PARTON MODEL EXPECTATIONS FOR WEAK PION PRODUCTION

BY NEUTRAL CURRENTS *)

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A B S T R A C T

A quark model of the Weinberg-Salam theory is used in conjunction with the parton fragmentation hypothesis of Feynman to obtain the distribution of a final state hadron H produced in semi-inclusive neutrino reactions. A general relation is obtained between the neutral to charged current ratios

$$R(H) = \frac{\sigma(\nu \rightarrow \nu + H)}{\sigma(\nu \rightarrow \mu^- + H)} \quad \text{and} \quad \bar{R}(H) = \frac{\sigma(\bar{\nu} \rightarrow \bar{\nu} + H)}{\sigma(\bar{\nu} \rightarrow \mu^+ + H)}$$

which is sensitive to the value of the parameter $\sin^2\theta$. Particular relations are derived among the ratios $R(\pi)$ and $\bar{R}(\pi)$. All predictions refer to the parton fragmentation region of phase space. Comparison is made with model independent bounds.

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1. INTRODUCTION

Evidence has been found at CERN-Gargamelle¹⁾ and at NAL²⁾ for the existence of neutral current reactions

$$\nu_\mu (\bar{\nu}_\mu) + N \rightarrow \nu_\mu (\bar{\nu}_\mu) + \text{hadrons} \quad (N = \text{nucleon})$$

The strength of the neutral current signal is remarkably high, the ratio of the inelastic cross-sections relative to the charged current reactions

$\nu_\mu (\bar{\nu}_\mu) + N \rightarrow \mu^- (\mu^+) + \text{hadrons}$ being^{1,3)*})

$$R = \frac{\sigma(\nu \rightarrow \nu)}{\sigma(\nu \rightarrow \mu^-)} = 0.23 \pm 0.03 \quad (1a)$$

$$\bar{R} = \frac{\sigma(\bar{\nu} \rightarrow \bar{\nu})}{\sigma(\bar{\nu} \rightarrow \mu^+)} = 0.45 \pm 0.09 \quad (1b)$$

Equally remarkable, these ratios are consistent with model-independent lower bounds derived on the basis of the Weinberg-Salam theory⁴⁾, as well as with more specific predictions obtained in a parton model³⁻⁷⁾. Because of the large neutral current signal, one anticipates that detailed information on the distribution of hadrons produced in the final state will become available. The question inevitably arises whether more incisive tests of the theory are possible using such information.

We consider in this paper the expectations of the Weinberg-Salam theory for the semi-inclusive reactions $\nu_\mu (\bar{\nu}_\mu) + N \rightarrow \nu_\mu (\bar{\nu}_\mu) + \pi + \text{anything}$. The model employed is an extension of the parton model used in Ref. 5 for the analysis of neutral currents in inclusive neutrino scattering. The distribution of the final state hadron is obtained following the fragmentation hypothesis of Feynman⁸⁾, as described in detail by Gronau, Ravndal and Zarmi⁹⁾. The model enables predictions to be made about the ratio of inclusive pion cross-sections induced by neutral and charged currents, in terms of the parameter $\sin^2 \Theta$ of the theory. The predictions obtained refer to a special kinematical domain -- the parton fragmentation region -- which is defined in Section 2. The model is expected to be applicable at high energies, and while it is at present largely untested, its validity can in principle be checked by independent measurements in electroproduction ($e + N \rightarrow e + \pi + \text{anything}$) and in charged current neutrino reactions [$\nu_\mu (\bar{\nu}_\mu) + N \rightarrow \mu^- (\mu^+) + \pi + \text{anything}$].

