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HIGH ENERGY PION PHOTOPRODUCTION AMPLITUDES
FROM FIXED - T DISPERSION RELATIONS AND DUALITY

I.M. Barbour *)
CERN - Geneva

and

R.G. Moorhouse
Department of Natural Philosophy
Glasgow University, Glasgow

A B S T R A C T

Contributions to the high energy imaginary parts of the charged and neutral pion photoproduction amplitudes from degenerate ρ and A_2 exchanges and ω and B exchanges are parametrized in accord with the dual absorption model and the asymptotic phase relation. These contributions together with the imaginary parts of the amplitudes found at low energies from partial wave analyses are then used to evaluate the high energy real parts of the photoproduction amplitudes from fixed- t dispersion relations. A fit is made to data on pion photoproduction at 3.4 and 16 GeV incident photon energies, for momentum transfers up to $-1(\text{GeV}/c)^2$. It is shown that much of the data is reproduced both qualitatively and quantitatively by the model. The high energy imaginary parts of the photoproduction amplitudes given by the fit are shown to be in accord with their average low energy behaviour.

*) On leave of absence from the University of Glasgow.

1. INTRODUCTION

In this paper we intend to show that the data for $|t| \leq 0.8(\text{GeV}/c)^2$ on high energy charged and neutral pion photoproduction can be explained within the context of a model based on the assumptions of

- a) a simple power law behaviour for the high energy imaginary parts of the photoproduction amplitudes so that the imaginary parts of the invariant amplitudes, $A(s, t)$ are of the form

$$\text{Im } A(s, t) = b(t) \left(\frac{s}{s_0} \right)^{\alpha(t)-1} + \bar{b}(t) \left(\frac{s}{s_0} \right)^{\bar{\alpha}(t)-1} \quad (1)$$

where $\alpha(t)$ is a simple ρ, ω, A_2 ("exchange degenerate") natural parity trajectory and $\bar{\alpha}(t)$ in the second term, which is zero or small, is a π, B ("exchange degenerate") unnatural parity trajectory;

- b) the $b(t)$ in Eq. (1) have a "Bessel function like" t behaviour, simply related to the helicity flip of the s channel helicity amplitudes as in the dual absorptive model of Harari ¹⁾;
- c) a low energy behaviour for the imaginary parts of the amplitudes consistent with that given by the analyses of Moorhouse, Oberlack and Rosenfeld ²⁾ and by Walker ³⁾;
- d) the real parts of the high energy photoproduction amplitudes are calculated by fixed t dispersion relations, with input the imaginary parts of the amplitudes given by a), b) and c).

The basic idea of our model is to propose a simple behaviour for the high-energy imaginary parts of the amplitudes. All the complications which in other theories ⁴⁾ are generated by a formalism involving cuts in the angular momentum plane, occur in our model in the real part and are simply calculated by fixed $-t$ dispersion relations. It is thus an extension of the idea of the dual absorption model of Harari ¹⁾ and it has already been proposed and used with some success in the context of pion-nucleon charge exchange scattering by Ghandour and Moorhouse ⁵⁾ and by Coirier et al. ⁶⁾.

Part of this program has already been implemented in an earlier paper ⁷⁾ in which we showed that assumptions c) and d) are consistent with the data for high energy charged pion photoproduction in the extreme

forward direction ($t \sim -\mu^2$). We also pointed out that away from the forward region, for $-0.2 \geq t \geq -0.8(\text{GeV}/c)^2$, there is a strong cancellation between the Born terms and the contributions from the low energy resonances through the fixed $-t$ dispersion relations. It is in this region that we expect the high energy behaviour of the imaginary part of the photoproduction amplitudes to contribute significantly to the real parts.

Hontebeyrie et al., ⁸⁾ have used a similar model to calculate the high energy charged pion photoproduction differential cross-sections. However, they require a large real contribution from the unnatural parity B exchange amplitudes at $t \sim -0.6(\text{GeV}/c)^2$ in order to reproduce the observed π^-/π^+ ratio. We show below that this ratio follows naturally from strongly exchange degenerate ρ and A_2 exchanges. We also show that the polarized target asymmetry T^+ need not have the zero at $t \sim -0.6(\text{GeV}/c)^2$ as predicted by their fit and not seen in the data.

Argyres et al., ⁹⁾ have shown that the zero in the single helicity flip amplitudes at $t \sim -0.5(\text{GeV}/c)^2$ together with natural parity exchange and Walker's low energy amplitudes gives an asymmetry $T^0 \simeq -0.5$ at $t \sim -0.5(\text{GeV}/c)^2$ and incident photon energy of 4 GeV, in agreement with the data. They have recently extended ¹⁰⁾ this calculation into a detailed amplitude analysis of π^0 photoproduction at 4 GeV.

