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A SOLUBLE REALISTIC MODEL FOR LOW ENERGY $\pi\pi$ SCATTERING

N. Johannesson

CERN -- Geneva

A B S T R A C T

Using analyticity, crossing and unitarity and making a few approximations, we predict the $\pi\pi$ scattering amplitudes in terms of the S wave scattering lengths from threshold up to 800 MeV.

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1 - INTRODUCTION

This paper deals with a soluble model for low energy $\pi\pi$ scattering based on the three basic principles : analyticity, crossing and unitarity. Experimentally there seem to be only three important amplitudes in the region up to 1 GeV, namely the $I = 0$ S wave A_0 , the $I = 1$ P wave A_1 and the $I = 2$ S wave A_2 . We know from the Chew-Mandelstam equations that it is necessary to bring in some information on higher waves and inelastic channels in order to have solutions consistent with experiments and the three basic principles ¹⁾.

One approximate way of taking this into account has been developed by Shirkov and coworkers ¹⁾. The equations for the three amplitudes take the form

$$A_i(z) = \sum_{j=0}^2 a_{ij} v_j(z) + \frac{1}{\pi} \int_1^{\infty} \frac{\text{Im } A_i(z')}{z' - z} dz' + \sum_{j=0}^2 b_{ij} \frac{1}{\pi} \int_1^{\infty} \frac{\text{Im } A_j(z')}{z' + z} dz', \quad i = 0, 1, 2 \quad (1.1)$$

where, with units such that the pion mass equals unity, $z = (s-2)/2$

$$a_{ij} = \begin{pmatrix} -1 & 1 & 5/3 \\ -1/3 & 1/6 & -5/18 \\ -1 & -1/2 & 1/6 \end{pmatrix} \quad (1.2)$$

and

$$b_{ij} = \begin{pmatrix} 1/3 & -3 & 5/3 \\ -1/9 & 1/2 & 5/18 \\ 1/3 & 3/2 & 1/6 \end{pmatrix} \quad (1.3)$$

The terms $v_i(z)$ are the high energy contributions describing the short-range repulsions corresponding to the t channel isospin i . These functions are of slow variation in the low energy region and moreover one can show that v_1 and v_2 are small ²⁾. Our model for these terms consists, as in Refs. 1) and 3), in putting $v_1 = v_2 = 0$ and $v_0 = \text{const.}$ Finally, the normalization of the amplitudes is such that the unitarity condition is

$$\text{Im } A_i(z) = K(z) |A_i(z)|^2, \quad K(z) = \left(\frac{z-1}{z+1} \right)^{1/2} \quad (1.4)$$

We will only consider solutions to these equations that are such that the integrals in (1.1) are convergent.

These equations have been solved analytically under the assumption $A_2 = 0$ and this yields a solution which depends on two parameters ¹⁾. If these are fixed from PCAC one obtains a ρ resonance at 940 MeV with a width of 260 MeV. For the $I = 0$ phase shifts one finds typically 25° - 40° in the ρ mass region.

In a subsequent paper ³⁾ it was shown that the introduction of experimental information on A_2 lowers the value of the ρ resonance parameters while the $I = 0$ phase shifts gets bigger. Typical figures are now $m_\rho = 750$ - 800 MeV, $\Gamma_\rho = 200$ - 230 MeV and $\delta_0^0(m_\rho) = 60^\circ$ - 70° .

In this paper we present calculations that predict all three amplitudes once the two S wave scattering lengths, a_0 and a_2 are given.

The fact that the scattering lengths are free parameters is quite natural for $\pi\pi$ scattering where the nearby singularities, $z = -1$, involve information on the scattering lengths. This situation is contrary to, for instance πN , KN and πK scattering, where the singularities at $z = -1$ can be included in the short-range terms.

In order to perform the calculations we use the experimental fact that the amplitude A_2 is small in the physical region up to 1 GeV. As we do not explicitly consider the coupling to the $K\bar{K}$ channel, which is crucial from about 1 GeV, we cannot hope to obtain agreement with experiments more than up to around 800 MeV.

2 - THE METHOD OF SOLUTION

We start from Eq. (1.1) where, as mentioned in the Introduction, we put

$$v_1 = v_2 = 0 \quad (2.1a)$$

and

$$\text{Im } R_2(z) = 0 \quad ; \quad z \gg 1 \quad (2.1b)$$

Eq. (1.1) then appears as

$$R_0(z) = -v_0 + I_0(z) + 1/3 J_0(z) - 3 J_1(z) \quad (2.2)$$

$$R_1(z) = -1/3 v_0 + I_1(z) - 1/9 J_0(z) + 1/2 J_1(z) \quad (2.3)$$

$$R_2(z) = -v_0 + 1/3 J_0(z) + 3/2 J_1(z) \quad (2.4)$$

where

$$I_i(z) = J_i(-z) = \frac{1}{\pi} \int_1^{\infty} \frac{\text{Im } R_i(z')}{z' - z} dz' \quad (2.5)$$

