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SOME APPLICATIONS OF ISOSPIN RELATIONS OF INCLUSIVE REACTIONS

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A B S T R A C T

The isospin equalities of inclusive cross-sections are combined with the Mueller formalism to obtain relations of fragmentation and pionization vertices and useful simplified relations of inclusive cross-sections. Isospin equalities of two-particle correlation functions are derived for particles of various isospins with the produced particles in the fragmentation region, or one in the fragmentation region and the other in the pionization region, or both in the pionization region.

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1. INTRODUCTION

The isospin equalities of inclusive cross-sections listed in a previous paper ¹⁾ are combined with the Mueller-Regge approach to inclusive reactions ²⁾. In Section 2, factorization is assumed and relations of fragmentation and pionization vertices, and simplified equations of inclusive cross-sections in certain kinematic regions are obtained. The above vertices are independent of the reactions in which they occur and therefore relations among them are interesting. The results of factorization in inclusive cross-sections with the Pomeron and the usual set of natural parity trajectories f , ω , ρ and A_2 seem to be fairly good ^{3),4)}, except probably in the central region where new singularities may be needed ⁵⁾.

In Section 3 certain results of Section 2 are used to derive isospin relations of two particle correlation functions ⁶⁾, when the difference between the rapidity of one of the particles of the initial state, having, say, isospin I , and the rapidities of the other particles, is very large. The produced particles may be in the fragmentation region, or one may be in the fragmentation region and the other in the central region. For $I=0$ these relations are exact isospin equations of correlation functions, i.e., they are independent of factorization, and hold in all kinematic regions. Also isospin relations of two particle correlation functions in the pionization region are obtained for particles of isospins $(\frac{1}{2}, 1)$, $(1, 1)$, $(\frac{1}{2}, \frac{3}{2})$ and $(1, \frac{3}{2})$. In Section 4, it is shown that the isospin equalities of the vertices give useful relations of inclusive cross-sections at finite rapidity differences.

2. COMBINATION OF ISOSPIN EQUALITIES WITH THE MUELLER FORMALISM

Let $f_i(I_2^3, I_3^3, \dots, I_n^3) \equiv f_i(I_2^3 \rightarrow I_3^3, \dots, I_n^3)$ be the fragmentation vertex of particle $a_2(I_2, I_2^3)$ with isospin I_2 and third component of isospin I_2^3 , into particles $a_3(I_3, I_3^3), \dots, a_n(I_n, I_n^3)$. We shall consider the trajectories $i = P, f, \omega, \rho$ and A_2 . From isospin invariance and the relation ⁷⁾ $(I_\ell I_k I_n, -I_\ell^3, -I_k^3, -I_n^3) = (-1)^{I_\ell + I_k - I_n} (I_\ell I_k I_n, I_\ell^3 I_k^3 I_n^3)$, where $(I_\ell I_k I_n, I_\ell^3 I_k^3 I_n^3)$ is a Clebsch-Gordan coefficient, we get

$$f_i(-I_2^3, -I_3^3, \dots, -I_n^3) = (-1)^I f_i(I_2^3, I_3^3, \dots, I_n^3) \quad (2.1)$$

where $I = 0$ for $i = P, f, \omega$ and $I = 1$ for $i = \rho, A_2$. Let us consider an isospin equality of inclusive cross-sections in the fragmentation region of one of the particles of the initial state, and let us express each cross-section in the Mueller-Regge form. Using Eq. (2.1), we find that in most cases the contribution of the isovector trajectories ρ and A_2 cancel out in the isospin equalities, leaving a sum of contributions coming from $i = P, f$ and ω . By taking the limit $s \rightarrow \infty$ the contribution of the Pomeron trajectory can be easily separated giving a relation of the vertices f_P . Identical relations for f_f and f_ω are obtained by changing the particle of the initial state which is not in the fragmentation vertex, i.e., the particle $a_1(I_1, I_1^3)$. For example, if this particle is an isosinglet, the relation for the f_f 's can be obtained by taking $a_1(0,0) = \eta(549)$, in which case only P and f are present, and using the relation for the f_P 's already found. Then by taking $a_1(0,0) = \Lambda(1115)$, in which case the ω trajectory is also present, an identical relation for the f_ω 's is derived. A similar approach is applied in all other cases. For example, if $a_1(I_1, I_1^3) = a_1(\frac{1}{2}, I_1^3)$ we may assume that a_1 is first a nucleon and then an anti-nucleon, and use the relations $\gamma_f(p) = \gamma_f(n) = \gamma_f(\bar{p}) = \gamma_f(\bar{n})$, $\gamma_\omega(p) = \gamma_\omega(n) = -\gamma_\omega(\bar{p}) = -\gamma_\omega(\bar{n})$, $\gamma_\rho(p) = -\gamma_\rho(n) = -\gamma_\rho(\bar{p}) = \gamma_\rho(\bar{n})$, $\gamma_{A_2}(p) = -\gamma_{A_2}(n) = \gamma_{A_2}(\bar{p}) = -\gamma_{A_2}(\bar{n})$ of the couplings of these particles to the various trajectories.