A model-independent analysis of weak pion production by neutral currents has been made by Albright, Lee, Paschos and Wolfenstein¹⁰⁾, who have derived bounds on the semi-inclusive cross-sections in terms of quantities that can be derived from electroproduction data. A significant aspect of those bounds is that they hold for any isoscalar target and any kinematical region and include automatically

*) These are, strictly, ratios obtained after certain kinematical cuts.
See Refs. 1, 3, 5.

nuclear effects such as rescattering which are known to be important at low energies. By contrast, the considerations of the present paper are tied to a definite (and somewhat speculative) model of hadron production and apply to a special kinematical domain. Further, we consider scattering to occur from isolated nucleons, assuming that nuclear effects are either small at high energies or can be corrected for in a reliable way. The advantage of using a model is, of course, that it yields sharp predictions about measurable quantities that can serve as a guide in the interpretation of experimental data. In particular, the predictions we derive are expressed entirely in terms of neutrino cross-sections, and can be tested without knowledge of the corresponding electroproduction data. Ultimately, the neutral current predictions of this paper have significance only to the extent that the parton model expectations are borne out in electroproduction and in charged neutrino production. In Section 5 we compare our results with the bounds of Ref. 10, principally as a check on the self-consistency of the model and of the approximations employed.

2. PARTON MODEL FOR INCLUSIVE HADRON PRODUCTION

The parton model^{8,9)} suggests certain distinctive features for the distribution of a hadron H produced in reactions of the type

$$\ell + N \longrightarrow \ell' + H + \text{anything} \quad (2)$$

where ℓ and ℓ' are the initial and final leptons and N the target nucleon. These features are simplest to describe in a reference frame in which the nucleon momentum p and the momentum transfer q (i.e. the momentum of the virtual intermediate boson) have the components

$$\begin{aligned} p &= (P, 0, 0, P) \\ q &= (0, 0, 0, -2xP) \end{aligned} \quad , P \rightarrow \infty \quad (3)$$

where $x = -q^2/2q \cdot p$. The nucleon momentum p is supposed to be shared in various fractions among the parton constituents of the nucleon (see Fig. 1). Energy-momentum conservation dictates that the virtual boson will be absorbed by a parton whose four-momentum is

$$xp = (xP, 0, 0, xP) \quad (4)$$

the scattered parton emerging in the negative z direction with four-momentum $(xP, 0, 0, -xP)$. Hadrons observed in the final state are supposed to be "fragmentation" products of the scattered parton and of the residual nucleon. The assumption is made that among these hadrons there will be a group of left-moving particles possessing limited transverse momenta and relatively large longitudinal momenta. Such hadrons are associated with the left-moving scattered parton and are termed "parton fragments". It is with these that the analysis of this paper will be concerned.

In general, the distribution of a hadron H of momentum h created in the reaction (2) may be expected to depend on q^2 , $q \cdot p$, $p \cdot h$, and $q \cdot h$. The hypothesis of Feynman is that if H is a parton fragment, its distribution is a fraction only of

$$x = \frac{-q^2}{2p \cdot q}, \quad z = \frac{h \cdot p}{q \cdot p}$$

and the transverse momentum h_T relative to $\vec{q}$. Furthermore, the distribution in the variables x and z (integrated over h_T) is supposed to have the form

$$\frac{d\sigma}{dx dz} = \sum_j \left(\frac{d\sigma}{dx} \right)_j D_j^H(z) \quad (5)$$

where $(d\sigma/dx)_j$ is the inclusive cross section corresponding to a (final) scattered parton of type j and $D_j^H(z)$ is a "fragmentation function" expressing the probability for such a parton to give rise to a hadron H. The variable z, in the reference frame of Eq. (3), is the fraction of the scattered parton's carried by the observed hadron. In the lab. frame, z is the energy of H divided by the total energy transfer ν . The important implication of the Feynman hypothesis is scaling in the variables x and z, and factorisation of the x and z dependence in each term of Eq. (5). This equation is the basis of all predictions about inclusive hadron production in the parton fragmentation region.

