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FAST LEADING PARTICLES AND
LONG RANGE RAPIDITY CORRELATIONS BETWEEN PIONS

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A B S T R A C T

The long-range rapidity correlations between two produced particles due to the existence of very fast leading particles are calculated using the high energy data on the single diffraction dissociation as input. The method of calculation is to a large extent analytic. The long range correlations are found to be equally important as short range effects in interpreting experimental data on two-particle correlations. The results are not sensitive to the detailed properties of diffraction dissociation.

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1. INTRODUCTION

Recently, the two-particle rapidity correlations have been studied experimentally at the CERN ISR ¹⁾. They can be understood ^{2),3),4)} as a superposition of long-range effects due to the existence of the inelastic diffraction and short-range effects which can be interpreted in a picture of (dominantly) small clusters of particles produced in an approximately independent way. However, the actual amount of the long-range and short-range correlations is not yet precisely determined. With sufficiently accurate data on single and two-particle distributions over the ISR energy range the separation of the two types of correlations could be performed in an almost model independent way. One attempt in this direction has already been done ⁵⁾ on the basis of the new ISR results at $\sqrt{s} = 23$ and 62 GeV ¹⁾.

In our earlier calculation of these long-range correlations ³⁾ we have used a simple model of diffraction with a sharp cut-off in the mass distribution of the excited system. Consequently, the contribution of diffraction to the pion production in the central rapidity region is assumed to be negligible. According to the recent ISR and NAL data ⁶⁾, an important part of the leading particle peak corresponds, however, to relatively high excitation masses. Thus, the diffractive production of pions in the central rapidity region should not be totally neglected and a more refined analysis is necessary.

In this paper we present a new quantitative estimation of the long range effects. Our analysis is based on the recent NAL and ISR data on the inelastic diffraction. The role of the low mass and high mass diffractive excitations is examined in some detail. We calculate the one and two-pion rapidity distributions using the two-nucleon distribution as an input and assuming that the properties of the pion distributions depend in a universal way on the energy available for pions. Our method of calculation is to a large extent analytic and it is described in the Appendix.

The present analysis shows that the refined treatment of diffraction dissociation does not change the earlier conclusion ³⁾ that both long-range and short-range effects are important in understanding the overall two-particle correlations.

2. FAST LEADING PARTICLES AND LONG-RANGE CORRELATIONS BETWEEN PIONS

Two-particle rapidity correlations are generally expressed in terms of the correlation function ^{*)}

$$C_2(y_1, y_2) = \Psi_2(y_1, y_2) - \Psi(y_1) \Psi(y_2) \quad (1)$$

or in terms of the ratio

$$R_2(y_1, y_2) = C_2(y_1, y_2) / (\Psi(y_1) \Psi(y_2)) \quad (2)$$

where y denotes the cms longitudinal rapidity. We want to make here a separation between short-range and long-range correlations. A typical mechanism leading to short-range correlations is the resonance formation or more generally clustering. The long-range correlations, instead, are due to variations in the global structure of the events. In the present discussion we concentrate on the long-range correlations due to the existence of fast leading particles. We make therefore the assumptions that the global structure is essentially determined by the x values ($x = 2p_L / \sqrt{s}$) of the two nucleons ^{**) (we discuss here only proton-proton scattering and neglect baryon-antibaryon production) and that the pion distribution of a mesonic system is determined completely by the mass of this system. More specifically we calculate the one and two-pion distributions from the formulae:}

^{*)} We define the single and two-particle rapidity densities

$$\Psi(y) = \frac{1}{\sigma} \frac{d\sigma}{dy}, \quad \Psi_2(y_1, y_2) = \frac{1}{\sigma} \frac{d^2\sigma}{dy_1 dy_2}$$

with $\sigma = \sigma_{\text{inelastic}}$.

