Archives

MUON-ELECTRON UNIVERSALITY AT HIGH ENERGIES AND HADRONICCORRECTIONS TO $e^+e^- \rightarrow \mu^+\mu^-$ G. Karl ^{*)}

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ABSTRACT

It is argued that unitarity and muon-electron universality are sufficient to determine the contribution of hadrons to the imaginary part of the forward amplitude for the process $e^+e^- \rightarrow \mu^+\mu^-$. With these assumptions $\text{Im}T_1^h(e^+e^- \rightarrow \mu^+\mu^-) = \text{so}(e^+e^- \rightarrow \text{hadrons})$ to all orders in α . Some consequences of this formula are discussed.

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Recent experiments ¹⁾ have revealed that the annihilation cross-section of electron-positron pairs into hadrons at high energy

$$e^+ e^- \rightarrow \text{hadrons} \quad (1)$$

is remarkably large when compared to the cross-section for purely leptonic processes, such as

$$e^+ e^- \rightarrow \mu^+ \mu^- \quad (2)$$

While this fact carries implications for theories of hadronic constituents, we will concentrate here on the modifications to be expected for T_1 , the amplitude ²⁾ of process (2) from its value computable in leptodynamics ³⁾. One can describe these modifications in QED as changes in the photon propagator caused by the hadronic processes (1). We will instead apply unitarity plus an assumption which expresses electron-muon universality at high energies to derive the contribution of hadronic processes (1) to the imaginary part of the amplitude of process (2). It seems that the assumptions used here are somewhat weaker than in the usual treatment and the amount of formal manipulations is quite small, so that the final formula, with the correct normalization factors is obtained quickly. The difficulty with applying unitarity to process (2) is that it is an inelastic process. We therefore need some extra input. We assume a strong version of electron-muon universality at high energies. Namely, we assume that, given an amplitude for a process involving high energy electrons and/or muons it equals the amplitude of the process where all electrons are interchanged with muons and conversely. More precisely a μ^- of momentum $\vec{p}$, helicity λ is replaced by an e^- of momentum $\vec{p}$ and helicity λ and similarly for μ^+, e^-, e^+ . By high energy we mean that the energies are high enough to set ⁴⁾ the ratios $m_e/E \sim m_\mu/E \sim 0$. Usually universality is stated as meaning that the only difference between electrons and muons is their mass. At high energy even this distinction disappears. Although we stated this rule generally we only need two instances of it. Firstly, we take the amplitude T_1 for process (2) at high energy to equal the amplitude T_2 for the process :

$$\mu^+ \mu^- \rightarrow e^+ e^- \quad (3)$$

at the same energy, momenta and helicities as in (2). The equality ⁵⁾

$$T_1 = T_2 \quad (4)$$

can be verified in the Born approximation $T_1^{(B)} = T_2^{(B)}$, where in fact only the helicity states $(+-)$ and $(-+)$ give non-zero amplitudes in the limit discussed here ^{6),5)}. The equality (4) is assumed to hold true even if the corrections to the Born amplitudes $T_1^{(B)}$ and $T_2^{(B)}$ are large.

The second application of this universality rule is to take the amplitude for process (1), leading to a definite hadronic state h , equal at high energy to the amplitude for the process $\mu^+\mu^- \rightarrow h$ where again the momentum and helicity of $\mu^+(\mu^-)$ are the same as the momentum and helicity of $e^+(e^-)$ in process (1)

$$T(e^+e^- \rightarrow h) = T(\mu^+\mu^- \rightarrow h) \quad (5)$$

With the specific universality assumptions (4) and (5), we can apply the usual unitarity equations to the process (2) in the forward direction ⁷⁾

$$\langle \mu^+\mu^- | T | e^+e^- \rangle - \langle \mu^+\mu^- | T^\dagger | e^+e^- \rangle = i \langle \mu^+\mu^- | T^\dagger T | e^+e^- \rangle \quad (6)$$

where using equation (4) we have

$$\langle \mu^+\mu^- | T^\dagger | e^+e^- \rangle = \langle e^+e^- | T | \mu^+\mu^- \rangle^* = \langle \mu^+\mu^- | T | e^+e^- \rangle^* \quad (7)$$

Inserting a complete set of intermediate states on the right-hand side of equation (6) one obtains

$$2 \operatorname{Im} T_i(E, \theta=0) = \sum_n \langle n | T | \mu^+\mu^- \rangle^* \langle n | T | e^+e^- \rangle \quad (8)$$

where E denotes the energy of an electron (in the c. of m. frame), θ the scattering angle, and the helicity indices are suppressed, though to be specific we consider the amplitude $(+;-;-)$ say.