In this paper we succeed, within the above model, in obtaining a consistent fit to the data on charged and neutral pion photoproduction for incident photon energies between 3 and 16 GeV and for momentum transfers up to $|t| \sim 0.8(\text{GeV}/c)^2$.

2. THE MODEL

We write unsubtracted fixed t dispersion relations for the invariant amplitudes $A_i^{+, -, 0}$ ($i=1, \dots, 4$) describing the processes

$$\gamma p \rightarrow \pi^+ n \quad ; \quad \gamma n \rightarrow \pi^- p \quad ; \quad \gamma p \rightarrow \pi^0 p$$

$$A_i^{+, -, 0}(s, t) = B_i^{+, -, 0}(s, u) + \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} ds' \left\{ \frac{\text{Im } A_i^{+, -, 0}(s', t)}{s' - s} + \overset{(2)}{\xi_i} \frac{\text{Im } A_i^{+, -, 0}(s', t)}{s' - u} \right\}$$

with $\xi_i = +1$ for $i=1,2,4$ and $\xi_3 = -1$, M is the nucleon mass and μ the pion mass. The Born terms $B_i^{+, -, 0}$ are given by Berends et al. ¹¹⁾.

We split the integration region into two parts

- 1) $(M + \mu)^2 \leq s' \leq \bar{s} \sim 3 \text{ GeV}^2$: the low energy region in which we take the imaginary parts of the photoproduction amplitudes as given by the analyses of Walker ³⁾ and of Moorhouse et al. ²⁾.

As shown in a previous paper ⁷⁾, the high energy behaviour of the real part of the photoproduction amplitudes depends sensitively upon a strong unidirectional cancellation between the Born terms and the low energy resonances through the fixed- t dispersion relations. Hence, any fit to the high energy data must depend strongly upon the low energy amplitudes. We therefore allowed the couplings in the phenomenological analyses to vary about their stated values.

In an earlier paper ⁷⁾ we noted that the $f_{37}(1920)$ can make an important contribution to the unidirectional cancellation between the resonance contributions and the Born terms for $|t| < 0.6(\text{GeV}/c)^2$. Hence, our fit should be relatively sensitive to this contribution and it would be a flaw in the model should an unrealistic contribution from the $f_{37}(1920)$ be required in order to fit the high energy data. We therefore added this resonance with a magnetic coupling to be determined by the fit. The analyses of Moorhouse et al. ²⁾ gives a value for this coupling (consistent with quark model predictions) against which we can compare. Note that the sign of the coupling is determined in our model purely by the necessity that this contribution cancel with the low energy resonance contributions and Born terms.

- 2) $s' > \bar{s}$: the high energy region where we assume that the high energy behaviour of the invariant amplitudes and equivalently (for large s) the s channel helicity amplitudes $H_n(s, t)$ is given by a simple power law.

From Eq. (1) we have

$$\text{Im } H_n(s, t) = \beta_n(t) \left(\frac{s}{s_0} \right)^{\alpha(t) - 1/2} + \bar{\beta}_n(t) \left(\frac{s}{s_0} \right)^{\bar{\alpha}(t) - 1/2} \quad (3)$$

where $n=0, \pm 1, 2$ corresponds to the non-flip, single-flip and double-flip amplitudes respectively.

The s channel helicity amplitudes are simply related to the invariant amplitudes $A_i(s,t)$ for s large and t/s small by

$$\begin{aligned} H_0 &\simeq -\frac{\sqrt{s}}{2\sqrt{2}}(2A_1 + tA_2) & H_1 &\simeq \frac{\sqrt{s}}{2\sqrt{2}}\sqrt{-t}(A_4 - A_3) \\ H_2 &\simeq -\frac{\sqrt{s}}{2\sqrt{2}}tA_2 & H_{-1} &\simeq \frac{\sqrt{s}}{2\sqrt{2}}\sqrt{-t}(A_4 + A_3) \end{aligned} \quad (4)$$

To leading order in s, natural parity t channel exchanges contribute to the combinations

$$H_0 - H_2 \quad \text{and} \quad H_1 + H_{-1} \quad (5a)$$

and unnatural parity t channel exchanges to

$$H_0 + H_2 \quad \text{and} \quad H_1 - H_{-1} \quad (5b)$$

The polarized photon asymmetries for π^+ and π^0 photo-production are consistent with the dominance of natural parity exchange for $|t| > 0.1(\text{GeV}/c)^2$. We therefore assume that