In some cases, the contribution of the trajectories ρ and A_2 does not cancel out in the isospin equalities. However, combinations of isospin equalities lead to equations involving only a sum of contributions of the trajectories P, f, ω or the trajectories ρ, A_2 . This was found in several cases we checked. Identical relations involving the f_P 's or the f_{A_2} 's are obtained as before. The couplings of the $I=1$ trajectories to particles $a_1(I_1, I_1^3)$ are not as simple as in the case of $I=0$, for example $\gamma_i(\frac{3}{2}) = \sqrt{3} \gamma_i(\frac{1}{2})$ for $i = \rho, A_2$, etc.

We observe the following :

- a) the isospin relations of fragmentation vertices $f_i(I_2^3, I_3^3, \dots, I_n^3)$ for $i = P, f, \omega$ are identical to the isospin relations of inclusive cross-sections $\sigma(0, I_2^3, I_3^3, \dots, I_n^3)$ of processes $0 + I_2 \rightarrow I_3 + \dots + I_n$, in the notation of Ref. 1), in which $\sigma(I_1^3, I_2^3, I_3^3, \dots, I_n^3) \equiv \sigma(I_1^3, I_2^3 \rightarrow I_3^3, \dots, I_n^3)$, and are obtained from them by the replacement $\sigma(0, I_2^3, I_3^3, \dots, I_n^3) \rightarrow f_i(I_2^3, I_3^3, \dots, I_n^3)$. Therefore ¹⁾ they are also obtained from the isospin relations of the cross-sections of processes involving the isospins $I_2, I_3, \dots, I_n$ by the replacement $\sigma(I_2^3, -I_3^3, I_4^3, \dots, I_n^3) \rightarrow f_i(I_2^3, I_3^3, I_4^3, \dots, I_n^3)$.

- b) if in a relation of the vertices $f_i(I_2^3, I_3^3, \dots, I_n^3)$ we make the replacement $I_2^3 \rightarrow -I_k^3$, $I_k^3 \rightarrow -I_2^3$, we obtain the corresponding relation for the fragmentation of particle with isospin I_k into particles with isospins $I_3, \dots, I_{k-1}, I_2, I_{k+1}, \dots, I_n$. In the vertices f_i an arbitrary number of isosinglets may be added without affecting their equalities. Also, we may have $I_k = 0$.
- c) the isospin relations among the fragmentation and the pionization vertices are independent of any exoticity criterion of inclusive reactions ⁸⁾.
- d) at very high rapidity differences between one of the particles of the initial state, of arbitrary isospin I , and the observed particles of the final state, the isospin relations among the inclusive cross-sections become identical to the isospin relations among the corresponding f_P 's, since in such cases only the Pomeron contributes. Therefore the isospin relations of the f_P 's may be used to get such equations. For example, we get in the notation of Ref. 1) with the above restrictions

$$\sigma(I_1^3, \frac{1}{2}, 1) + \sigma(I_2^3, \frac{1}{2}, -1) = 2 \sigma(I_3^3, \frac{1}{2}, 0) \quad (2.2)$$

where I_1^3, I_2^3 and I_3^3 are arbitrarily chosen third components of I , and 0 is the vanishing third component of the isospin 1. Equations like the above one, which for $I = 0$ become exact isospin relations, can be used in some cases to give an estimate of cross-sections, for example, the cross-section $\sigma(I_{3/2}^3 \rightarrow 0, X)$, in terms of other cross-sections which are measured much easier.

