $\alpha \rightarrow \beta$ are given by

$$F^W(\alpha \rightarrow \beta) = \frac{g_W^2}{2} \frac{1}{(2\pi)^4} \int d^4 q \frac{\delta_{\lambda\mu} + q_{\lambda} q_{\mu} / m_W^2}{q^2 + m_W^2} j_{\lambda\mu}^W \quad (39)$$

whereas $\tau^W(\alpha \rightarrow \beta)$ and $\tau^{\phi}(\alpha \rightarrow \beta)$ are given by

$$\tau^W(\alpha \rightarrow \beta) = \frac{g_W^2}{2} \frac{1}{(2\pi)^4} \int d^4 q \frac{\delta_{\lambda\mu} + q_{\lambda} q_{\mu} / m_W^2}{q^2 + m_W^2} g_{\lambda\mu}^W \quad (40a)$$

$$\tau^\phi(\alpha \rightarrow \beta) = \frac{g_W^2}{m_W^2} \frac{1}{(2\pi)^4} \int d^4 q \frac{1}{q^2 + m_\phi^2} g^\phi \quad (40b)$$

$\mathcal{F}_{\lambda\mu}^W$, $\mathcal{G}_{\lambda\mu}^W$ and $\mathcal{G}^\phi$ are as given in Eqs. (22d), (20a) and (20b), respectively, but with J_μ^W , $\bar{J}_\mu^W$, h_μ^W , $\bar{g}_\mu^W$, J_ϕ and j_ϕ as given in Eqs. (33), (34), (36), (37), (32) and (38).

Now following exactly the same method as in Section 2, it is easy to see that for the observed non-leptonic decays

$$\hat{F}^W(\alpha \rightarrow \beta) = \frac{2 \sin \theta_c \cos \theta_c}{\sin^2 \theta_c - \cos^2 \theta_c} \hat{\tau}^W(\alpha \rightarrow \beta) \quad (41)$$

$$\hat{\tau}_f^W(\alpha \rightarrow \beta) = - \hat{\tau}_f^\phi(\alpha \rightarrow \beta), \quad (42)$$

so that we have

$$\hat{F}_f^W(\alpha \rightarrow \beta) = \tan 2\theta_c \hat{\tau}_f^\phi(\alpha \rightarrow \beta). \quad (43)$$

We see from Eqs. (40b), (32b) and (38a) that $\hat{\tau}_f^\phi$ is determined by

$$J_\phi(x) j_\phi(0) = [m_4 S^c(x) + \epsilon_0 S_0(x) + \epsilon_8 S_8(x) + \epsilon_3 S_3(x)] \\ \times \left[-\left(\frac{2}{3}\right)^{1/2} \epsilon_0 \left[\left(1 - \frac{c}{2\sqrt{2}}\right) P_7(0) + \frac{3c}{2\sqrt{2}} S_7(0) \right] \right] \quad (44)$$

We observe that the coefficient of S_6 contains the parameter c which violates $SU(3)$. Thus the parity-violating part of the amplitude arises to the extent of $SU(3)$ violation compared to the parity-conserving part. Secondly the matrix elements for the observed $|\Delta S| = 1$ non-leptonic decays exhibit octet dominance if the term $m_4 S^c(x) + \epsilon_0 S_0(x)$ which is an $SU(3)$ singlet, dominates over the $SU(3)$ violating part $\epsilon_8 S_8$; $\epsilon_3 S_3$ violates isospin and since $\epsilon_3 \ll \epsilon_8$ can be neglected. In general, however, the violation of the octet dominance rule is of first order in $SU(3)$ breaking for parity-conserving decay and is of second order in $SU(3)$ breaking for parity-violating decay.

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