^{**) A similar assumption was used in a phenomenological analysis of multi-particle production at lower energies ⁷⁾. It is also inherent in several models of particle production (e.g., in the uncorrelated jet model ⁸⁾).}

$$\Psi(y) = \int \rho(x_1, x_2) D^{(1)}(y, x_1, x_2) dx_1 dx_2 \quad (3)$$

$$\Psi_2(y_1, y_2) = \int \rho(x_1, x_2) D^{(2)}(y_1, y_2, x_1, x_2) dx_1 dx_2 \quad (4)$$

where $\rho(x_1, x_2)$ is the normalized two-nucleon distribution and $D^{(1)}$ and $D^{(2)}$ are the one and two-pion distributions for fixed x_1 and x_2 . Since all transverse momenta are small, it is obvious that the values of x_1 and x_2 determine the mass of the produced mesonic system M_x and the CM rapidity $\emptyset$ of its center ⁹⁾ [formulae (A.3) and (A.4) of the Appendix]. Our last assumption means that the one-pion distribution $D^{(1)}(y, x_1, x_2)$ depends only on the two variables $y - \emptyset$ and M_x . We assume further that it develops at high masses M_x a plateau with a constant (s and M_x independent) height Ψ_0 . In the appendix we show that with these assumptions, and for $x_1 > 0$, $x_2 < 0$ one can write $D^{(1)}$ approximately in the form

$$D^{(1)}(y, x_1, x_2) = \Psi_0 D(y - y_r) D(y_l - y) \quad (5)$$

where y_l and y_r are the effective limits of the rapidity plateau ($y_l < y_r$) for fixed x_1 and x_2 and y_l (y_r) depends only on x_2 and s (x_1 and s). $D(y)$ is a cut-off function which has the nature of a rounded off step function: $D(y) \cong \theta(-y)$ (see the Appendix). When $y_r > 0$ and $y_l < 0$ formula (5) follows from the observation that at high energies and not too high multiplicities energy and momentum are conserved practically independently in both c.m. hemispheres. The approximation of formula (5) is, however, valid also when y_1 and y_2 have the same sign as long as $x_1 > 0$, $x_2 < 0$ and $(1-x_1)(1+x_2) \gg m_p^2/s$. Next, we separate the correlations (present at fixed x_1 and x_2) from the two-pion distribution $D^{(2)}$ by writing

$$D^{(2)}(y_1, y_2, x_1, x_2) = D^{(1)}(y_1, x_1, x_2) D^{(1)}(y_2, x_1, x_2) \quad (6)$$

$$+ C_2(y_1, y_2, x_1, x_2)$$

We expect that C_2 is dominantly due to short-range correlations but in any case only the first term in (6) is of interest for us as long as we study the long range effects due to the existence of fast leading particles. We notice that in the approximation $D(y) = \theta(-y)$ and for $y_1 < y_2$ the first term in formula (6) is given simply by

$$\psi_0^2 D(y_2 - y_1) D(y_1 - y_1) .$$

The more general case is considered in the Appendix [formula (A.9)].

In the framework of our scheme the calculation of inclusive one and two-pion distributions requires the knowledge of the two-nucleon distribution. There is unfortunately very little information on nucleon-nucleon correlations at high energies, we know only the single proton distributions. In the spirit of the two-component approach we take $\rho(x_1, x_2)$ as a sum of non-diffractive and diffractive terms (for the latter only single diffraction dissociation is considered) and furthermore we assume the absence of the nucleon-nucleon correlations within each of the components ^{*}):

$$\begin{aligned} \rho(x_1, x_2) = & \alpha_\pi \rho_\pi(x_1) \rho_\pi(-x_2) \\ & + \frac{\alpha_D}{2} (\rho_{D1}(x_1) \rho_{D2}(-x_2) + \rho_{D2}(x_1) \rho_{D1}(-x_2)) \end{aligned} \quad (7)$$

Here π refers to the non-diffractive (pionization) component and D to the diffractive component; $\rho_{D1}(x)$ is the distribution of non-excited protons and is peaked at $x \approx 1$. The distributions $\rho_{D2}(x)$, which correspond to nucleons originating from a diffractively excited system, and $\rho_\pi(x)$ are expected to be relatively flat around $x = 0.5$ and drop towards $x = 1$. All functions are normalized to $\int_0^1 \rho(x) dx = 1$.

