Archives

TRANSVERSE MOMENTUM BOUNDS AND SCALING
IN THE HYDRODYNAMICAL MODEL

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ABSTRACT

It is shown that the equation of state of an ideal relativistic gas, as applied in the hydrodynamical model, leads not only to deviations from scaling in longitudinal rapidity distributions, but also to an average transverse momentum increasing asymptotically as a power of the incident energy. To prevent such an increase, one must use the equation of state of an interacting gas, in which the velocity of sound becomes asymptotically equal to that of light. This then also restores scaling (up to logarithmic terms) in longitudinal rapidity.

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The hydrodynamical model^{1,2)} for multiparticle production has recently attracted renewed interest^{3,4)}, when it became evident that in the pionization region of hadronic interactions, scaling for longitudinal momentum distributions, if it occurs at all, certainly sets in much later than in the fragmentation region; there are some indications for an energy dependence of pionization even at the highest ISR energies⁵⁾. The hydrodynamical model in Landau's formulation in fact contains such a behaviour as an inherent asymptotic feature: in the central region, it never leads to scaling. The aim of this note is to point out that these deviations from scaling are, however, intimately connected with the energy dependence of the transverse momentum distribution, and in particular, that they are not compatible with the constancy of the average transverse momentum as observed in accelerator data up to ISR energies⁶⁾.

The hydrodynamical description of multiparticle production in a collision of two hadrons (for simplicity identical and of mass M) can be divided into three steps: In the first, at the instant of the collision, all the initial energy W is released in a small volume $V=V_0(2M/W)$, Lorentz-contracted in the beam direction. The hadronic matter inside this volume is supposed to be governed by the equation of state of an ideal relativistic gas

$$p = \frac{1}{3} \epsilon \quad (1)$$

p denoting the pressure and $\epsilon = W/V$ the energy density. In the second step, the system expands; the expansion is taken as adiabatic and to be determined by the hydrodynamical equation for an ideal fluid. As the pressure gradient, because of the Lorentz-contraction, is greatest along the beam, the expansion occurs mainly in the beam direction. In the final stage, the system has expanded until all constituents are separated by a distance comparable to the range of hadronic forces; strong interactions cease and the system breaks up into the observed secondaries. - The model thus combines the statistical approach of the Fermi model⁷⁾ with the concept of expanding hadronic matter proposed by Pomeranchuk⁸⁾ to justify a free gas approach; it adds hydrodynamical expansion, because of the Lorentz-contraction dominantly in the longitudinal direction, and hence provides one of the first attempts⁹⁾ to arrive at a jet structure of hadronic interactions.

This jet structure has since then not only been strongly confirmed, but also much more precisely specified. On one hand, we have as one of the best established features of multihadron production, the constancy of the average transverse momenta over the full energy interval available with present accelerators⁶⁾. On the other hand, the energy independence of longitudinal momentum distributions as functions of the scaled variable $x=2p_L/W$ appears also rather well established for $x \neq 0$ ¹⁰⁾. In addition, we have such features as leading particle effects, factorization of e.g. pion vs. proton induced reactions, etc. One is thus today faced with the question of whether a hydrodynamical approach is still tenable in the light of this more detailed information.

While most features connected with fragmentation in the widest sense are at least not in contradiction to such a picture, it seems to us that the observed $\langle p_T \rangle$ behaviour is in fact not compatible with that predicted by the hydrodynamical model, at least not with the form of solutions so far discussed. Already the approximate results of Milekhin¹¹⁾ had suggested such difficulties; he found

$$\langle p_T \rangle \sim W^{1/4} \quad (2)$$

a behaviour which today contradicts both accelerator and cosmic ray data. Our aim here will be on one hand to study the energy dependence of $\langle p_T \rangle$ and its connection with deviations from scaling in Landau's formulation of the model, on the other to investigate alternatives to the equation of state (1) and their consequences on longitudinal and transverse distributions.