$$\begin{aligned} \text{Im } H_n^{\pm,0}(s,t) &= \beta_n^{\pm,0}(t) \left(\frac{s}{s_0}\right)^{\alpha(t)-\frac{1}{2}} \quad n = \pm 1 \\ \text{Im } H_n^{\pm,0}(s,t) &= \beta_n^{\pm,0}(t) \left(\frac{s}{s_0}\right)^{\alpha(t)-\frac{1}{2}} + \bar{\beta}_n^{\pm,0}(t) \left(\frac{s}{s_0}\right)^{\bar{\alpha}(t)-\frac{1}{2}} \quad n = 0, 2 \end{aligned} \quad (6)$$

where $\alpha(t)$ is a simple exchange degenerate trajectory

$$\alpha(t) \simeq .5 + t$$

describing the natural parity exchanges ρ , ω , and A_2 . Any unnatural parity exchange contributions we assume to be described by an elementary pion exchange in the Born terms and other contributions to the real part from the dispersion relations together with a small imaginary part from B exchange with trajectory $\bar{\alpha}(t) \sim t$. Note that G parity conservation requires that π and B exchanges do not contribute to the single flip amplitudes.

The amplitudes for π^+ , π^- and π^0 production of protons are then given by the identifications

$$\begin{aligned}\beta_m^\pm(t) &= \sqrt{2} \left(\beta_m^{\rho}(t) \pm \beta_m^{A_2}(t) \right) & \bar{\beta}_m^\pm(t) &= \sqrt{2} \beta_m^B(t) \\ \beta_m^0(t) &= \beta_m^{\rho}(t) + \beta_m^{\omega}(t) & \bar{\beta}_m^0(t) &= \beta_m^B(t)\end{aligned}\quad (7)$$

In the dual absorption model of Harari ¹⁾, the s channel helicity amplitudes have imaginary parts with a Bessel function-like t behaviour

$$\text{Im } H_{\Delta\lambda} \sim J_{|\Delta\lambda|}(R\sqrt{-t}) \quad ; \quad R \sim 1 \text{ fm} \quad (8)$$

If the imaginary part of the non-flip amplitude $H_0(s,t)$ had the t dependence of Eq. (8), $\sim J_0(R\sqrt{-t})$ and were large, then this would manifest itself through an energy dependence in the near forward direction of the form

$$\frac{d\sigma}{dt} \sim 1/s^{2-2\alpha(0)} \sim 1/s$$

rather than the observed $1/s^2$ behaviour. With a parametrization of the non-flip amplitude of the standard form

$$\text{Im } H_0(s,t) = \gamma_0 e^{\lambda_0 t} J_0(R\sqrt{-t}) \left(\frac{s}{s_0} \right)^{\alpha(t) - \frac{1}{2}} \quad (9)$$

we found that we could not obtain a satisfactory fit to all of the data. In particular, a fit to the π^+ and π^- differential cross-sections and π^+ polarized target asymmetry at $t \sim -0.5(\text{GeV}/c)^2$ resulted in the above incorrect energy dependence of the differential cross-sections in the near forward direction.

Hontebeyrie et al. ⁸⁾ have also used the parametrization, Eq. (9), for the non-flip amplitudes in their fit to the charged pion differential cross-sections and found that their fit required $H_0 \ll H_2$ for $t \sim -0.6(\text{GeV}/c)^2$. In this momentum transfer region we would argue that the observed π^+ polarized photon asymmetry and the π^-/π^+ production ratio require that the exchange degenerate natural parity ρ and A_2 amplitudes

must dominate. We would therefore expect any unnatural parity exchange amplitudes to be small and from Eq. (5b) $H_0 \simeq -H_2$ (as also found by Worden⁴⁾ in his analyses). Hence the results, in the work of Hontebeyrie et al.,⁸⁾ of parametrizing $\text{Im } H_0(s, t)$ proportional to J_0 as in Eq. (9) are unsatisfactory. Neutral pion photoproduction is predominantly single flip and is relatively insensitive to the parametrization of the non-flip amplitudes.

The original formulation of the dual absorption model is therefore unsatisfactory in that the imaginary part of the non-flip amplitude in photoproduction is not given by Eq. (9). A good fit to the data for $|t| < 0.8(\text{GeV}/c)^2$ can be obtained with a " J_0 " behaviour for the non-flip amplitude but it requires a parametrization giving a smaller intercept at $t=0$ than that given by Eq. (9). However, the present photoproduction data require only that the imaginary part of the non-flip amplitude be small for $|t| < 0.25(\text{GeV}/c)^2$ and it is not sensitive (at the present level of analysis) to its detailed form in that region. For this reason we choose to parametrize the imaginary part of the non-flip amplitude as

$$\text{Im } H_0(s, t) = g_0 e^{\lambda_0 t} J_2(R\sqrt{-t}) \left(\frac{s}{s_0} \right)^{\alpha(t) - \frac{1}{2}} \quad (10)$$

for s large.