Observations a) and b) allow one to write down the isospin relations among the f_i 's, $i = P, f, w$ directly from the isospin relations of the inclusive cross-sections. For example, we get

$$f_i(1 \rightarrow \frac{1}{2}) + f_i(-1 \rightarrow \frac{1}{2}) = 2 f_i(0 \rightarrow \frac{1}{2}) \quad (2.3)$$

$$f_i(\frac{1}{2} \rightarrow 1) + f_i(\frac{1}{2} \rightarrow -1) = 2 f_i(\frac{1}{2} \rightarrow 0) \quad (2.4)$$

$$f_i(0 \rightarrow \frac{1}{2}, 1) + f_i(0 \rightarrow \frac{1}{2}, -1) = 2f_i(0 \rightarrow \frac{1}{2}, 0) \quad (2.5)$$

which in our compact notation can be written as

$$f_i(\frac{1}{2}, 1) + f_i(\frac{1}{2}, -1) = 2f_i(\frac{1}{2}, 0) \quad (2.6)$$

Equation (2.4) was obtained before ⁹⁾ for $i = P$, and under more restrictive assumptions for $i = f$. From the isospin equalities of the cross-sections only a few relations of the vertices f_i , $i = P, A_2$ can be obtained. We list some of them

$$f_i(\frac{3}{2}, 0) = 3f_i(\frac{1}{2}, 0) \quad (2.7)$$

$$f_i(\frac{1}{2}, \frac{3}{2}) + 3f_i(\frac{1}{2}, -\frac{1}{2}) = f_i(\frac{1}{2}, -\frac{3}{2}) + 3f_i(\frac{1}{2}, \frac{1}{2}) \quad (2.8)$$

$$f_i(1, \frac{3}{2}) + f_i(0, \frac{3}{2}) + f_i(-1, \frac{3}{2}) = 3f_i(1, \frac{1}{2}) + 3f_i(0, \frac{1}{2}) + 3f_i(-1, \frac{1}{2}) \quad (2.9)$$

which can be derived from the inclusive processes involving the isospins $(\frac{1}{2}, \frac{3}{2})$, $(\frac{1}{2}, \frac{1}{2}, \frac{3}{2})$ and $(\frac{1}{2}, 1, \frac{3}{2})$, respectively. In Eqs. (2.3)-(2.9), 0, 0 and $\pm\frac{1}{2}$ are third components of the isospins 0, 1 and $\frac{3}{2}$, respectively.

Isospin relations among the pionization vertices $f_{ij}(I_3^3, I_4^3, \dots)$, $i, j = P, f, \omega, \rho$ and A_2 , can be derived similarly from the isospin equalities of inclusive cross-sections and the relation

$$f_{ij}(-I_3^3, -I_4^3, \dots) = (-1)^{I_i + I_j} f_{ij}(I_3^3, I_4^3, \dots) \quad (2.10)$$

where $I_i, I_j = 0$ for $i, j = P, f, \omega$ and $I_i, I_j = 1$ for $i, j = \rho, A_2$. If $i, j = P, f, \omega$ the relations among the vertices $f_{ij}(I_3^3, I_4^3, \dots)$ are identical to the isospin relations among the inclusive cross-sections $\sigma(-I_3^3, -I_4^3, I_5^3, \dots)$ in the notation of Ref. 1), and can be obtained from them by the replacement $\sigma(-I_3^3, -I_4^3, I_5^3, \dots) \rightarrow f_{ij}(I_3^3, I_4^3, \dots)$. A few relations of the vertices $f_{ij}(I_3^3, I_4^3, \dots)$ for $i = \rho, A_2$ and $j = P, f, \omega$ or vice versa are also obtained. Some examples of the last case are the following :

