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QUARK ADDITIVITY FOR MASS SPLITTINGS

IN THE $p_z \rightarrow \infty$ FRAME

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A B S T R A C T

Quark additivity for $SU(3)$ mass splittings at $p_z \rightarrow \infty$ is found to give substantial corrections to naïve additivity, leading to a good agreement with the observed pattern.

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Quarks ¹⁾ have been given a double rôle in hadron physics. In their first rôle they are seen as basic carriers of weak and electromagnetic charges. In their second rôle they are seen as building blocks of the observed hadrons, baryons being three-quark states, mesons being quark-antiquark pairs. Deep inelastic phenomena have confirmed the view, originally based on the analysis of current algebra at infinite momentum, that these two rôles correspond to different, although related, kinds of quarks ²⁾. The recent work of Melosh ³⁾ has shown that this phenomenon is in fact natural and unescapable, since it occurs even for freely propagating quarks.

The Melosh transform, while leading to good results in the treatment of hadron spectroscopy, seems to be too simple to yield a correct description of deep inelastic phenomena. In a recent paper ⁴⁾ the relation between constituent and current quarks has been approached from a more phenomenological point of view, and it has been found that the present experimental information is consistent with the following picture : at $p_z \rightarrow \infty$ the nucleon is a loosely bound state of three constituent quarks, which are themselves made up of current (point-like) quarks and neutral objects (gluons). Structure functions of the nucleon in deep inelastic scattering can be computed in the impulse approximation on constituent quarks. This study has yielded detailed information both on the wave function of the three constituents within a nucleon in the $p_z \rightarrow \infty$ frame and on the structure functions of the constituents themselves.

We recall that the basic assumption in Ref. 4) is that deep inelastic phenomena are additive on constituent quarks. In the language of the light cone ⁵⁾ this means that certain observables which are bilinear in current quark fields (bilocal operators) remain essentially bilinear also when expressed in terms of constituent quark fields.

We will assume that this is true in general, i.e., also for those bilocal operators which are not simply measurable in deep inelastic scattering. This implies that observables bilinear in the current quark fields are additive on constituent quarks in the $p_z \rightarrow \infty$ frame.

In this paper we apply these ideas to the study of mass differences within $SU(3)$ multiplets. The present approach represents a substantial amelioration over the additivity hypothesis formulated within the old static quark model ⁶⁾ for two different reasons : it clarifies in which frame the additivity is to be applied and it gives large corrections to the naïve additivity, bringing the model into a good agreement with the experimental data.

Any bilinear, spin independent, quantity will be determined by its value on constituent quarks and by the distributions $Q_i^H(x)$ which define the probability of finding a constituent of type i ($i=p,n,\bar{\lambda},\bar{p},\bar{n},\bar{\lambda}$) inside a hadron of type H with a fraction x of the total momentum in the $p_z \rightarrow \infty$ frame. The quantities of interest will be expressed in terms of the moments of these distributions :

$$M_i^H(\lambda) = \int_0^1 x^\lambda Q_i^H(x) dx \quad (1)$$

We will assume the total momentum of the hadron, in the $p_z \rightarrow \infty$ frame, to be carried by constituent quarks, which leads to the following sum rules :

$$\sum_i M_i^H(1) = 1 \quad (\text{any } H) \quad (2)$$

Other constraints on the $M_i^H(\lambda)$ are given by the known composition of hadrons in terms of quarks, $SU(3)$ invariance, and by positivity arguments. We will assume the following form for the $SU(3) \otimes SU(3)$ breaking term in the strong interaction Hamiltonian ^{7),8)} :

$$\theta_{00} = \bar{\theta}_{00} + \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_3 u_3 \quad (3)$$

where, in terms of current quarks,

$$u_i = \bar{q} \lambda_i q \quad (4)$$

λ_i being the Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}}I$, and $\bar{\theta}_{00}$ is $SU(3) \otimes SU(3)$ invariant.

In the $p_z \rightarrow \infty$ frame, the matrix elements of the u_i between constituent quarks of momentum $x p_z$ will be given by

$$\langle i, x p_z | u_k | i, x p_z \rangle = \frac{U^{(k)}}{x p_z} (\lambda_k)_{ii} \quad (5)$$

$k = 0, 3, 8$

where $SU(3)$ symmetry requires $U^{(3)} = U^{(8)}$.

Let us evaluate the matrix element of u_8 between proton states :

$$\begin{aligned}
 m_{\text{proton}} \langle \text{proton}, P_z=0 | u_8 | \text{proton}, P_z=0 \rangle &\equiv \\
 &\equiv \lim_{P_z \rightarrow \infty} |P_z| \langle \text{proton}, P_z | u_8 | \text{proton}, P_z \rangle = \\
 &= U^{(8)} \frac{1}{\sqrt{3}} \left[\int_0^1 \frac{\phi_p^{\text{proton}}(x) + \phi_n^{\text{proton}}(x)}{x} dx \right] \quad (6) \\
 &= U^{(8)} \frac{1}{\sqrt{3}} \left[M_p^{\text{proton}}(-1) + M_n^{\text{proton}}(-1) \right]
 \end{aligned}$$

In general, we find :

$$m_H \langle H, P_z=0 | u_k | H, P_z=0 \rangle = U^{(k)} \sum_i (\lambda_k)_{ii} M_i^H(-1) \quad (7)$$

Using this equation, and assuming a knowledge of quark distribution within hadrons, the mass splittings due to $SU(3)$ breaking can be computed for any $SU(3)$ multiplet in terms of the single constant $\epsilon_8 U^{(8)}$.