It should be noted that this parametrization

- a) away from the forward region is approximately that of Eq. (9) since $J_0 \sim -J_2$ for $|t| > 0.3(\text{GeV}/c)^2$;
- b) allows a clear distinction [through Eq. (5)] between the natural and unnatural parity exchange contributions to the s channel helicity amplitudes since the non-flip and double-flip amplitudes now have the same t dependence *)

*) Here we differ from Hontebeyrie et al.⁸⁾ in that they made use of the differing t dependence of their H_0 and H_2 to allow the B exchange amplitudes to give a contribution to the natural parity exchange amplitudes and hence obtain the observed π^-/π^+ cross-section ratios.

Our s channel helicity amplitudes were therefore parametrized as

$$\begin{aligned} \text{Im } H_m^{\pm,0}(s,t) = & \gamma_m^{\pm,0} e^{\lambda_0 t} J_2(R\sqrt{-t}) \left(\frac{s}{s_0}\right)^{\alpha(t)-\frac{1}{2}} \\ & + \bar{\gamma}_m^{\pm,0} e^{\bar{\lambda}_0 t} J_2(R\sqrt{-t}) \left(\frac{s}{s_0}\right)^{\bar{\alpha}(t)-\frac{1}{2}} \end{aligned} \quad n=0,2 \quad (11)$$

and

$$\text{Im } H_m^{\pm,0}(s,t) = \gamma_m^{\pm,0} e^{\lambda_1 t} J_1(R\sqrt{-t}) \left(\frac{s}{s_0}\right)^{\alpha(t)-\frac{1}{2}} \quad n=\pm 1$$

with

$$\gamma_0^{\pm,0} + \gamma_2^{\pm,0} = \gamma_1^{\pm,0} - \gamma_{-1}^{\pm,0} = 0 \quad (12a)$$

corresponding to natural parity exchange and

$$\bar{\gamma}_0^{\pm,0} - \bar{\gamma}_2^{\pm,0} = 0 \quad (12b)$$

corresponding to unnatural parity exchange.

Since the model incorporates all the analyticity properties of the scattering amplitudes, the finite energy sum rules are automatically satisfied. This has been discussed in some detail by Hontebeyrie et al. ⁸⁾.

One further constraint that the model must satisfy is that of the asymptotic phase relation which follows from the choice of a power law for the high energy behaviour of the imaginary parts of the amplitudes. This need only be satisfied asymptotically, but most models of high energy photoproduction satisfy it identically by multiplying the Regge residue functions by appropriate ghost killing factors. This factor, for example, introduces a zero at $t \sim -0.5(\text{GeV}/c)^2$ in the imaginary part of the non-flip amplitude for A_2 exchange. We shall show below that the high energy behaviour of the imaginary parts of the photoproduction amplitudes obtained from our fit is consistent with their low energy behaviour as found by Walker ³⁾ and by Moorhouse et al. ²⁾. This low energy behaviour shows no evidence for this zero in the imaginary part of the non-flip A_2 exchange amplitudes. The zero would also require that T^+ , the polarized target asymmetry in π^+ photoproduction, be zero or small at $t \sim -0.5$, again in contradiction with the data.

We, on the other hand, assume a power law behaviour for $\text{Im } A(s, t)$ only for $s > \bar{s}$. This assumption introduces a logarithmic modification to the energy dependence of the $\text{Re } A(s, t)$ and consequently this modifies the asymptotic phase relation for the cases in which the ratio of real to imaginary parts of the amplitudes can be infinite ¹²⁾. For the crossing odd amplitudes (A_2 exchange) instead of (for t/s small)

$$\lim_{s \rightarrow \infty} \frac{\text{Re } A(s, t)}{\text{Im } A(s, t)} = \cot \frac{\pi \alpha(t)}{2} \quad (13a)$$

and the consequent zero in $\text{Im } A(s, t)$ at $\alpha(t) = 0$, our assumption gives

$$\lim_{s \rightarrow \infty} \frac{\text{Re } A(s, t)}{\text{Im } A(s, t)} = \cot \frac{\pi \alpha(t)}{2} - \frac{2}{\pi} \left(\frac{\bar{s}}{s} \right)^{\alpha(t)} \sum_{n=0}^{\infty} \frac{\left(\bar{s}/s \right)^{2n}}{2n + \alpha(t)} \quad (13b)$$

When $\alpha(t) = 0$, this ratio is finite ^{*)} and hence at asymptotic energies the imaginary part of the amplitude is non-zero for $t \sim -0.5(\text{GeV}/c)^2$.