$$f_{ij}(\frac{3}{2}) = 3f_{ij}(\frac{1}{2}) \quad (2.11)$$

$$f_{ij}\left(\frac{1}{2}, \frac{3}{2}\right) + 3f_{ij}\left(\frac{1}{2}, -\frac{1}{2}\right) = f_{ij}\left(\frac{1}{2}, -\frac{3}{2}\right) + 3f_{ij}\left(\frac{1}{2}, \frac{1}{2}\right) \quad (2.12)$$

$$f_{ij}\left(1, \frac{3}{2}\right) + f_{ij}\left(0, \frac{3}{2}\right) + f_{ij}\left(-1, \frac{3}{2}\right) = 3f_{ij}\left(1, \frac{1}{2}\right) + 3f_{ij}\left(0, \frac{1}{2}\right) + 3f_{ij}\left(-1, \frac{1}{2}\right) \quad (2.13)$$

3. ISOSPIN RELATIONS OF TWO PARTICLE CORRELATION FUNCTIONS

In this section we shall use some results of the previous section to derive isospin equalities among two particle correlation functions. The relations among the correlation functions for $I \neq 0$, where I is specified below, are consequences of factorization and are valid in certain kinematic regions. If, however, $I = 0$, these relations become exact, i.e., they are consequences of isospin invariance only, and hold in all kinematic regions. In this case no factorization is employed in their derivation.

Consider inclusive reactions of the type $I + \frac{1}{2} \rightarrow 1 + X$, and $I + \frac{1}{2} \rightarrow \frac{1}{2} + 1 + X$, with I arbitrary. For such processes, we get from Eq. (2.2) and a similar one for the processes $I + \frac{1}{2} \rightarrow \frac{1}{2} + 1 + X$

$$\sigma\left(I^3, \frac{1}{2} \rightarrow 1, X\right) + \sigma\left(I^3, \frac{1}{2} \rightarrow -1, X\right) = 2\sigma\left(I^3, \frac{1}{2} \rightarrow 0, X\right) \quad (3.1)$$

$$\begin{aligned} &\sigma\left(I^3, \frac{1}{2} \rightarrow \frac{1}{2}, 1, X\right) + \sigma\left(I^3, \frac{1}{2} \rightarrow -\frac{1}{2}, 1, X\right) + \sigma\left(I^3, \frac{1}{2} \rightarrow \frac{1}{2}, -1, X\right) + \\ &+ \sigma\left(I^3, \frac{1}{2} \rightarrow -\frac{1}{2}, -1, X\right) = 2\sigma\left(I^3, \frac{1}{2} \rightarrow \frac{1}{2}, 0, X\right) + 2\sigma\left(I^3, \frac{1}{2} \rightarrow -\frac{1}{2}, 0, X\right) \end{aligned} \quad (3.2)$$

The correlation function $C(a_1 a_2 \rightarrow a_3 a_4 X)$ is defined by ¹⁰⁾

$$C(a_1 a_2 \rightarrow a_3 a_4 X) = \frac{1}{\sigma} \sigma(a_1 a_2 \rightarrow a_3 a_4 X) - \frac{1}{\sigma^2} \sigma(a_1 a_2 \rightarrow a_3 X) \sigma(a_1 a_2 \rightarrow a_4 X) \quad (3.3)$$

As normalizing cross-section σ , one can choose ⁶⁾ the total cross-section σ_T for the process $a_1 + a_2 \rightarrow X$, as is usually done in the inclusive Regge model, or the inelastic cross-section for this process, etc. Of course σ depends only on the particles of the initial state. Introducing correlation functions in Eq. (3.2) by means of Eq. (3.3) and using Eq. (3.1), we get

$$C(I^3, \frac{1}{2} \rightarrow \frac{1}{2}, 1, X) + C(I^3, \frac{1}{2} \rightarrow -\frac{1}{2}, 1, X) + C(I^3, \frac{1}{2} \rightarrow \frac{1}{2}, -1, X) + \\ + C(I^3, \frac{1}{2} \rightarrow -\frac{1}{2}, -1, X) = 2C(I^3, \frac{1}{2} \rightarrow \frac{1}{2}, 0, X) + 2C(I^3, \frac{1}{2} \rightarrow -\frac{1}{2}, 0, X) \quad (3.4)$$

For $I \neq 0$, Eqs. (3.1), (3.2) and therefore (3.4) follow from isospin invariance and factorization, if the difference between the rapidity of the particle with isospin I and the rapidity of each of the observed particles of the final state is very large. This implies the limit $s \rightarrow \infty$. The rapidities of the particles with isospins $\frac{1}{2}$, $\frac{1}{2}$ and 1 are arbitrary otherwise. Equation (3.4) is similar to the isospin relation among the two particle fragmentation vertices involving the same combination of isospins, and corresponding to $i = P$. The same observation applies to all subsequent equations of the same kind.