If we restrict ourselves to hadronic states h only, in the right-hand side of equation (8) and denote by a superscript h the corresponding contribution to the left-hand side, we obtain

$$\begin{aligned} 2 \operatorname{Im} T_i^h(E, \theta=0) &= \sum_h \langle h | T | \mu^+\mu^- \rangle^* \langle h | T | e^+e^- \rangle \\ &= \sum_h |\langle h | T | e^+e^- \rangle|^2 \end{aligned} \quad (9)$$

where we have made use of equation (5) in the last step. When translated into cross-sections ⁸⁾, equation (9) reads

$$\text{Im } T_1^h(E, \theta=0) = S \sigma(e^+e^- \rightarrow h) = S \sigma_0 \quad (10)$$

where $s = 4E^2$ and the helicity of the electron-positron pair in σ_0 on the right-hand side of (10) is the same as that in the amplitude T_1 on the left-hand side. This is the result we wanted to obtain. Taking σ_0 (unpolarized) $= 2 \times 10^{-32} \text{ cm}^2 = 0.5 \times 10^{-4} \text{ GeV}^{-2}$ at $E = 2.5 \text{ GeV}$ and assuming that the cross-section retains this value at some higher energies as well ⁹⁾ we can compute $\text{Im } T_1^h$ generated at various high energies using equation (10). Assuming annihilation from the helicity states $(+-)$ and $(-+)$ only, and with equal rates, we get $\sigma_0(+-) \sim 10^{-4} \text{ GeV}^{-2}$. We then obtain

$$\begin{array}{ll} \text{at } E = 2.5 \text{ GeV} & \text{Im } T_1^h = 2.5 \times 10^{-3} \\ E = 5 \text{ GeV} & \text{Im } T_1^h = 10^{-2} \\ E = 10 \text{ GeV} & \text{Im } T_1^h = 4 \times 10^{-2} \\ E = 20 \text{ GeV} & \text{Im } T_1^h = .16 \end{array}$$

For comparison, the (real) amplitude in the Born approximation is $T_1^{(B)}[(+-); \theta = 0] = 8 \pi \alpha = 0.17$. We see that at $E = 2.5 \text{ GeV}$ the hadronic contribution is still comparable to $\alpha T_1^{(B)}$ whereas at 10 GeV the imaginary part generated by hadronic processes is 25 % of the Born amplitude, while at $E = 20 \text{ GeV}$ the two quantities are equal. One would expect this imaginary part to be present not only in the forward direction but also in the whole $J = 1$ amplitude [to the extent process (1) occurs through the state $J = 1$]. Such a sizeable imaginary part could be detected by polarization measurements and also eventually in the total cross-section. The observation of imaginary parts of this size in a purely leptonic process (2) would constitute prima facie evidence for the breakdown of leptodynamics. On the other hand, should this prediction fail to be true one would disprove experimentally the assumptions leading to (10). Trusting unitarity one would suspect that the version of universality assumed here was at fault. There would be interest in such a conclusion; it could be interpreted to mean that the intermediate photon "remembers" whether it was created by electrons or muons. No matter how unlikely this possibility seems a priori, it is worth trying to verify it.

The difficulty about verifying experimentally equation (10) is that one can only measure $\text{Im } T_1$ and not separately its hadronic contribution, so