The crossing even amplitudes (ρ and ω exchange) still have a finite ratio of real to imaginary parts with

$$\lim_{s \rightarrow \infty} \frac{\text{Re } A(s, t)}{\text{Im } A(s, t)} = \tan \frac{\pi \alpha(t)}{2} \quad \text{for } \alpha(t) > -1$$

Consequently, for $|t| < 1(\text{GeV}/c)^2$ they have an asymptotic energy dependence which is unaltered by the assumption of a power law behaviour in their imaginary parts for $s > \bar{s}$.

*) At $\alpha(t) = 0$ we have from Eq. (13b)

$$\lim_{s \rightarrow \infty} \frac{\text{Re } A(s, t)}{\text{Im } A(s, t)} = -\frac{2}{\pi} \log \frac{\bar{s}}{s}$$

and consequently $\text{Re } A(s, t) \sim (1/s) \log s$ rather than $\sim 1/s$ as given by Eq. (13a). For $\alpha(t) < 0$, Eq. (13b) gives a more complicated logarithmic modification to the asymptotic energy behaviour of the real part.

This mechanism then allows us to choose exchange degenerate ρ and A_2 Regge amplitudes with imaginary parts non-zero around $t \sim -0.5(\text{GeV}/c)^2$. If the imaginary parts of the ρ and A_2 exchange amplitudes cancel in $K^+p \rightarrow K^{*+}p$, then $SU(3)$ and vector dominance give

$$\gamma_n^\rho = \frac{1}{3} \gamma_n^{A_2} \quad \lambda_n^\rho = \lambda_n^{A_2} \quad (14)$$

These relations also follow from a quark impulse model¹³⁾ for charged pion photoproduction and should lead to the observed cross-section ratio

$$\frac{d\sigma}{dt}(\gamma n \rightarrow \pi^- p) / \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) \simeq .35$$

for $-0.2 > t > -0.8(\text{GeV}/c)^2$. In this region the imaginary parts of the single flip amplitudes should be small so we impose only the constraint

$$\gamma_0^\rho = \gamma_2^\rho = \frac{1}{3} \gamma_0^{A_2} = \frac{1}{3} \gamma_2^{A_2}$$

and

$$\lambda_0^\rho = \lambda_0^{A_2} \quad (15)$$

It is this interpretation of the asymptotic phase relation which gives rise to the differences between our model and that of Hontebeyrie et al.⁸⁾. Both models are based on the four assumptions outlined in the Introduction but whereas Hontebeyrie et al., get the charged pion data in terms of ρ , A_2 , π and B exchanges (+ Born terms) with the B exchange amplitude dominating the imaginary parts at high energy, we interpret the data in terms of a dominant strongly exchange degenerate ρ and A_2 exchange contribution to the high energy imaginary parts.

3. RESULTS

We have attempted to fit the available data at 3.4 and 16 GeV incident photon energies using the model described above. However, any detailed quantitative comparison between the model's predictions and experiment is strongly restricted by our present lack of knowledge of the low to medium energy partial waves above 1.2 GeV photon energy.

In particular, we found that the low energy analyses of Moorhouse et al., ²⁾ and of Walker ³⁾ tended to give, through the fixed t dispersion relation, high energy real parts which made

$$\frac{d\sigma_{\parallel}}{dt} \sim |A_1 + t A_2|^2 - t |A_3|^2 \quad (16)$$

too large. G parity conservation requires that the unnatural parity exchange Regge poles, π and B , should have no non-flip coupling to nucleons. This in turn requires at high energies they should not contribute to $\text{Im } A_3(s, t)$. Natural parity exchanges should also not contribute at high energies. Hence, in the model $A_3(s, t)$ should be a real amplitude asymptotically, determined purely by the Born term and the low energy resonances. The data for π^+ and π^0 photoproduction are consistent with the unnatural parity exchange amplitudes being small away from the extreme forward direction where, as we see from Eq. (16), A_3 cannot contribute significantly. We therefore put $A_3(s, t) = 0$ throughout our fit. One can see from Fig. 1 that the evaluation of $\text{Re } A_3^{\pi^+}(s, t)$ would depend strongly upon cancellations between the Born term, the $P_{33}(1236)$ and the contributions from resonances above $s = 2(\text{GeV})^2$.

We also noted that we could obtain different fits to the data according as to whether one parametrized the s channel helicity amplitudes as in Eq. (11) (simple helicity amplitude parametrization) or used this parametrization together with Eq. (4) as a parametrization for the high energy imaginary parts of the invariant amplitudes (simple invariant amplitude parametrization). Both prescriptions would be equivalent if Eq. (4) were exact but since the low to medium energy partial waves are only known up to $s \sim 3(\text{GeV})^2$ and Eq. (4) is only valid for s large (and t/s small) and one must use the exact relations elsewhere, they each provide a different continuation from high to low energies. The constraint that the high energy behaviour of the amplitudes be reasonably consistent with their average low energy behaviour led us to adopt the simple invariant amplitude parametrization.