When $I = 0$, Eq. (3.4), like Eqs. (3.1) and (3.2), becomes exact and holds in all kinematic regions. It is derived without introducing factorization. Reactions of the type $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 1 + X$ are accessible experimentally, for example with deuteron or α particle targets.

The initial states of all reactions whose cross-sections are connected by Eq. (3.2) are the same. In the following, to derive the relations of correlation functions, we shall also use isospin equalities involving reactions with different initial states, namely with the states $a_1(I, I^3) + a_2(1, 1)$ and $a_1(I, I^3) + a_2(1, 0)$. In such cases after the introduction of correlation functions in the equalities, the terms which are products of two single particle inclusive cross-sections do not cancel out, unless the normalizing cross-sections are the same for both initial states. In these cases, we shall take $\sigma = \sigma_T$, and we shall assume factorization. Then we get in the limit $s \rightarrow \infty$

$$\sigma_T(I^3, 1) = \sigma_T(I^3, 0) \quad (3.5)$$

However, for $I = 0$ the above relation is a consequence of isospin invariance only, i.e., it is independent of factorization and the limit $s \rightarrow \infty$. Therefore, the isospin relations of correlation functions for different initial states are exact if $I = 0$.

For the processes $I+1 \rightarrow \frac{1}{2} + X$ and $I+1 \rightarrow \frac{1}{2} + \frac{1}{2} + X$ we get from Eq. (2.2) and a similar one for the processes $I+1 \rightarrow \frac{1}{2} + \frac{1}{2} + X$

$$\sigma(I^3, 1 \rightarrow \frac{1}{2}, X) + \sigma(I^3, 1 \rightarrow -\frac{1}{2}, X) = 2\sigma(I^3, 0 \rightarrow \frac{1}{2}, X) \quad (3.6)$$

$$\begin{aligned} & \sigma(I^3, 1 \rightarrow \frac{1}{2}, \frac{1}{2}, X) + \sigma(I^3, 1 \rightarrow \frac{1}{2}, -\frac{1}{2}, X) + \sigma(I^3, 1 \rightarrow -\frac{1}{2}, -\frac{1}{2}, X) + \\ & + \sigma(I^3, 1 \rightarrow -\frac{1}{2}, \frac{1}{2}, X) = 2\sigma(I^3, 0 \rightarrow \frac{1}{2}, \frac{1}{2}, X) + 2\sigma(I^3, 0 \rightarrow \frac{1}{2}, -\frac{1}{2}, X) \end{aligned} \quad (3.7)$$

From Eqs. (3.3), (3.5), (3.7) and the relation

$$\sigma(I^3, 0 \rightarrow \frac{1}{2}, X) = \sigma(I^3, 0 \rightarrow -\frac{1}{2}, X) \quad (3.8)$$

which is valid in our kinematic region, we obtain

$$\begin{aligned} & C(I^3, 1 \rightarrow \frac{1}{2}, \frac{1}{2}, X) + C(I^3, 1 \rightarrow \frac{1}{2}, -\frac{1}{2}, X) + C(I^3, 1 \rightarrow -\frac{1}{2}, -\frac{1}{2}, X) + \\ & + C(I^3, 1 \rightarrow -\frac{1}{2}, \frac{1}{2}, X) = 2C(I^3, 0 \rightarrow \frac{1}{2}, \frac{1}{2}, X) + 2C(I^3, 0 \rightarrow \frac{1}{2}, -\frac{1}{2}, X) \end{aligned} \quad (3.9)$$