we will consider also the expected contributions from non-hadronic channels n , in equation (8). Two cases can be distinguished : channels containing only leptons ¹⁰⁾ (i.e., e^+e^- or $\mu^+\mu^-$) and mixed channels containing both leptons and hadrons. The mixed channels contribute amplitudes which are (in terms of graphs) at least one power of α higher than the hadronic channels (assuming 1 photon annihilation), and so their contribution to $\text{Im}T_1$ is suppressed by a factor of $\alpha^2 \sim 10^{-4}$. In addition one can say that the overlap in the RHS of (8) between $T(e^+e^- \rightarrow e^+e^-h)$ and $T(\mu^+\mu^- \rightarrow e^+e^-h)$ is expected to be small as the two amplitudes are large in different regions of phase space ¹¹⁾. In any case, detailed experimental information on e^+e^- annihilation when available can be used to put an upper bound on the contribution of mixed channels in (8). The expected contribution of the leptonic channels on the right-hand side of (8) is more delicate. I cannot compute their contribution directly because of well-known divergence difficulties. However, one would expect to start with, that if this calculation could be carried out, it would generate an imaginary part $\text{Im}T_1^{\text{leptons}}$ in accordance with leptodynamics, i.e., of order $\alpha T_1^{(B)}$, which is smaller than $\text{Im}T_1^h$ already at $E = 5$ GeV, not to speak of higher energies. However, as the energy increases, $\text{Im}T_1^h$ will increase and therefore we can no longer expect (at these higher energies) the leptonic channels to continue to contribute an amount of order $\alpha T_1^{(B)}$ to $\text{Im}T_1$. However, to start with, say up to about $E \sim 10$ GeV, I would expect these "second order" corrections to the leptonic contributions to be small and the imaginary part of T_1 be well approximated by $\text{Im}T_1^h$.

Clearly the measurement of $\text{Im}T_1$ at energies E between 5-10 GeV would be of great interest. Equally clearly it seems that the long arm of unitarity deserves further theoretical investigation, especially at these high energies where the leptons begin to know the hadrons more intimately.

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FOOTNOTES AND REFERENCES

- 1) B. Richter. Talk at Irvine, Calif. (Dec.73). I am grateful to Ch. Llewellyn Smith for information about this talk;
L. Litke et al. Phys.Rev.Letters 30, 1189 (1973).
- 2) I use the normalization of amplitudes T and phase space described in H. Pilkuhn, the interaction of hadrons, North Holland (1967). In this normalization the differential cross-section for $2 \rightarrow 2$ scattering is given in terms of the dimensionless amplitude T by :

$$\frac{d\sigma}{d\Omega} = \frac{1}{4s} \left| \frac{T(s, \theta)}{4\pi} \right|^2 \quad \text{in GeV}^2/\text{steradian}$$

and if there are helicity indices then this formula applies to a definite initial and final helicity state.
- 3) Leptodynamics is QED restricted to leptons. I have borrowed this term from J.S. Bell and E. de Rafael, Nuclear Phys. B11, 611 (1969), who discuss corrections due to processes (1) in the gyromagnetic ratio g_μ of muons. The corrections to g_μ are rather small and high precision experiments are necessary to detect them. In contrast, at sufficiently high energy the corrections we discuss are equal to the whole leptodynamic amplitude.
- 4) I am ignoring any logarithms of (E/m_μ) which might be present. See however, footnote 5), which shows that for $e^+e^- \rightarrow \mu^+\mu^-$ even the logarithms present must be the same in the "large" amplitude; while the "small" amplitudes vanish rigorously in the high energy limit even if logarithms are present.
- 5) For the specific "large" amplitudes $T_1(+;-;-)$ and $T_1(+;-;-)$ one can use TCP invariance to obtain their equality (up to a phase) to the amplitudes $T_2(+;-;-)$, $T_2(+;-;-)$ at any energy. However, to settle the phase one has to resort to some universality statement, which furthermore is also needed later on. So I settle with the rather stronger statement used in the text. I am indebted to Ch. Llewellyn Smith for raising this point.
- 6) R. Gatto, Ergebnisse der Exakten Naturwissenschaften, Vol. 39, p.106 Springer Verlag, Berlin.
- 7) We could use partial-wave unitarity if we are willing to guess which angular momenta contribute to (1) and (2).
- 8) See Ref. 2) page 38, Eq. (4.2).
- 9) If (1) occurs via $J = 1$ intermediate states only, then $\sigma_0(e^+e^-, \text{unpolarized})$ could retain the value $0.5 \times 10^{-4} \text{ GeV}^{-2}$ up to an energy of $E \sim 200 \text{ GeV}$ without violating the unitarity bound $\sigma \leq 3/4\pi\lambda^2$. See Ref. 6
- 10) In these channels, since the amplitude for $e^+e^- \rightarrow e^+e^-$ is unrelated to T_1 , I cannot make any statement about the sign of the contribution to $\text{Im}T_1$.
- 11) I am indebted to Ch. Llewellyn Smith for this remark.