For example, Hontebeyrie et al.⁸⁾ found that the average value of the low energy behaviour of the s channel helicity amplitudes

$$\int_{(M+\mu)^2}^{\bar{s}} \frac{I_m}{(M+\mu)^2} \left[H_m^+(s,t) - H_m^-(s,t) \right] ds \quad (17)$$

corresponding to A_2 exchange, did not have the Bessel function zeros of the dual absorptive model. If we adopt the simple invariant amplitude parametrization and parametrize the invariant amplitudes through Eqs. (4) and (11) then they should have asymptotically the t dependence

$$\begin{aligned} 2 A_1 + t A_2 &\sim J_0(R\sqrt{-t}) \\ t A_2 &\sim J_2(R\sqrt{-t}) ; \quad \sqrt{t} A_4 \sim J_1(R\sqrt{-t}) \end{aligned} \quad (18)$$

and, as shown in Figs. 2 and 3, this is not inconsistent with their "average" behaviour at low energies for $|t| < 0.7(\text{GeV}/c)^2$ especially for the single flip amplitudes. The isovector A_2 non-flip amplitude $A_1^\Theta(s,t)$ behaves very much like $J_2(R\sqrt{-t})$ together with a small admixture of $J_0(R\sqrt{-t})$, consistent with our high energy parametrization Eq. (10). Taking the appropriate moments of the even crossing ρ and ω exchange amplitudes does not alter their zero structure.

Note that we have subtracted out the contribution of the $P_{33}(1236)$ in the comparison. In the quark impulse model the spacial symmetry of the $P_{33}(1236)$ and the nucleon is such that the quark excited by the absorption of a photon need not be the quark which de-excites by pion emission. Hence these intermediate states must be considered separately. The model therefore requires the leading t channel exchanges to be dual in some way to the s channel exchanges except for the nucleon and $P_{33}(1236)$. This is consistent with the observations of Worden⁴⁾ and of Argyres et al.¹⁰⁾ in that the $P_{33}(1236)$ and the nucleon poles introduce into the F.E.S.R.'s contributions with a t structure different from that of the dual absorption model. Evaluation of the t dependence of Eq. (17) with the P_{33} contribution taken out does not remove the inconsistency between

the A_2 exchange contribution and the dual absorption model. The low energy behaviour of the photoproduction amplitudes therefore favours the simple invariant amplitude parametrization rather than the simple helicity amplitude parametrization as a continuation between high and low energies.

The results of our fit to charged and neutral pion photoproduction of neutrons and protons are summarized in Figs. 4-13. Many of the features of high energy photoproduction can be seen to be both qualitatively and quantitatively reproduced. The values of the parameters obtained for the high energy imaginary parts of the amplitudes are given in the Table. These parameters give the high energy behaviour of the invariant amplitudes through Eq. (4) for $s > \bar{s}$ ¹⁴).

Note that the charged pion results given by the model are not reliable for $|t| > 0.7(\text{GeV}/c)^2$. In this region the real part of the amplitudes is not given accurately by the model because of the massive cancellation between the Born terms, the low energy resonances and the Regge contributions. The low energy imaginary parts of the amplitudes are unphysical in this region and very sensitive to the details of the resonance shapes. No great weight, for example, should be placed on the structure in T^+ or in the factor of 2 discrepancy in $(s-M^2)^2 d\sigma/dt$ for $t < -0.7(\text{GeV}/c)^2$. Other more complicated contributions most probably become important in this region. These uncertainties must be inherent in any calculation of charged pion photoproduction using fixed- t dispersion relations. In neutral pion photoproduction the real part does not depend on any strong cancellation and hence the model should be valid for $|t| < 1(\text{GeV}/c)^2$.

The fit presented here has the strong constraint of exchange degenerate ρ and A_2 Regge poles with

$$\gamma_0^\rho = \frac{1}{3} \gamma_0^{A_2}$$

A quantitative improvement in the fit would obviously be obtained by relaxing this condition slightly, but it is interesting to note that the fit finds that the single-flip contributions are also exchange degenerate with

$$\gamma_1^\rho \approx \frac{1}{4.8} \gamma_1^{A_2}$$

The trajectory function $\alpha(t)$ for the natural parity exchanges was allowed to vary by 20% about an intercept of 0.5 and a slope of $1(\text{GeV}/c)^{-2}$.