Finally, in analogy with Eq. (2.2), we get for the processes $I + \frac{1}{2} \rightarrow 1 + 1 + X$

$$\begin{aligned} & \sigma(I^3, \frac{1}{2} \rightarrow 1, 1, X) + \sigma(I^3, \frac{1}{2} \rightarrow 1, -1, X) + \sigma(I^3, \frac{1}{2} \rightarrow -1, -1, X) + \sigma(I^3, \frac{1}{2} \rightarrow -1, 1, X) = \\ & = \sigma(I^3, \frac{1}{2} \rightarrow 0, 1, X) + \sigma(I^3, \frac{1}{2} \rightarrow 0, -1, X) + 2\sigma(I^3, \frac{1}{2} \rightarrow 0, 0, X) \end{aligned} \quad (3.10)$$

$$\sigma(I^3, \frac{1}{2} \rightarrow 1, 0, X) + \sigma(I^3, \frac{1}{2} \rightarrow -1, 0, X) = \sigma(I^3, \frac{1}{2} \rightarrow 0, 1, X) + \sigma(I^3, \frac{1}{2} \rightarrow 0, -1, X) \quad (3.11)$$

from which we get, using Eqs. (3.1) and (3.3)

$$\begin{aligned} & C(I^3, \frac{1}{2} \rightarrow 1, 1, X) + C(I^3, \frac{1}{2} \rightarrow 1, -1, X) + C(I^3, \frac{1}{2} \rightarrow -1, -1, X) + C(I^3, \frac{1}{2} \rightarrow -1, 1, X) = \\ & = C(I^3, \frac{1}{2} \rightarrow 0, 1, X) + C(I^3, \frac{1}{2} \rightarrow 0, -1, X) + 2C(I^3, \frac{1}{2} \rightarrow 0, 0, X) \end{aligned} \quad (3.12)$$

$$C(I^3, \frac{1}{2} \rightarrow 1, 0, X) + C(I^3, \frac{1}{2} \rightarrow -1, 0, X) = C(I^3, \frac{1}{2} \rightarrow 0, 1, X) + C(I^3, \frac{1}{2} \rightarrow 0, -1, X) \quad (3.13)$$

Equations (3.9), (3.12) and (3.13) are valid in the same kinematic region with Eq. (3.4). For $I = 0$ they become exact isospin relations.

Equation (3.12), which refers to correlations between two isospin 1 particles, for example between two pions, is particularly interesting. It tells that the charged-charged correlation functions are equal to the charged-neutral plus twice the neutral-neutral correlation functions.

Proceeding as before, we can write isospin equalities of correlation functions of various processes. In Tables I and II we give such equalities for the processes $I+1 \rightarrow \frac{1}{2}+1+X$ and $I+1 \rightarrow 1+1+X$. These expressions are valid in the same kinematic region with Eqs. (3.4), (3.9), (3.12) and (3.13), and become exact if $I = 0$.

It should be emphasized that in all previous relations of correlation functions, the only restriction of rapidities, in the case $I \neq 0$, is that the difference between the rapidity of the particle with isospin I and the rapidity of each of the other three particles is very large. The rapidities of the other three particles are otherwise arbitrary. For example, one of the produced particles may be in the pionization region and the other in the fragmentation region.

The correlation functions in the case of fragmentation of one particle into two or more fragments are interesting in the framework of fragmentation models¹¹⁾⁻¹³⁾. Also recently measurements of two particle correlation functions in high energy proton-proton collisions at the CERN ISR, with one particle in the pionization region and the other a proton in the fragmentation region, have been reported¹⁴⁾.

We shall also give isospin relations of two particle correlation functions if both produced particles a_3 and a_4 are in the pionization region, with very large rapidity difference from both particles of the initial state, so that only the Pomeron contributes in the Mueller diagram. In this case if we introduce correlation functions $C(I_3^3, I_4^3)$ into the isospin equalities, we find that the contributions of the terms $\sigma(a_1 a_2 \rightarrow a_3 X) \sigma(a_1 a_2 \rightarrow a_4 X)$ cancel out, and the relations among the functions $C(I_3, I_4)$ become identical to the isospin relations among inclusive cross-sections involving the isospins I_3 and I_4 . In this way for $I_3 = \frac{1}{2}$ and $I_4 = 1$ we find the relation.