The choice made in formula (7) allows one to write $\psi(y)$ in the form (see Appendix):

$$\psi(y) = \psi_0 \left[\alpha_\pi F_\pi(y) F_\pi(-y) + \frac{\alpha_D}{2} (F_{D1}(y) F_{D2}(-y) + F_{D2}(y) F_{D1}(-y)) \right] \quad (8)$$

^{*}) For the calculations of this paper the only essential assumption made here is that there is no enhancement of events with two very fast nucleons.

The functions $F(y)$ are given, in the approximation $D(y) = \theta(-y)$, by

$$F(y) = \int_0^{x_{\max}} dx \rho(x) \quad (9)$$

where

$$x_{\max} = 1 - e^{y-Y}$$

and $Y = \log(\sqrt{s}/M_0)$. M_0 is a constant which is related to the effective width of the plateau originating from a mesonic system with fixed mass [formula (A.5)]. When $D(y) \neq \theta(-y)$ we get $F(y)$ by convoluting the result of formula (9) with the derivative of $D(y)$ [formula (A.17)]. In the case $D(y) = \theta(-y)$ we have similarly

$$\begin{aligned} \psi_2(y_1, y_2) = & \psi_0^2 \left[\alpha_\pi F_\pi(y_2) F_\pi(-y_1) \right. \\ & + \frac{\alpha_D}{2} (F_{D1}(y_2) F_{D2}(-y_1) + F_{D2}(y_2) F_{D1}(-y_1)) \left. \right] \quad (10) \\ & + C_2^{SR}(y_1, y_2) \end{aligned}$$

where $C_2^{SR}(y_1, y_2)$ is the integral of $\rho(x_1, x_2) \cdot C_2(y_1, y_2, x_1, x_2)$ and is not considered further here. In the case of two-particle distribution the corrections present when $D(y) \neq \theta(-y)$ are somewhat more complicated and are discussed in the Appendix. They are also applied in the calculations of Chapter 4.

As the approximations involved are not valid for the production of low mass mesonic systems we cannot calculate reliably the one and two-pion distributions $\psi(y)$ and $\psi_2(y_1, y_2)$ in the fragmentation region ^{*)} where the contribution from low mass systems is non-negligible. In particular, the

^{*)} A quantitative estimate for the validity region is $|y| \lesssim \frac{1}{2} \log(s/10)$.
E.g., for $s = 930 \text{ GeV}^2$ we get $|y| \lesssim 2.3$.

correlation function may have a wrong behaviour in the fragmentation region as it is the difference of two comparable quantities.

Now we see that our earlier analysis ³⁾ corresponds to formulae (8) and (10) with the additional assumptions that $F_{\pi}(y)=1$ and $F_{D1}(y)=0$ over the whole central region. Then we obtain as expected on the basis of the old analysis $C_2^{LR}(y_1, y_2) = \alpha_D \alpha_{\pi}$ and $R_2^{LR}(y_1, y_2) = \alpha_D / \alpha_{\pi}$ over the validity region of the above assumptions.

One should note that the existence of long-range correlations is not directly related to the presence of several components in formula (7) but to the fact that not all of the functions $F(y)$ have the value one, i.e., to the fact that there are events that cannot produce pions at $y=0$, say. This can be seen most clearly when we have only one term in formula (7). Let us consider only the point $y_1=y_2=0$ and denote $F(0)=1-\varepsilon$. Then we see from formulae (8) and (10) that $C_2(0,0) = \psi_0^2 \varepsilon (1-\varepsilon)^2 (2-\varepsilon) + C_2^{SR}(0,0)$, i.e., to the lowest order in ε the long-range correlations are proportional to

$$\varepsilon = 1 - F(0) = \int \frac{1}{1 - \exp(-Y)} dx \varphi(x)$$

i.e., to the integral of $\varphi(x)$ over a small region near $x=1$. The correlations are thus negligible if $\varphi(x)$ is not peaked in this region.