The good agreement of the parameters obtained for the $F_{37}(1950)$ resonance (see the Table) with those obtained by the analyses of Moorhouse et al. ²⁾ is perhaps fortuitous since our fit using Walker's ³⁾ analysis is missing the $D_{33}(1760)$. The fit obtained did not require much change in Walker's partial widths. A comparison between the low energy amplitudes given by the fit and those obtained by one of the fits of Ref. 2) is shown in Figs. 14a and 14b for two of the dominant amplitudes $\text{Im } A_1 \pi^+(s,t)$ and $\text{Im } A_4 \pi^0(s,t)$. As can be seen from Figs. 3 and 14a there is no evidence in the average behaviour of the low energy data for a zero in the A_2 Regge residue γ_0 at $t \sim -0.5$ or $\alpha(t) = 0$.

The imaginary part of the single-flip amplitudes in charged pion photoproduction is predicted to be small. The polarized target asymmetry is therefore proportional to

$$\text{Re}(H_1) \times \text{Im}(H_2)$$

and the zero predicted at $t \sim -0.7(\text{GeV}/c)^2$ is the reflection of a zero in the real part of the single flip amplitudes. As discussed above, the position of this zero is strongly dependent upon cancellations, but should occur for $|t| < 1(\text{GeV}/c)^2$. The model predicts that $T^\pm$ should be approximately energy independent. On the other hand, because of crossing and the dominance of ω exchange, the polarized target asymmetry T^0 should have an energy dependence $\sim E_\gamma^{-1}$ at $t \sim -0.5(\text{GeV}/c)^2$. The zero in T^0 at $t \sim -0.3(\text{GeV}/c)^2$ is a reflection of the dominance of the single flip amplitudes in this region and is again due to a zero in their real part. Argyres et al. ¹⁰⁾ find a similar zero in their analysis at 4 GeV and point out that it might well be a consequence of constraining $A_3(s,t)$ to be zero.

Our results show that all of the data above 3 GeV can be explained consistently in terms of the Born amplitudes, the low energy multipole amplitudes, and ρ , ω , A_2 and B Regge exchanges. There is a strong unidirectional cancellation in the fixed t dispersion relations between the Born terms and the low energy resonance contributions to the real parts of the charged pion photoproduction amplitudes. This allows the contributions from the ρ and A_2 Regge poles to be important for $|t| > 0.2(\text{GeV}/c)^2$. Exchange degeneracy, or the quark impulse model, then gives the observed π^+/π^- cross-section ratio and together with the dual absorptive model is consistent with the observed polarized target asymmetry.

Neutral pion photoproduction is given essentially by ω exchange and is not as sensitive to the details of the low energy contributions to the real part.

The exchange degeneracy for the ρ and A_2 exchanges

$$g^{\rho} = \frac{1}{3} g^{A_2}$$

requires that the A_2 residues be non-zero at $\alpha(t)=0$. We have shown that this is not inconsistent with either the asymptotic phase relation or the average behaviour of the low energy imaginary parts of the amplitudes. An extension of the analyses of the low energy photoproduction amplitudes beyond their present limit of $E_{\gamma} \sim 1200$ MeV should further constrain their high energy t dependence and also indicate whether our assumption of exchange degenerate ρ and A_2 exchanges in the ratio 1:3 is correct or not. It would also allow a fit to be made to the high energy data which would be less dependent upon whether the simple invariant amplitude or simple helicity amplitude parametrization is taken to describe the average behaviour of the amplitudes in the intermediate energy region.

T A B L E

	γ_0	γ_1	λ_0	λ_1
ρ	-1.55	0.26	-1.27	7.64
A_2	-4.66	1.20	-1.27	7.48
ω	-1.60	9.77	0.23	-1.12
B	-0.65	0	-6.64	-

The parameters used for the fit illustrated. Units are $\sqrt{\mu b}$ for γ_n and $(\text{GeV}/c)^{-2}$ for λ_n . The constraints were $\gamma_0^p = 1/3$ $\gamma_0^{A_2}$ and $\lambda_0^p = \lambda_0^{A_2}$. The natural parity trajectory $\alpha(t) = 0.44 + 0.89t$ with unnatural parity trajectory constrained to be $\alpha(t) - 0.5$. $R = 1\text{fm}$, $s_0 = 1.92 \text{ GeV}^2$, $\bar{s} = 3.60 \text{ GeV}^2$ and the resonance coupling of the $f_{37}(1950)$ was found to be

$$A_{3/2}^+ = \sqrt{\frac{5}{3}} A_{1/2}^+ = -100 (\text{GeV}^{-1/2} \times 10^{-3})$$

in the notation of Ref. 2). The cross-section is given by

$$(s - M^2)^2 \frac{d\sigma}{dt} = s \sum_{i=1}^4 |H_i|^2$$

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FIGURE CAPTIONS

Figure 1 :

The low energy behaviour of the imaginary part of the invariant amplitude $A_3(s,t)$ in π^+ photoproduction for $t = -0.25$ and -0.75 (GeV/c)²

————— from Walker ³⁾ plus $f_{37}(1920)$
 - - - - - from Moorhouse et al. ²⁾.