The identification of a hadron as a parton fragment requires that it satisfy the criterion of being "left-moving" in the reference frame defined by Eq. (3), and of possessing longitudinal momentum large compared with transverse momentum. The criterion of "left-moving" reduces in the lab. frame to the condition that the hadron possesses a ratio of longitudinal momentum to energy

$$\frac{k \cdot \hat{q}}{E_H} > \frac{\nu}{\nu + Mx}$$

M being the nucleon mass. For a more precise definition of the parton fragmentation region we refer to Refs. 8 and 9.

In the next section we use the model of Ref. 5 in conjunction with Eq. (5) to calculate the inclusive distribution of any parton fragment H produced in charged and neutral current reactions.

3. INCLUSIVE CROSS-SECTIONS FOR HADRON PRODUCTION BY CHARGED AND NEUTRAL CURRENTS

In Ref. 5, a four quark realisation of the Weinberg-Salam theory was used together with a quark model of the nucleon to obtain inclusive cross-sections for neutrino reactions induced by charged and neutral currents. The major contributions to the total cross-sections comes from the valence quarks p and n, the core contribution being a small fraction. The distributions $d\sigma/dx$ are also dominated by the valence quarks as long as one excludes the diffractive region close to $x = 0$ ¹¹). In the present discussion we neglect all core effects, since the error

involved is probably commensurate with the approximations inherent in our model for hadron production. The inclusive cross-sections $d\sigma/dx$ for proton and neutron targets are then given, in units of G^2ME/π , by

$$\begin{aligned}
 \frac{d\sigma^P(\nu \rightarrow \mu^-)}{dx} &= 2x d(x) \quad , \quad \frac{d\sigma^N(\nu \rightarrow \mu^-)}{dx} = 2xu(x) \quad , \\
 \frac{d\sigma^P(\bar{\nu} \rightarrow \mu^+)}{dx} &= \frac{2}{3} xu(x) \quad , \quad \frac{d\sigma^N(\bar{\nu} \rightarrow \mu^+)}{dx} = \frac{2}{3} x d(x) \quad , \\
 \frac{d\sigma^P(\nu \rightarrow \nu)}{dx} &= 2x (u(x)F + d(x)F') \quad , \\
 \frac{d\sigma^N(\nu \rightarrow \nu)}{dx} &= 2x (d(x)F + u(x)F') \quad , \\
 \frac{d\sigma^P(\bar{\nu} \rightarrow \bar{\nu})}{dx} &= 2x (u(x)\bar{F} + d(x)\bar{F}') \quad , \\
 \frac{d\sigma^N(\bar{\nu} \rightarrow \bar{\nu})}{dx} &= 2x (d(x)\bar{F} + u(x)\bar{F}') \quad (6)
 \end{aligned}$$

where the notation is that of Ref. 5, and $F, F', \bar{F}, \bar{F}'$ are functions of the parameter $\sin^2\theta$,

$$F = \frac{1}{4} - \frac{2}{3} \sin^2\theta + \frac{16}{27} \sin^4\theta \quad ,$$

$$F' = \frac{1}{4} - \frac{1}{3} \sin^2 \theta + \frac{4}{27} \sin^4 \theta ,$$

$$\bar{F} = \frac{1}{12} - \frac{2}{9} \sin^2 \theta + \frac{16}{27} \sin^4 \theta ,$$

$$\bar{F}' = \frac{1}{12} - \frac{1}{9} \sin^2 \theta + \frac{4}{27} \sin^4 \theta . \quad (7)$$

[Note that $F, F', \bar{F}, \bar{F}'$ are the integrals over y of the functions $f(y), f'(y), \bar{f}(y), \bar{f}'(y)$ defined in Eq. (9) of Ref. 5].