$$C\left(\frac{1}{2}, 1\right) + C\left(\frac{1}{2}, -1\right) = C\left(\frac{1}{2}, 0\right) + C\left(-\frac{1}{2}, 0\right) = 2 C\left(\frac{1}{2}, 0\right) \quad (3.14)$$

which expresses the charged-neutral and neutral-neutral correlation functions in terms of charged-charged correlation functions. Also we find that the correlation functions of two $I = 1$ particles satisfy the relations

$$C(1,1) + C(1,-1) = C(1,0) + C(0,0) \quad (3.15)$$

$$C(1,0) = C(0,1) \quad (3.16)$$

the correlation functions of an $I = \frac{1}{2}$ and an $I = \frac{3}{2}$ particle satisfy the relations

$$C\left(\frac{1}{2}, \frac{3}{2}\right) + C\left(\frac{1}{2}, -\frac{1}{2}\right) = 2C\left(\frac{1}{2}, \frac{1}{2}\right) \quad (3.17)$$

$$C\left(\frac{1}{2}, -\frac{3}{2}\right) + C\left(\frac{1}{2}, \frac{1}{2}\right) = 2C\left(\frac{1}{2}, -\frac{1}{2}\right) \quad (3.18)$$

where the meaning of $\frac{1}{2}'$ is explained after Eq. (2.9), and the correlation functions of an $I = 1$ and an $I = \frac{3}{2}$ particle satisfy the relations

$$C\left(1, \frac{3}{2}\right) + C\left(1, \frac{1}{2}\right) + C\left(-1, \frac{1}{2}\right) + C\left(-1, \frac{3}{2}\right) = 2C\left(0, \frac{3}{2}\right) + 2C\left(0, \frac{1}{2}\right) \quad (3.19)$$

$$C\left(1, \frac{3}{2}\right) + C\left(0, \frac{3}{2}\right) + C\left(-1, \frac{3}{2}\right) = C\left(1, \frac{1}{2}\right) + C\left(0, \frac{1}{2}\right) + C\left(-1, \frac{1}{2}\right) \quad (3.20)$$

$$C\left(1, \frac{3}{2}\right) + 3C\left(-1, \frac{1}{2}\right) = C\left(-1, \frac{3}{2}\right) + 3C\left(1, \frac{1}{2}\right) \quad (3.21)$$

Of course, we have $C(-I_3^3, -I_4^3) = C(I_3^3, I_4^3)$.

Equations (3.14)-(3.21) are valid for any difference between the rapidities y_3 and y_4 of the particles in the pionization region. They contain both the short-range part and the long-range part of the correlation functions. If $|y_3 - y_4|$ is large, we may assume that the poles P, f, ω, ρ and A_2 are exchanged between a_3 and a_4 , and we find that all above relations are satisfied. Therefore they may be interpreted also as relations of the short-range part of correlation functions, with the correlation length $\ell = 2$. Measurements of two particle correlation functions with both particles in the pionization region have been reported ¹⁵⁾.

Isospin relations of correlation functions of two particles with isospins higher than those considered before, or of three or more particles, can be derived in a similar fashion.

4. ISOSPIN EQUALITIES AT FINITE RAPIDITY DIFFERENCES

The relations of the fragmentation and the pionization vertices can be used to derive equalities of inclusive cross-sections, valid at finite rapidity differences. Such relations are valid for arbitrary isospin I of one of the particles of the initial state and for some small values of I they coincide with exact isospin equalities. We shall not investigate this approach systematically but we shall only give an example to illustrate it.

From Eqs. (2.1) and (2.4) we obtain

$$\begin{aligned} \sigma(I^3, \frac{1}{2} \rightarrow 1, X) + \sigma(I^3, -\frac{1}{2} \rightarrow -1, X) + \sigma(I^3, \frac{1}{2} \rightarrow -1, X) + \sigma(I^3, -\frac{1}{2} \rightarrow 1, X) = \\ = 2\sigma(I^3, \frac{1}{2} \rightarrow 0, X) + 2\sigma(I^3, -\frac{1}{2} \rightarrow 0, X) \end{aligned} \quad (4.1)$$