We form the linear combinations

$$\tilde{R}_0(z) = R_0(z) - R_2(z) = I_0 - 9/2 J_1 \quad (2.6)$$

$$\tilde{R}_1(z) = R_1(z) - 1/3 R_2(z) = I_1 - 2/9 J_0 \quad (2.7)$$

$\tilde{A}_0$ and $\tilde{A}_1$ then have the simple crossing property

$$\tilde{A}_0(-z) = -9/2 \tilde{A}_1(z) \quad (2.8)$$

For $\tilde{A}_0$ we have

$$\tilde{A}_0(1) = A_0(1) - A_2(1) = a_0 - a_2 \quad (2.9)$$

and

$$\tilde{A}_0(-1) = -9/2 \tilde{A}_1(1) = -9/2 [A_1(1) - 1/3 A_2(1)] = 3/2 a_2 \quad (2.10)$$

Hence from (2.8), (2.9) and (2.10) we know the values of $\tilde{A}_0$ and $\tilde{A}_1$ at $z = \pm 1$. For $|z| \leq 1$ we make as a starting point a linear interpolation for $\tilde{A}_0$ and $\tilde{A}_1$ through their known values at $z = \pm 1$. For instance, for $\tilde{A}_0$ we find

$$\tilde{A}_0^{\text{lin}}(z) = 1/2 (a_0 - 5/2 a_2) \left(z + \frac{2a_0 + a_2}{2a_0 - 5a_2} \right) \quad (2.11)$$

Equations (2.2) and (2.3) can now be written on the form ¹⁾

$$\tilde{A}_0^{-1}(z) = \tilde{A}_0^{\text{lin}-1}(z) - U_0(z) + 2/9 U_1(-z) + \text{C.D.D.} \quad (2.12)$$

$$\tilde{A}_1^{-1}(z) = \tilde{A}_1^{\text{lin}-1}(z) - U_1(z) + 9/2 U_0(-z) + \text{C.D.D.} \quad (2.13)$$

where

$$U_i(z) = \frac{z}{\pi} \int_1^\infty \frac{\text{Im} \tilde{A}_i^{-1}(z')}{z'(z'-z)} dz' \quad (2.14)$$

The CDD terms correspond to possible zeros on the real axis. They constitute our only ambiguities and will be excluded in the subsequent calculations.

Equations (2.4), (2.12) and (2.13) will now be solved in an iterative way. We find a zeroth order solution to (2.12) and (2.13) by using the smallness of A_2 in Eq. (2.14)

$$\text{Im } \tilde{A}_i^{-1}(z) \simeq \text{Im } \tilde{A}_i^{-1}(z) = -K(z) \quad (2.15)$$

Hence we obtain the zeroth order solutions to (2.12) and (2.13) in analytic form with a_0 and a_2 as parameters. The zeroth order solution is then inserted into (2.4) to gain information on A_2 . Equation (2.4) contains the unknown quantity v_0 which, however, can be found by studying the equation for $z = 1$, i.e.,

$$A_2(1) = a_2 = -v_0 + \frac{1}{3} J_0(1) + \frac{3}{2} J_1(1) \quad (2.16)$$

Once v_0 is known, we have a zeroth order solution for A_2 as well. Our zeroth order information can now be used to obtain a better approximation of Eq. (2.14).

The $\tilde{A}_0$ and $\tilde{A}_1$ calculated from the new expressions of $U_1(z)$ yields when inserted into (2.14) new values on v_0 and A_2 . The procedure is continued until the amplitudes $A_0 = \tilde{A}_0 + A_2$ and $A_1 = \tilde{A}_1 + \frac{1}{3} A_2$ are unitary. In practice A_0 and A_1 are unitary to a very high degree already after a few iterations.

Hence we end up with a unique prediction for low energy amplitudes in terms of scattering lengths. Just to give some idea of the quality of the solutions we show the results for $a_0 = 0.25$ and $a_2 = 0.01$ in Figs. 1, 2. For these values on a_0 and a_2 , v_0 turns out to be 0.4 in good agreement with various estimates of this quantity ¹⁾.

3 - DISCUSSION

In the above calculations, we have constructed a solution depending on the S wave scattering lengths. In Figs. 1 and 2, we have shown our results for a special choice of the scattering lengths. This choice is, however, not optimal and should just be regarded as a demonstration of the quality of our solutions. We feel that before doing any systematic variation of the scattering lengths and detailed comparison with experiment, one ought to incorporate information about the $K\bar{K}$ threshold. However, as can be seen from the Figures, our solutions contain all qualitative features that are found experimentally below 800 MeV.

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FIG.1 Predictions for the s-wave phase shifts corresponding to $a_0 = 0.25$ and $a_2 = -0.01$

FIG. 2 Prediction for the P - wave corresponding to $a_0 = 0.25$ and $a_2 = -0.01$