It is convenient to rewrite Eq. (7) in the form :

$$m_H \langle H, P_z=0 | u_k | H, P_z=0 \rangle = U^{(k)} \sum_i (\lambda_k)_{ii} M_i^H(0) \left\langle \frac{1}{x} \right\rangle_i^H \quad (8)$$

where $\left\langle \frac{1}{x} \right\rangle_i^H$ represents the mean value of $1/x$ for a constituent of kind i in the parton H . This factor represents the correction to the naïve quark additivity, since $M_i^H(0)$ is just the number of i quarks inside H .

Another feature of Eqs. (7) or (8) is the appearance of m_H in the left-hand side. This means that the comparison between different multiplets will result in quadratic rather than linear mass formulas.

Let us begin to apply Eq. (8) to the low-lying baryon states, the octet $(\frac{1}{2})^+$ and the decuplet $(\frac{3}{2})^+$. Since these states are classified in the same $SU(6)$ representation, we may, as a first approximation, assume that the quantity $\left\langle \frac{1}{x} \right\rangle_i^H$, within this family, does not depend on either H or i .

This leads to a pure F type splitting within the octet and equal spacing in the octet and decuplet. The first prediction is only approximately true, since there is a small D type contribution. This can in fact be predicted on the basis of electroproduction data. If SU(6) were exact (i.e., equal distributions for p quarks and n quarks within a nucleon), one would obtain the inequality ⁴⁾ :

$$\frac{F_2^{e \text{ Neutron}}(x)}{F_2^{e \text{ Proton}}(x)} \geq \frac{2}{3} \quad (9)$$

which is badly broken near threshold. This can be interpreted by assuming that in a proton the p quarks are, on the average, faster than the n quarks, so that :

$$\left\langle \frac{1}{x} \right\rangle_p^{\text{Proton}} < \left\langle \frac{1}{x} \right\rangle_n^{\text{proton}} \quad (10)$$

One easily sees that this implies, through Eq. (8), a negative D/F for mass splittings. We have computed this quantity using the wave functions proposed in Ref. 4) and we find

$$\left(\frac{D}{F} \right)_{\delta M^2, \text{Baryon Octet}} = -0.3 \quad (11)$$

in excellent agreement with the experimental value -0.31.

The other prediction (equal spacing for octet and decuplet) is best checked through the relation :

$$\Xi^2 - N^2 = \Xi^{*2} - \Delta^2 \quad (12)$$

which should not be very sensitive to the small D part.

The same argument applies to the low-lying pseudoscalar and vector mesons, where we are led to the relation

$$m_K^2 - m_\pi^2 = m_{K^*}^2 - m_\rho^2 \quad (13)$$

Some caution is needed in the pseudoscalar case where mass splittings are of the same order as the mean mass of the multiplet. However, Eq. (13) can be justified using the treatment of Gell-Mann, Oakes and Renner ⁷⁾ from which it follows that :

$$m_K^2 - m_{\pi}^2 \simeq 2 m_K \langle K | \epsilon_8 u_8 | K \rangle - 2 m_{\pi} \langle \pi | \epsilon_8 u_8 | \pi \rangle \quad (14)$$

Relations (12) and (13) have been proposed many times ⁹⁾, and are well satisfied.

An essential correction to naïve additivity arises if we compare baryon and meson splittings, since we cannot expect constituent quarks to have the same distribution in both. Equation (8) leads to

$$\frac{1}{2} (\Xi^2 - N^2) \simeq (K^2 - p^2) \left[\langle \frac{1}{x} \rangle^{\text{Baryon}} / \langle \frac{1}{x} \rangle^{\text{Meson}} \right] \quad (15)$$

which reads :

$$0.42 \simeq 0.21 \left[\langle \frac{1}{x} \rangle^{\text{Baryon}} / \langle \frac{1}{x} \rangle^{\text{Meson}} \right] = 0.21 (1 + \delta) \quad (16)$$

The term in square brackets, which represents the correction to naïve additivity, should be roughly equal to the ratio of the number of quarks which would lead to $1 + \delta \approx \frac{3}{2}$. As we see the correction is in the good direction to achieve numerical agreement, experiment requiring a slightly larger correction $1 + \delta \approx 2$.

A better determination of the correction factor $1 + \delta$ would require some information on the distribution of constituents within a meson. Since we have no detailed experimental information on the meson wave function, comparable to that provided by deep inelastic scattering on nucleons, we tried a simple theoretical model where the meson wave function is treated on the same grounds as the nucleon one in Ref. 4), with the constraint that the average transverse momentum of constituents be the same for mesons and baryons. This results in $1 + \delta = 1.9$ which is perfect, but of course not very compelling in view of the unchecked assumptions which are needed.

It is interesting to recall that, as pointed out by Gell-Mann ¹⁰⁾ additivity implies

$$\frac{\langle \text{proton} | \varepsilon_0 u_0 | \text{proton} \rangle}{\langle \text{proton} | \varepsilon_8 u_8 | \text{proton} \rangle} = \frac{\langle \text{pion} | \varepsilon_0 u_0 | \text{pion} \rangle}{\langle \text{pion} | \varepsilon_8 u_8 | \text{pion} \rangle} \quad (16)$$

This, in turn, means that the σ term in pion-nucleon scattering should be rather small (~ 30 MeV), in agreement with the early determination of Ref. 11), and with some of the more recent determinations ¹²⁾.

In conclusion, we find that the $p_z \rightarrow \omega$ constituent quark description of hadrons which was successfully used in Ref. 4) to describe deep inelastic phenomena, also describes correctly the main features of SU(3) breaking mass splittings. In particular the observed D/F ratio in baryon mass splittings is explained in terms of the same configuration mixing introduced in Ref. 4) to interpret deep inelastic electroproduction.

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