Such a task is always confronted by one basic difficulty, the absence of a full analytic or even numerical solution of the hydrodynamical equation. As in all previous works, we here also take recourse to the approximate solution first obtained by Landau^{1,2)}. Using the equation of state (1), he finds for the normalized single particle distribution

$$N(\lambda, W) \equiv \frac{1}{\sigma_{in}} \int dp \left(\frac{d^2 \sigma}{dp d\lambda} \right) = \frac{c_1}{\sqrt{2-L}} e^{\sqrt{L^2 - \lambda^2}} \quad (3)$$

where $L = \ln(W/2M)$, while $\lambda = -\ln \tan \theta/2$, with θ denoting the CMS angle between the momentum p of the secondary and the beam; σ_{in} is the total inelastic cross-section. In addition, the average energy of a secondary emitted at an angle θ is given by

$$\omega(\lambda, W) \equiv \frac{1}{\bar{N} \sigma_{in}} \int d\rho \rho_0 \left(\frac{d^2 \sigma}{d\rho d\lambda} \right) = \frac{c_2}{\bar{N} \sqrt{2\pi L}} e^{\lambda + \frac{4}{3} \sqrt{L^2 - \lambda^2}} \quad (4)$$

where $\bar{N} = \bar{N}(W)$ stands for the mean multiplicity. The particle number density $N(\lambda, W)$ and the energy density $E(\lambda, W) = \bar{N}(W) \omega(\lambda, W)$ must satisfy the sum rules

$$\int d\lambda N(\lambda, W) = \bar{N}(W) \quad (5)$$

$$\int d\lambda E(\lambda, W) = W \quad (6)$$

These relations fix the normalization factors c_1 and c_2 , which depend only on the total CMS energy W . Because of the adiabatic nature of the hydrodynamical expansion, $\bar{N}(W)$ is already determined by the equation of state alone; for (1), we have

$$\bar{N}(W) \sim (V W^3)^{1/4} \sim V_0^{1/4} W^{1/2} \quad (7)$$

Together with (5) and (6), this determines the energy dependence of c_1 and c_2 .

The average transverse momentum

$$\langle p_T \rangle \equiv \frac{1}{\bar{N} \sigma_{in}} \int d^3\rho |\vec{p}_T| \left(\frac{d^3 \sigma}{d^3 \rho} \right) \quad (8)$$

becomes at high energy ($m/W \rightarrow 0$) in terms of the variables used above

$$\langle p_T \rangle \approx \int d\lambda \sin \theta \omega(\lambda, W) \quad (9)$$

The evaluation of (9) will be given below in a more general context; the result is

$$\langle p_T \rangle \sim W^{1/6} \quad (10)$$

In fig. 1 this is compared to the experimentally observed values⁶⁾ of $\langle p_T \rangle$ and seen to be clearly incompatible with them.

The same solution (3-4), on the other hand, yields the mentioned deviations from scaling for longitudinal momentum distributions in the central region. In terms of the rapidity $y = (1/2) \ln((p_0 + p_L)/(p_0 - p_L))$ we have¹²⁾

$$\left(\frac{d\sigma}{dy}\right)_{y=0} \equiv \left\{ \int d^2 p_T \left(\frac{d^3 \sigma}{dy d^2 p_T} \right) \right\}_{y=0} = \left(\frac{d\sigma}{d\lambda}\right)_{\lambda=0} / \langle p/p_0 \rangle \quad (11)$$

which with (3) and (10) yields

$$\frac{1}{\sigma_{in}} \left(\frac{d\sigma}{dy}\right)_{y=0} \sim W^{1/2} / \sqrt{\log W} \quad (12)$$

i.e., the familiar⁴⁾ energy dependence of the rapidity distribution in the central region. We thus see that the deviation from scaling, (12), and the non-constant $\langle p_T \rangle$, (10), have the same origin: with the equation of state (1), hydrodynamic expansion occurs, though to a different extent, in both longitudinal and transverse directions.