To obtain the distribution of a hadron H in the parton fragmentation region, we substitute (6) in (5). Denoting by

$$\frac{d\sigma(l \rightarrow l' + H)}{dz dx}$$

the distribution for the general reaction (2), averaged over neutron and proton targets, we get

$$\frac{d\sigma(\nu \rightarrow \mu^- + H)}{dx dz} = x(u(x) + d(x)) D_u^H(z) \quad (8a)$$

$$\frac{d\sigma(\bar{\nu} \rightarrow \mu^+ + H)}{dx dz} = x(u(x) + d(x)) D_d^H(z) \quad (8b)$$

$$\frac{d\sigma(\nu \rightarrow \nu + H)}{dx dz} = x(u(x) + d(x)) (F D_u^H(z) + F' D_d^H(z)) \quad (8c)$$

$$\frac{d\sigma(\bar{\nu} \rightarrow \bar{\nu} + H)}{dx dz} = x(u(x) + d(x)) (\bar{F} D_u^H(z) + \bar{F}' D_d^H(z)) \quad (8d)$$

The significant features of Eq. (8) are: (i) the x and z dependence factorizes in all four cases, (ii) the x dependence is the same for all reactions. These are characteristics of the model which can be checked even by studying the charged current reactions by themselves.

From the point of view of testing the Weinberg-Salam theory, the quantities of interest are the ratios of the neutral current and charged current distributions which are obtained directly from Eq. (8). To make optimum use of the statistics, it may be convenient to compare the integral of the distributions $d\sigma/dx dz$, either over the whole range of x and z , or over a partial range which is

the same for the neutral and charged current reactions. We then obtain the following ratios for the production of a hadron H in the parton fragmentation region

$$R(H) = \frac{\sigma(\nu \rightarrow \nu + H)}{\sigma(\nu \rightarrow \mu^- + H)} = F + F' \frac{\mathcal{D}_d^H}{\mathcal{D}_u^H} \quad (9a)$$

$$\bar{R}(H) = \frac{\sigma(\bar{\nu} \rightarrow \bar{\nu} + H)}{\sigma(\bar{\nu} \rightarrow \mu^+ + H)} = 3 \left[\bar{F}' + \bar{F} \frac{\mathcal{D}_u^H}{\mathcal{D}_d^H} \right] \quad (9b)$$

where $\mathcal{D}_{u,d}^H = \int dz D_{u,d}^H(z)$. Equation (9) implies immediately the following lower bounds for the production of any hadron H:

$$R(H) \geq F = \frac{1}{4} - \frac{2}{3} \sin^2 \theta + \frac{16}{27} \sin^4 \theta \quad (10a)$$

$$\bar{R}(H) \geq 3\bar{F}' = \frac{1}{4} - \frac{1}{3} \sin^2 \theta + \frac{4}{9} \sin^4 \theta \quad (10b)$$

Further, eliminating the unknown quantity $\mathcal{D}_d^H/\mathcal{D}_u^H$ from Eq. (9), we obtain the following relation between $R(H)$ and $\bar{R}(H)$

$$(R(H) - F)(\bar{R}(H) - 3\bar{F}') = 3\bar{F}F' \quad (11)$$

This relation is plotted in Fig. 1 for four values of $\sin^2 \theta$: 0.2, 0.25, 0.4, 0.5. Eqs. (10) and (11) constitute our first major results.

In the next section, we specialize to the case of pion production.

4. PREDICTIONS FOR PION PRODUCTION

In deriving predictions for weak pion production, we take advantage of the fact that the fragmentation functions D_u^π, D_d^π (or their integrals $\mathcal{D}_u^\pi, \mathcal{D}_d^\pi$) satisfy a number of relations following from isospin invariance⁸⁾. Thus

$$\mathcal{D}_u^{\pi^+} = \mathcal{D}_d^{\pi^-} ; \quad \mathcal{D}_d^{\pi^+} = \mathcal{D}_u^{\pi^-} ; \quad \mathcal{D}_u^{\pi^0} = \mathcal{D}_d^{\pi^0} = \frac{1}{2} (\mathcal{D}_u^{\pi^+} + \mathcal{D}_d^{\pi^+}) \quad (12)$$