Figure 2 :

The "average" values, as a function of t , of the low energy behaviour of

- a) $A_4^S(s,t) \simeq A_4^P$
- b) $A_4^\ominus(s,t) \simeq A_4^{A_2}$
- c) $A_4^\oplus(s,t) \simeq A_4^\omega$ - at high energies, as calculated from Walker ³⁾ plus $f_{37}(1920)$. The black dots display the expected zero of each amplitude from Harari's prescription ¹⁾.

Figure 3 :

As in Fig. 2, for the non-flip combination $2A_1 + tA_2$

- a) $(2A_1^S + tA_2^S) \sim (\rho) + (B)$
- b) $(2A_1^\ominus + tA_2^\ominus) \sim (A_2) + (\pi)$

The isovector combination corresponding to ω exchange is small and not shown. The "average" behaviour of tA_2 is approximately $J_2(R\sqrt{-t})$ for each of the three isospin combinations $(\oplus, \ominus, S)$.

Figure 4 :

The differential cross-section for π^+ photoproduction at

- a) 3.4 GeV and
- b) 16 GeV.

Data from Refs. 15), 16) and 17) at 3.4 GeV and Ref. 18) at 16 GeV.

Figure 5 :

The π^+ polarized photon asymmetry $A^+ = (\sigma_{\perp} - \sigma_{\parallel}) / (\sigma_{\perp} + \sigma_{\parallel})$ at

a) 3.4 GeV and

b) 16 GeV.

Data $\downarrow$ from Ref. 19) and $\downarrow$ from Ref. 20) at 3.4 and 3 GeV respectively and from Ref. 21) at 16 GeV.

Figure 6 :

The π^+ polarized target asymmetry at 16 GeV. Data from Ref. 22) at 16 GeV.

Figure 7 :

The differential cross-section for π^- photoproduction at

a) 3.4 GeV and

b) 16 GeV.

Data from Ref. 17) at 3.4 GeV.

Figure 8 :

The π^- polarized photon asymmetry A^- at

a) 3.4 GeV and

b) 16 GeV.

Data $\downarrow$ from Refs. 23) and $\downarrow$ from Ref. 24) at 3 and 3.4 GeV respectively and from Ref. 21) at 16 GeV.

Figure 9 :

Ratio of differential cross-sections for π^- and π^+ photoproduction

a) $R^C = (d\sigma/dt)(\pi^-) / (d\sigma/dt)(\pi^+)$ at 16 GeV

b) $R_{\perp}^C = (d\sigma_{\perp}/dt)(\pi^-) / (d\sigma_{\perp}/dt)(\pi^+)$ at 3.4 GeV.

Data from Ref. 25) at 16 GeV and Ref. 20) at 3 GeV.

Figure 10 :

The differential cross-section for π^0 photoproduction of protons at

a) 3.4 GeV and

b) 16 GeV.

Data from Ref. 26) at 4 GeV and Ref. 27) at 15 GeV.

Figure 11 :

The π^0 polarized photon asymmetry A^0

----- 3.4 GeV theory
 ----- 16 GeV theory
 Data $\bullet$ from Ref. 28) at 3 GeV and $\times$ from Ref. 27) at 10 GeV.

Figure 12 :

The π^0 polarized target asymmetry

----- 3.4 GeV theory
 ----- 16 GeV theory
 Data from Ref. 29) at 4 GeV.

Figure 13 :

Ratio of differential cross-sections for π^0 photoproduction of neutrons and protons

$$R^n = \frac{d\sigma/dt(\pi^0 n)}{d\sigma/dt(\pi^0 p)}$$

at 3.4 GeV. Data from Ref. 26) at 4 GeV.

Figure 14 :

- a) The low energy behaviour of the imaginary part of $A_1(s,t)$ in π^+ photoproduction for $t = -0.25$ and $-0.75(\text{GeV}/c)^2$

----- from Walker ³⁾ plus $f_{37}(1920)$
 ----- from Moorhouse et al. ²⁾. [The $P_{33}(1236)$ contribution is suppressed in each case.]
 ----- Regge high energy contribution as found in the fit (see the Table) continued down to low energies.

- b) as in a) for the amplitude $A_4(s,t)$ in π^0 photoproduction of protons.

FIG.1

FIG. 3a

Polarized photon asymmetry $A^+(\gamma p \rightarrow \pi^+ n)$

FIG.5a

FIG.5b

FIG.6

Polarized photon asymmetry $A^-(\gamma n \rightarrow \pi^- p)$

FIG. 8a

FIG. 8b

FIG. 9a

FIG. 9b

FIG. 11

FIG.12

FIG. 13

FIG. 14a