Before going to more quantitative calculations of correlations we discuss in the next section the properties of diffraction dissociation in order to determine the function $\varphi_{D1}(x)$ which is giving the leading particle peak near $x=1$ and is therefore expected to be the main source of long-range correlations.

3. EXCITATION MASS SPECTRUM IN THE SINGLE DIFFRACTION DISSOCIATION

On the basis of the discussion of the previous section (in particular the examples) it is evident that the strength of the long range correlation is mainly determined by the amount of diffraction and by the excitation mass spectrum [or equivalently by the shape of the x distribution $\varphi_{D1}(x)$ of non-excited protons].

Furthermore, we see that if we do not try to push the analysis too far towards the fragmentation regions, the spectrum of low mass excitations is always integrated over and the results in the central region

depend on it only through the value of the integral ^{*}). This situation is very fortunate as the only direct experimental information on the low mass region at high energies is just the value of this integral. However, in order to extend our calculation eventually over the whole kinematic range and to calculate also the correction terms, we specify in this section the distribution

$$\rho(M) \equiv \frac{M}{s} \sin \alpha_D \rho_{D1}(x) \quad (\text{where } 1-x \approx \frac{M^2}{s}) \quad (11)$$

for all values of M .

Qualitatively we see from the NAL and ISR data that the peak in the proton spectrum near $x=1$ has a tail consistent in shape with a $1/(1-x)$ dependence [or a $1/M$ dependence in $\rho(M)$]. Especially, at small t values we have in addition evidence ¹⁰⁾ for low mass excitations with a mass spectrum falling more rapidly than $1/M$. We have fixed our mass spectrum for $16 \text{ GeV}^2 < M^2 < 52 \text{ GeV}^2$ using the ISR experimental data at $s = 930 \text{ GeV}^2$, $t = -0.35 \text{ GeV}^{-2}$ ¹¹⁾ and assuming an exponential t dependence with mass dependent slope $B = a + b \log(s/M^2)$ where $a = 3.5 \text{ GeV}^{-2}$ and $b = 0.7 \text{ GeV}^{-2}$ (thus we assume that the shrinkage of the t distribution is similar to that observed for elastic scattering). For $M^2 > 52 \text{ GeV}^2$ we assume a $1/M$ dependence at $t=0$ and a t dependence as above. In the absence of suitable high energy experiments we have determined the spectrum in the low mass region ($M^2 < 4 \text{ GeV}^2$) on the basis of the 24 GeV/c experiment ¹²⁾ assuming that the energy dependence is given by the shrinkage of the t distribution with the same slope formula as used at high masses.

*) If we want e.g., to calculate the correlations over the whole validity region of our method $[|y| \lesssim \frac{1}{2} \log(s/10)]$, we need the missing mass distribution between $4 \text{ GeV}^2 < M^2 < 0.45s$ and the integral over the region $M^2 < 4 \text{ GeV}^2$ (the numbers are based on $M_0 = 1.3 \text{ GeV}$, see next Section). The value of long-range correlations at $y_1 = y_2 = 0$ depends instead essentially only on the integral from the threshold up to $M^2 = M_0 \sqrt{s}$.

To correct for the non-diffractive background present at low energies we have an additional reduction of 20% in the mass spectrum. Finally we have joined the low mass and high mass regions smoothly to get the complete mass spectra shown in Fig. 1.

As an essential check we have integrated our mass spectrum at 303 GeV/c from threshold to $M^2 = 57 \text{ GeV}^2$. The result 6.8 mb (for twice the integral) is in good agreement with the experimental NAL result ¹³⁾ of about 6.5 mb (with an uncertainty of about 1 mb due mainly to difficulties in determining the two-prong contribution). We note also that the high mass parametrization $[g(M) \sim 1/[M(a+b \log s/M^2)]]$ when extrapolated to small M values (we take the $N\pi$ threshold for definiteness) gives the cross-section of only 3.6 mb for $M^2 < 57 \text{ GeV}^2$ at 300 GeV/c.