The experimental disagreement with (10) implies that the hydrodynamical model in its original form cannot be maintained. From the theoretical side, this conclusion is perhaps not totally surprising: with the equation of state (1) - which, together with the hydrodynamical equation, forms the basis of the approach - we have assumed hadronic matter to behave as an ideal relativistic gas. While $p/\epsilon = 1/3$ represents an upper bound for the speed of sound in the presence of electromagnetic interactions¹³⁾, this does not need to remain so once hadronic forces come into play¹⁴⁾. We therefore want to consider the hydrodynamical approach, using the equation of state of a strongly interacting gas¹⁴⁾

$$p = c_0^2 \epsilon, \quad 0 < c_0^2 < 1 \quad (13)$$

with particular attention directed at the average transverse momentum and its connection with scaling near $y=0$.

Because of the adiabatic hydrodynamical expansion, the general equation of state (13) gives immediately^{1,15)}

$$\bar{N}(W) \sim V (W/V)^{1/(1+c_0^2)} \sim W^{\frac{1-c_0^2}{1+c_0^2}} V_0^{c_0^2/(1+c_0^2)} \quad (14)$$

for the average particle number. - The solution of the hydrodynamical equation is carried out as before^{1,2)}, but with (13) we now obtain

$$N(\lambda, W) = \frac{\tilde{c}_1}{\sqrt{2\pi L}} e^{\frac{1-c_0^2}{2c_0^2} \sqrt{L^2 - \lambda^2}} \quad (15)$$

for the particle number density, and

$$\omega(\lambda, W) = \frac{\tilde{c}_2}{\bar{N} \sqrt{2\pi L}} e^{\left\{ \lambda + \frac{1-c_0^4}{2c_0^2} \sqrt{L^2 - \lambda^2} \right\}} \quad (16)$$

for the energy density, with the same notation as above. Equations (5) and (6) again determine the energy dependence of $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$; with (14), we find to leading power in W

$$\tilde{\sigma}_1 \approx W^{-(1-c_0^2)^2/2c_0^2(1+c_0^2)} \quad (17)$$

$$\tilde{\sigma}_2 \approx W^{1-(\frac{1+c_0^4}{2c_0^2})} \quad (18)$$

The average transverse momentum of a secondary is given by (8); using

$$\sin \theta = \operatorname{sech} \lambda \quad (19)$$

we have

$$\langle p_T \rangle \approx \frac{\tilde{\sigma}_2}{N \sqrt{2\pi L}} \int_0^L d\lambda e^{\left\{ \frac{1-c_0^4}{2c_0^2} \sqrt{L^2-\lambda^2} \right\}} \quad (20)$$

and hence to leading power in W

$$\langle p_T \rangle \sim W^{c_0^2(1-c_0^2)/(1+c_0^2)} \quad (21)$$

which for $c_0^2=1/3$ reduces to (10). - The amount of scale-breaking in the longitudinal distribution at $y=0$ is, using (11), now given by

$$\frac{1}{\sigma_{in}} \left(\frac{d\sigma}{dy} \right)_{y=0} \approx \frac{\tilde{\sigma}_1}{\sqrt{2\pi L}} e^{(\frac{1-c_0^2}{2c_0^2})L} \quad (22)$$

which with (17) yields

$$\frac{1}{\sigma_{in}} \left(\frac{d\sigma}{dy} \right)_{y=0} \sim W^{(1-c_0^2)/(1+c_0^2)} / \sqrt{\log W} \quad (23)$$

From (21) and (23) we see how the behaviour of $\langle p_T \rangle$ and the violation of scaling in the central region are interrelated for general equations of state of the form (13).

In equations (17) - (22) we have retained the full energy dependence, including logarithmic terms. As Landau's approximate solution (15)-(16) is, however, at best correct up to logarithmic terms, our results are also really meaningful only up to such accuracy.