As a consequence, we obtain from Eq. (9) the following results for the ratio of pion production on neutral and charged currents:

$$R(\pi^+) = F + F' \frac{\mathcal{D}_d^{\pi^+}}{\mathcal{D}_u^{\pi^+}} , \quad R(\pi^-) = F + F' \frac{\mathcal{D}_u^{\pi^+}}{\mathcal{D}_d^{\pi^+}} ,$$

$$\bar{R}(\pi^+) = 3 \left[\bar{F}' + \bar{F} \frac{\mathcal{D}_u^{\pi^+}}{\mathcal{D}_d^{\pi^+}} \right], \quad \bar{R}(\pi^-) = 3 \left[\bar{F}' + \bar{F} \frac{\mathcal{D}_d^{\pi^+}}{\mathcal{D}_u^{\pi^+}} \right] \quad (13)$$

Eliminating the unknown parameters $\mathcal{D}_d^{\pi^+}/\mathcal{D}_u^{\pi^+}$, we get our main results for charged pions

$$(R(\pi^+) - F)(R(\pi^-) - F) = F'^2, \quad (14a)$$

$$(\bar{R}(\pi^+) - 3\bar{F}')(\bar{R}(\pi^-) - 3\bar{F}') = 9\bar{F}^2, \quad (14b)$$

$$R(\pi^+) - F = \frac{F'}{3\bar{F}} (\bar{R}(\pi^-) - 3\bar{F}') , \quad (14c)$$

$$R(\pi^-) - F = \frac{F'}{3\bar{F}} (\bar{R}(\pi^+) - 3\bar{F}') . \quad (14d)$$

Relation (14a) correlates the neutrino-induced ratios $R(\pi^+)$ and $R(\pi^-)$ and is plotted in Fig. 3 for $\sin^2\theta = 0.2, 0.3, 0.4, 0.5$. Equation (14b) is the analogous relation for antineutrino reactions and is shown in Fig. 4. The linear correlation of $R(\pi^+)$ and $\bar{R}(\pi^-)$, expressed by Eq. (14c), is plotted in Fig. 5. An identical correlation exists between $R(\pi^-)$ and $\bar{R}(\pi^+)$ [Eq. (14d)]. Taken together, these correlations are a stringent test of the Weinberg-Salam theory, since it is required that measurements of $R(\pi^+)$, $R(\pi^-)$, $\bar{R}(\pi^+)$, $\bar{R}(\pi^-)$ (as well as the total cross-section ratios R and $\bar{R}$) be explained by the same value of the parameter $\sin^2\theta$.

Predictions can also be made for neutral pion production. From Eqs. (9), we have

$$R(\pi^0) = F + F' = \frac{1}{2} - \sin^2\theta + \frac{20}{27} \sin^4\theta$$

$$\bar{R}(\pi^0) = \bar{F} + \bar{F}' = \frac{1}{2} - \sin^2\theta + \frac{20}{9} \sin^4\theta$$

These predictions are the same as those for the total cross-section ratios R and $\bar{R}$ [Eq. (14) of Ref. 5].

Similarly, if we define

$$R(\pi^{ch}) = \frac{\sigma(\nu \rightarrow \nu + \pi^+) + \sigma(\nu \rightarrow \nu + \pi^-)}{\sigma(\nu \rightarrow \mu^- + \pi^+) + \sigma(\nu \rightarrow \mu^- + \pi^-)} \quad (16a)$$

$$\bar{R}(\pi^{ch.}) = \frac{\sigma(\bar{\nu} \rightarrow \bar{\nu} + \pi^+) + \sigma(\bar{\nu} \rightarrow \bar{\nu} + \pi^-)}{\sigma(\bar{\nu} \rightarrow \mu^+ + \pi^+) + \sigma(\bar{\nu} \rightarrow \mu^+ + \pi^-)} \quad (16b)$$

we find

$$R(\pi^{ch}) = R(\pi^0) \quad ; \quad \bar{R}(\pi^{ch}) = \bar{R}(\pi^0) \quad (17)$$