For simplicity we use the spectra shown in Fig. 1 up to the absolute kinematical limit. This choice is done for practical convenience and we do not attach to it any physical significance. When we integrate our missing mass spectrum over the whole kinematic region, we get at $s = 930 \text{ GeV}^2$ $\sigma_D = 10.4 \text{ mb}$ which corresponds to $\alpha_D = 0.31$ (we use the value $\sigma_{\text{inel}} = 33.5 \text{ mb}$). The shape of the mass spectrum $g_{D1}(M)$ for $M \gtrsim 1/2 \sqrt{s}$ has, however, no effect on the final result for correlations.

4. RESULTS AND DISCUSSION

According to Section 2 the calculation of the one and two-pion distributions in our approach requires the knowledge of the following functions and parameters:

- i) one-nucleon distributions $g_\pi(x)$, $g_{D1}(x)$ and $g_{D2}(x)$;
- ii) relative amounts of diffraction and pionization α_D and α_π ;
- iii) parameters M_0 and ψ_0 and;
- iv) cut-off function $D(y)$.

The proton distribution $g_{D1}(x)$ as well as α_D and α_π were specified in the previous section. We expect that the spectra $g_{D2}(x)$ and $g_\pi(x)$ have small values when x is close to one. Therefore, their details are of little importance for our calculations. For simplicity we take them to be identical and we choose the parametrization ¹⁴⁾

$g_\pi(x) = C \cdot x^{2.5} (1-x)^2$. The parameters M_0 and ψ_0 are connected by $M_0 = \langle m_\perp \rangle \psi_0$ (see the Appendix) and they are related to the plateau value of the one-pion distribution. The experimental value $\psi(y=0) \approx 2$ for

charged particles is reproduced with $\Upsilon_0 = 3.6$, which gives $M_0 = 1.3$ GeV. We choose to parametrize the cut-off function by

$$D(y) = \frac{1}{2} (1 - \operatorname{erf}(y/\omega)) \quad (12)$$

where

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

The parameter ω is related directly to the shape of the single particle distribution in the fragmentation region. A comparison with experimental data shows that it should be of the order of one¹⁵⁾ and we choose $\omega = 1$. Most of the calculations are made at $s = 930$ GeV² because we have the most direct experimental information about proton spectrum around that energy.

As an intermediate step of our calculation we present in Fig. 2 the functions $F_{D1}(y)$ and $F_{\pi}(y)$ which can be compared with the simple examples shown in Fig. 4. One and two-particle rapidity distributions calculated according to the formulae (A.12) and (A.14) lead to the correlation function $R(y_1, y_2)$ shown in Fig. 3. These correlations are independent of the charges of the two pions. We see that the long-range correlations have a maximal value of about 0.2 when $y_1 \approx y_2 \approx 0$. In Fig. 3 we show also results obtained using the Θ function approximation for the cut-off function $D(y)$ which corresponds to $\omega = 0$ in formula (12). We see that the results are not sensitive to the actual value of the parameter ω .

As mentioned in Section 2, our approximations are valid only when $|y| \lesssim 2.3$ for $s = 930$ GeV². Outside that region the contribution of the low mass excitations to the pion distributions starts to be important. Our approximations lead to too low multiplicities for low mass excitations. In addition, the factorization of the two-nucleon distribution must also break for these events. We know from experimental data that the decay of the low mass excitations is essentially isotropic. We have, therefore, attempted to correct our calculations by using an isotropic decay of the excited system when the missing mass to one proton $M \leq 2.5$ GeV and applying the methods of the present paper for the remaining events. From the results shown in Fig. 3 we see that the correction is important for $|y| > 2.5$ and that the corrected correlation function crosses zero around $|y| \approx 4$. As the details of our procedure are quite arbitrary we cannot trust the detailed shape of the correlation function in the fragmentation region.

However, we see that with such corrections we are in qualitative agreement with experimental data even in that region.