From (21) it is evident that an asymptotic increase of $\langle p_T \rangle$ as a power of the incident energy can be avoided only if for $W \rightarrow \infty$ either $c_0^2 \rightarrow 0$ or $c_0^2 \rightarrow 1$. For $c_0^2 \rightarrow 0$ (a case realized e.g. in the Pomeranchuk model⁸⁾ or in the statistical bootstrap model¹⁶⁾) there is asymptotically

no hydrodynamical expansion, and hence the entire approach breaks down.

Let us therefore consider $c_0^2 \rightarrow 1$; with

$$c_0^2 = 1 - 2g(w) \quad ; \quad \lim_{w \rightarrow \infty} g(w) = 0 \quad (24)$$

we obtain for the multiplicity (14)

$$\bar{N}(w) \sim W^{g(w)} \quad (25)$$

and hence an asymptotic increase slower than any power of W ; the specific choice

$$g(w) = \log L / L \quad (26)$$

results in the familiar $\bar{N} \sim \log W$. It is moreover evident from (21) and (23) that $c_0^2 \rightarrow 1$ yields (up to logarithmic terms) a constant $\langle p_T \rangle$ and a scaling longitudinal rapidity distribution, i.e., a description¹⁷⁾ which on this level is equivalent to that given by the multiperipheral model.

We have thus seen that kinematical arguments (Lorentz contraction) alone cannot account for the hadronic jet structure; in addition, one must include strong interaction dynamics to obtain an equation of state which gives $c_0^2 \rightarrow 1$, instead of the $c_0^2 = 1/3$ resulting from an ideal three-dimensional relativistic gas. An example of a strongly interacting system with $c_0^2 \rightarrow 1$ was already given by Zeldovich¹⁴⁾; more recently, such systems have become of particular interest for the description of very high density objects in astrophysics^{18,19)}.

Formally, the equation of state (13) can be looked upon as arising from an ideal one-dimensional gas; the generating function

$$G(R, w) = \sum_{N=1}^{\infty} \frac{R^N}{N!} \int_0^{\infty} \prod_{i=1}^N d\eta_i \delta\left(\sum_i \eta_i - w\right) \quad (27)$$

immediately leads to $p = \epsilon$. It must be emphasized, however, that relation (27) does not refer to physical particles; it is simply an analogue to obtain the equation of state of an interacting three-dimensional system²⁰⁾.

The Feynman-Wilson fluid analogy²¹⁾, in which peripheral multiparticle production dynamics are represented by a one-dimensional analogue system in rapidity space, also leads to $c_0^2 \rightarrow 1$ ¹⁷⁾; here the analogue describes the full production dynamics. In the hydrodynamical model, however, interaction dynamics, or an analogue such as (27), are needed only to determine the equation of state of a physical hadronic systems²²⁾. Once this equation is given, multiparticle production is completely

fixed by the hydrodynamical equation together with the initial conditions provided by the collision process (Lorentz contraction).

Hence the asymptotic requirement $c_0^2 \rightarrow 1$, needed to bring the hydrodynamical model in accord with data from hadron-hadron collisions, becomes of particular interest when applied to problems with changed boundary conditions, e.g., for e^+e^- annihilation. While here the Feynman picture is generally taken to retain its essential characteristics from hadronic collisions²³⁾, the hydrodynamical description - because of the absence of any Lorentz contraction - becomes significantly different: with $c_0^2 \rightarrow 1$, we now have

$$\bar{N}(w) \sim w^{1/2} \quad (28)$$

instead of (25). The resulting rapidity distributions of the secondaries do not exhibit the scaling behaviour found for longitudinal distributions from hadron-hadron collisions; they will furthermore also differ from those previously obtained for e^+e^- annihilation with $c_0^2 = 1/3$ ^{24,25)}.

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FIGURE CAPTION

The average transverse momenta of positive pions (X) from accelerator data and of charged (Δ) and neutral particles (O) from cosmic ray data, as compiled in Ref. 6).