Another quantity of interest is the $\pi^+ - \pi^-$ asymmetry in the various reactions, which we define as

$$A(\ell \rightarrow \ell') = \frac{\sigma(\ell \rightarrow \ell' + \pi^+) - \sigma(\ell \rightarrow \ell' + \pi^-)}{\sigma(\ell \rightarrow \ell' + \pi^+) + \sigma(\ell \rightarrow \ell' + \pi^-)} \quad (18)$$

The model predicts

$$\begin{aligned} A(\nu \rightarrow \mu^-) : A(\bar{\nu} \rightarrow \mu^+) : A(\nu \rightarrow \nu) : A(\bar{\nu} \rightarrow \bar{\nu}) \\ = 1 : -1 : \frac{F - F'}{F + F'} : \frac{\bar{F} - \bar{F}'}{\bar{F} + \bar{F}'} \end{aligned} \quad (19)$$

Note that $A(\nu \rightarrow \mu^-) = [\mathcal{D}_u^{\pi^+} - \mathcal{D}_u^{\pi^-}] / [\mathcal{D}_u^{\pi^+} + \mathcal{D}_u^{\pi^-}]$, and is, in all probability, positive⁸⁾. Thus the sign of the asymmetry is also determined for all four reactions.

5. COMPARISON WITH MODEL-INDEPENDENT BOUNDS

The analysis of Albright, Lee, Paschos and Wolfenstein¹⁰⁾ leads to inequalities involving the cross-sections for weak pion production as well as the cross-sections for pion electroproduction. The electroproduction analogue of the inclusive neutrino cross-section $d\sigma/dx$ [Eq. (6)] which occurs in these inequalities is

$$V_{e.m.}(x) = \frac{G^2}{\pi} \frac{1}{4\pi\alpha^2} \int q^4 \frac{d\sigma(e \rightarrow e)}{dx dy} dy \quad (20)$$

Within our parton model, assuming dominance by p and n quarks, this quantity is given by (using units of $G^2 ME/\pi$):

$$V_{e.m.}^P(x) = \frac{20}{27} x u(x) \quad (21a)$$

$$V_{e.m.}^N(x) = \frac{20}{27} x d(x) \quad (21b)$$

where the superscript P(N) refers to proton (neutron) target. The corresponding distribution for the production of a hadron H in the parton fragmentation region, averaged over neutron and proton targets, is, within this model:

$$V_{e.m.}^H(x, z) = \frac{10}{27} x (u(x) D_u^H(z) + d(x) D_d^H(z)) \quad (22)$$

For pion production in particular,

$$\begin{aligned} V_{e.m.}^{\pi^+}(x, z) &= \frac{10}{27} x (u(x) D_u^{\pi^+}(z) + d(x) D_d^{\pi^+}(z)) \\ V_{e.m.}^{\pi^-}(x, z) &= \frac{10}{27} x (u(x) D_d^{\pi^+}(z) + d(x) D_u^{\pi^+}(z)) \\ V_{e.m.}^{\pi^0}(x, z) &= \frac{5}{27} x (u(x) + d(x)) (D_u^{\pi^+}(z) + D_d^{\pi^+}(z)) \\ V_{e.m.}^{ch.}(x, z) &= V_{e.m.}^{\pi^+}(x, z) + V_{e.m.}^{\pi^-}(x, z) \\ &= \frac{10}{27} x (u(x) + d(x)) (D_u^{\pi^+}(z) + D_d^{\pi^+}(z)) \end{aligned} \quad (23)$$