The relatively flat shape of the calculated correlation function cannot be seen directly in the experimental data even at the highest ISR energies when one of the particles is observed at $y \approx 0$. This is due to the fact that the range of the short range correlations is comparable to the half width of the central rapidity region. However, the situation is more favourable when one of the particles is observed at the edge of the central region. Then the long range correlations should dominate when the other particle is in the opposite hemisphere. The experimental data are in agreement with this expectation ^{1),5)}.

We end this section with a brief discussion of the energy dependence of the long range correlations. The main change in the long range part of the correlation function with increasing energy is the broadening of the central region where $R(y_1, y_2)$ is approximately constant. The value of the correlation function in that region is energy independent essentially to the same extent as the ratio σ_D / σ_{inel} . Since the short range correlations are expected to be energy independent, the overall correlations should show an energy dependence only outside the central region of the lowest energy considered. This prediction seems also to be in agreement with the experimental data ¹⁾.

5. CONCLUDING REMARKS

We have calculated the long range correlations between secondary particles due to the existence of fast leading particles. Using the high energy data on the single diffraction dissociation as an input we obtain for the correlation function $R(y_1, y_2)$ a value of about 0.2 in the central rapidity region. According to a recent analysis ⁵⁾ the new ISR data (in particular their energy dependence) seem to indicate that the long range effects may be even stronger [$R(0,0) \approx 0.3$]. This would mean that either the present estimate of the amount of low mass single diffraction dissociation is too small or, more likely, that there are other sources of important long range effects. One obvious possibility is the existence of low mass double diffraction dissociation. A cross-section of e.g., 1 mb for the double diffraction dissociation would lead to an increase of the correlation function $R(y_1, y_2)$ by about 0.03 at $y_1 \approx y_2 \approx 0$.

The remaining correlations present in the data can be understood as short-range correlations in a model of independent production of clusters of particles. The required size of the clusters depends on the strength of the long range effects. The maximum of the long range correlations in the range $R(0,0) = 0.2 - 0.3$ corresponds approximately to $3.6 - 2.8$ particles per cluster ⁴⁾ (on the average, assuming Poisson-like fluctuations).

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A P P E N D I X

In this Appendix we present a method of calculating inclusive one and two-pion distributions $\psi(y)$ and $\psi_2(y_1, y_2)$ from the following formulae:

$$\psi(y) = \int dx_1 dx_2 \rho(x_1, x_2) D^{(1)}(y, x_1, x_2) \quad (\text{A.1})$$

$$\psi_2(y_1, y_2) = \int dx_1 dx_2 \rho(x_1, x_2) D^{(2)}(y_1, y_2, x_1, x_2) \quad (\text{A.2})$$

where $\rho(x_1, x_2)$ is the two-nucleon x distribution, $D^{(1)}$ and $D^{(2)}$ are the inclusive one and two-pion rapidity distributions for fixed x_1 and x_2 . The calculation is based on the following assumptions:

- i) $D^{(1)}$ depends only on $y - \phi$ and M_x . M_x is the mass of the produced mesonic system and ϕ the c.m.s. rapidity of the center of this system.
- ii) $D^{(1)}(y, x_1, x_2)$ develops a plateau with a mass independent height ψ_0 , when M_x increases.
- iii) The normalized two-nucleon distribution $\rho(x_1, x_2)$ vanishes, when $x_1 < 0$ or $x_2 > 0$.
- iv) We take

$$D^{(2)}(y_1, y_2, x_1, x_2) = D^{(1)}(y_1, x_1, x_2) D^{(1)}(y_2, x_1, x_2),$$

i.e., correlations present at fixed x_1 and x_2 are not included in the calculation.

The mass and rapidity of the produced system can be calculated from formulae ⁹⁾

$$M_x = s(1-x_1)(1+x_2) \quad (\text{A.3})$$

$$\phi = \frac{1}{2} \log [(1-x_1)/(1+x_2)] \quad (\text{A.4})$$

when $x_1 > 0$, $x_2 < 0$, $1-x_1 \gg m_p^2/s$, $1+x_2 \gg m_p^2/s$ and $(1-x_1)(1+x_2) \gg \langle p_{\perp}^2 \rangle_p/s$ ($\langle p_{\perp}^2 \rangle_p$ is the average squared transverse momentum of the protons).

From the energy independence of the plateau height and from the energy conservation sum rule follows that the effective length of the distribution $D^{(1)}$ is given by

$$\Delta = 2 \log (M_x / M_0) \quad (\text{for } M_x \gg M_0) \quad (\text{A.5})$$

where $M_0 = \langle m_{\perp} \rangle \psi_0$ and $\langle m_{\perp} \rangle$ is the average longitudinal mass of pions (experimentally $\langle m_{\perp} \rangle \approx 0.36$ GeV). The result is obtained by assuming a sharp cut-off of the distribution $D^{(1)}$ at $y = \phi \pm \frac{1}{2} \Delta$ and a factorization of the dependences on transverse momentum and rapidity. The approximations involved in these two assumptions cancel to a large extent because the fastest pions have in general low transverse momenta.

Combining formulae (A.3), (A.4) and (A.5) we see that for fixed x_1 and x_2 the pion distributions extend effectively from

$$y_l = \phi - \frac{1}{2} \Delta = -Y - \log(1+x_2) \quad (\text{A.6a})$$

to

$$y_r = \phi + \frac{1}{2} \Delta = Y + \log(1-x_1) \quad (\text{A.6b})$$

where $Y = \log(\sqrt{s}/M_0)$. Thus y_l depends only on x_2 and y_r only on x_1 , and we can approximate the pion distribution by

$$D^{(1)}(y, x_1, x_2) = \psi_0 D(y - y_r) D(y_l - y). \quad (\text{A.7})$$

The function $D(y)$ has the properties

$$D(y) = \begin{cases} 1 & \text{for } y \ll 0 \\ 0 & \text{for } y \gg 0. \end{cases}$$

As a rough approximation we have thus

$$D(y) \approx \Theta(-y). \quad (\text{A.8})$$

We have similarly (for $y_1 \leq y_2$):

$$D^{(2)}(y_1, y_2, x_1, x_2) = \psi_0^2 [D(y_2 - y_1) - E(y_2, y_1, y_1)] [D(y_1 - y_2) - E(-y_1, -y_2, -y_2)]. \quad (\text{A.9})$$

The function (defined for $y_1 \leq y_2$)

$$E(y_2, y_1, y) = D(y_2 - y) [1 - D(y_1 - y)] \quad (\text{A.10})$$

is non-zero only for $y_1 \approx y_2 \approx y$. In the approximation $D(y) = \theta(-y)$ we have $E = 0$.

We see from formulae (A.7) and (A.9) that $D^{(1)}(y, x_1, x_2)$ and $D^{(2)}(y_1, y_2, x_1, x_2)$ can be expressed as products of two functions, each of which depends on the x value of one of the protons only. Thus we can express $\psi(y)$ and $\psi_2(y_1, y_2)$ in terms of one-dimensional integrals approximating the two-nucleon distribution by the form

$$\rho(x_1, x_2) = \sum_i \alpha_i \rho_i^R(x_1) \rho_i^L(-x_2) \quad (\text{A.11})$$

where the functions $\rho_i(x)$ are normalized to one in the interval $0 \leq x \leq 1$. The subscripts R and L refer to right and left hemisphere. The expansion is naturally useful only if we can expect that a small number of terms is sufficient.

Using formulae (A.1), (A.7) and (A.11) we have immediately

$$\psi(y) = \psi_0 \sum_i \alpha_i F_i^R(y) F_i^L(-y) \quad (\text{A.12})$$

with

$$F_i^{L,R}(y) = \int_0^1 dx \rho_i^{L,R}(x) D(y - Y - \log(1-x)). \quad (\text{A.13})$$

Similarly, we have for the two-particle density from formulae (A.2), (A.9) and (A.11) ($y_1 \leq y_2$)

$$\begin{aligned} \psi_2(y_1, y_2) \\ = \psi_0^2 \sum_i \alpha_i (F_i^R(y_2) - G_i^R(y_2, y_1)) (F_i^L(-y_1) - G_i^L(-y_1, -y_2)), \end{aligned} \quad (\text{A.14})$$

where

$$G_i^{L,R}(y_2, y_1) = \int_0^1 dx \rho_i^{L,R}(x) E(y_2, y_1, Y + \log(1-x)). \quad (\text{A.15})$$