The inequalities of Ref. 10, involve, in addition, the ratios

$$R_1 = \frac{[d\sigma(\nu \rightarrow \nu + \pi^+) + d\sigma(\nu \rightarrow \nu + \pi^-)]/dx dz}{2 d\sigma(\nu \rightarrow \mu^- + \pi^0)/dx dz} \quad (24)$$

$$R_2 = \frac{d\sigma(\nu \rightarrow \nu + \pi^0)/dx dz}{[d\sigma(\nu \rightarrow \mu^- + \pi^+) + d\sigma(\nu \rightarrow \mu^- + \pi^-) - d\sigma(\nu \rightarrow \mu^- + \pi^0)]/dx dz} \quad (25)$$

which, in our model, are

$$R_1 = R_2 = F + F' = \frac{1}{2} - \sin^2 \theta + \frac{20}{27} \sin^4 \theta \quad (26)$$

The results of Ref. 10 may now be stated:

$$R_1 - \frac{1}{2} \left[1 - 2 \sin^2 \theta \left\{ \frac{V_{e.m.}^{ch}(x,z)}{2 d\sigma(\nu \rightarrow \mu^- + \pi^0) / dx dz} \right\}^{\frac{1}{2}} \right]^2 \geq 0 \quad (27a)$$

$$R_2 - \frac{1}{2} \left[1 - 2 \sin^2 \theta \left\{ \frac{V_{e.m.}^{\pi^0}(x,z)}{[d\sigma(\nu \rightarrow \mu^- + \pi^+) + d\sigma(\nu \rightarrow \mu^- + \pi^-) - d\sigma(\nu \rightarrow \mu^- + \pi^0)] / dx dz} \right\}^{\frac{1}{2}} \right]^2 \geq 0 \quad (27b)$$

Using Eqs. (8), (12), (23) and (24), one verifies that in our model the left-hand side of Eq. (27a) as well as Eq. (27b) is

$$\left(\sqrt{\frac{40}{27}} - 1 \right) \sin^2 \theta$$

which is positive and thus consistent with the above inequalities. This quantity is a measure of the contribution made by the isoscalar neutral current of the Weinberg-Salam theory, and the fact that it is small is a specific feature of our quark parton model.

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Figure captions

- Fig. 1 : Four-momentum of proton (p) and virtual boson (g) in an infinite momentum frame ($P \rightarrow \infty$).
- Fig. 2 : Relation between $R(H) = \sigma(\nu \rightarrow \nu + H)/\sigma(\nu \rightarrow \mu^- + H)$ and $\bar{R}(H) = \sigma(\bar{\nu} \rightarrow \bar{\nu} + H)/\sigma(\bar{\nu} \rightarrow \mu^+ + H)$ for some typical values of $\sin^2\theta$ [Eq. (11)].
- Fig. 3 : Relation between $R(\pi^+) = \sigma(\nu \rightarrow \nu + \pi^+)/\sigma(\nu \rightarrow \mu^- + \pi^+)$ and $R(\pi^-) = \sigma(\nu \rightarrow \nu + \pi^-)/\sigma(\nu \rightarrow \mu^- + \pi^-)$ for some typical values of $\sin^2\theta$ [Eq. (14a)].
- Fig. 4 : Relation between $\bar{R}(\pi^+) = \sigma(\bar{\nu} \rightarrow \bar{\nu} + \pi^+)/\sigma(\bar{\nu} \rightarrow \mu^+ + \pi^+)$ and $\bar{R}(\pi^-) = \sigma(\bar{\nu} \rightarrow \bar{\nu} + \pi^-)/\sigma(\bar{\nu} \rightarrow \mu^+ + \pi^-)$ for some typical values of $\sin^2\theta$ [Eq. (14b)].
- Fig. 5 : Correlation between $R(\pi^+)$ and $\bar{R}(\pi^-)$ for some values of $\sin^2\theta$ [Eq. (14c)]. An identical correlation exists between $R(\pi^-)$ and $\bar{R}(\pi^+)$ [Eq. (14d)].

Fig. 1

Fig. 2

Fig. 3

FIG. 4

Fig. 4

Fig. 5