Replacing $D(y)$ by $\Theta(-y)$ [formula (A.8)] in the definition of $F_i(y)$ we have

$$F_i^\Theta(y) = \int_0^{x_{\max}} dx \rho_i(x) \quad (\text{A.16})$$

with

$$x_{\max} = 1 - e^{y-Y}$$

[with this replacement $G_i(y_2, y_1) \equiv 0$].

Making in (A.13) the substitution $z = Y + \log(1-x)$ and integrating by parts, we have

$$F_i(y) = \int_{-\infty}^{\infty} F_i^\Theta(z) |D'(y-z)| dz \quad (\text{A.17})$$

i.e., we can calculate $F_i(y)$ also by convoluting $F_i^\Theta(y)$ with the derivative of the function $D(y)$.

To clarify the relation between the functions $\rho(x)$ and $F(y)$ we consider the simple case

$$\rho(x) = C (1-x)^\beta \quad (0 \leq x \leq 1 - M_{th}^2/s)$$

The constant C is determined by the normalization requirement $\int \rho(x) dx = 1$. For simplicity we use the approximation $D(y) = \Theta(-y)$. The more general case can be obtained using formula (A.17). Now we have from formula (A.16), when $\beta \neq -1$

$$F(y) = \frac{C}{\beta+1} (1 - \exp((\beta+1)(y-Y))) \quad (-Y - \log(M_0^2/M_{th}^2) < y < Y)$$

and when $\beta = -1$

$$F(y) = C(Y-y) \quad (-Y - \log(M_0^2/M_{th}^2) < y < Y).$$

The results corresponding to $\beta = -\frac{3}{2}, -1, 0$ and 1 are shown also in Fig.4. We see that $F(y)$ is always zero for $y > Y$ and one for $y < -Y - \log(M_0^2/M_{th}^2)$ and that $F(y) \approx 1$ over the whole central region when $\rho(x)$ vanishes at $x=1$. On the other hand $F(y) \approx 0$ in the central region if $\rho(x)$ is very strongly peaked near $x=1$.

FIGURE CAPTIONS

Figure 1 :

The missing mass distribution $\rho(M)$ for non-excited protons in inelastic diffraction for $s=570 \text{ GeV}^2$ (upper curve) and $s=930 \text{ GeV}^2$ (lower curve) ($p_{\text{lab}}=300 \text{ GeV/c}$ and 500 GeV/c).

Figure 2 :

The functions $F_{\pi}(y)$ and $F_{D1}(y)$ at $s=930 \text{ GeV}^2$. The corresponding functions calculated using the approximation $D(y)=\Theta(-y)$ are shown by dashed lines.

Figure 3 :

The two-particle rapidity correlations due to leading particle effect at $s=930 \text{ GeV}^2$. The solid line gives the results of the model described in the text. The dashed line is obtained by using isotropic decay for low mass excited systems ($M < 2.5 \text{ GeV}$). The dash-dotted line corresponds to the approximation $D(y)=\Theta(-y)$. The experimental data at $s=930 \text{ GeV}^2$ (16) are shown for comparison. Note that these data are measured using the variable $\eta = -\log \tan \Theta/2 \approx y$.

Figure 4 :

The function $F(y)$ at $s=930 \text{ GeV}^2$ for $D(y)=\Theta(-y)$ and $\rho(x)=C(1-x)^{\beta}$ with $\beta = -\frac{3}{2}, -1, 0$ and 1 . The function is normalized between $x=0$ and $x=1-(M_{\text{th}}/s)^2$ where M_{th} is the two-particle threshold. $F(y)=1$ for $y < -3.530$ and $F(y)=0$ for $y > Y=3.155$ ($M_0=1.3 \text{ GeV}$).

Fig. 1

Fig. 2

Fig. 3

Fig. 4