which is valid for arbitrary I^3 in the fragmentation region of the particle with isospin $\frac{1}{2}$. The contribution of the trajectories ρ and A_2 cancels out on both sides of the above equation, which for $I = 0$ or $\frac{1}{2}$ becomes identical to the exact isospin relation. To show the usefulness of this equation, assume that $I = I^3 = 1$. Then it can be written as follows :

$$\begin{aligned} \sigma(1, \frac{1}{2} \rightarrow 1, X) + \sigma(-1, \frac{1}{2} \rightarrow 1, X) + \sigma(1, \frac{1}{2} \rightarrow -1, X) + \sigma(-1, \frac{1}{2} \rightarrow -1, X) = \\ = 2\sigma(1, \frac{1}{2} \rightarrow 0, X) + 2\sigma(-1, \frac{1}{2} \rightarrow 0, X) \end{aligned} \quad (4.2)$$

All cross-sections on the left-hand side of Eq. (4.2) refer to reactions involving only charged particles. Therefore one can get a reasonable estimate of the cross-sections on the right-hand side, which refer to reactions with a neutral observed particle in the final state. The exact isospin relation of the above case is not as useful, because it has on its right-hand side, besides the terms $\sigma(1, \frac{1}{2} \rightarrow 0, X)$ and $\sigma(-1, \frac{1}{2} \rightarrow 0, X)$, the term $\sigma(0, \frac{1}{2} \rightarrow 0, X)$, which is much less accessible experimentally.

In a similar way, several useful equalities can be obtained for other cases.

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TABLE I : Isospin relations of correlation functions of processes $I+1 \rightarrow \frac{1}{2}+1+X$. For $I = 0$ these relations are exact, and hold in all kinematic regions. For $I \neq 0$ they are valid if the difference of rapidities of particle with isospin I and the produced particles is very large.

$$\begin{aligned} C(I^3, 1 \rightarrow \frac{1}{2}, 1, X) + C(I^3, 1 \rightarrow -\frac{1}{2}, -1, X) + C(I^3, 1 \rightarrow -\frac{1}{2}, 1, X) + C(I^3, 1 \rightarrow \frac{1}{2}, -1, X) = \\ = C(I^3, 1 \rightarrow \frac{1}{2}, 0, X) + C(I^3, 1 \rightarrow -\frac{1}{2}, 0, X) + 2 C(I^3, 0 \rightarrow \frac{1}{2}, 0, X) \end{aligned}$$

$$C(I^3, 1 \rightarrow \frac{1}{2}, 0, X) + C(I^3, 1 \rightarrow -\frac{1}{2}, 0, X) = C(I^3, 0 \rightarrow \frac{1}{2}, 1, X) + C(I^3, 0 \rightarrow \frac{1}{2}, -1, X)$$

TABLE II : Isospin relations of correlation functions of processes $I+1 \rightarrow 1+1+X$. For $I = 0$ these relations are exact and hold in all kinematic regions. For $I \neq 0$ they are valid if the difference of rapidities of particle with isospin I and the produced particles is very large.

$$\begin{aligned} C(I^3, 1 \rightarrow 1, 1, X) + C(I^3, 1 \rightarrow 1, -1, X) + C(I^3, 1 \rightarrow -1, 1, X) + C(I^3, 1 \rightarrow -1, -1, X) = \\ = C(I^3, 1 \rightarrow 0, 0, X) + C(I^3, 0 \rightarrow 1, 0, X) + C(I^3, 0 \rightarrow 0, 1, X) + C(I^3, 0 \rightarrow 0, 0, X) \end{aligned}$$

$$\begin{aligned} C(I^3, 1 \rightarrow 0, 1, X) + C(I^3, 1 \rightarrow 0, -1, X) + C(I^3, 1 \rightarrow 0, 0, X) = C(I^3, 0 \rightarrow 1, 1, X) + \\ + C(I^3, 0 \rightarrow 1, -1, X) + C(I^3, 0 \rightarrow 1, 0, X) \end{aligned}$$

$$\begin{aligned} C(I^3, 1 \rightarrow 1, 0, X) + C(I^3, 1 \rightarrow -1, 0, X) + C(I^3, 1 \rightarrow 0, 0, X) = C(I^3, 0 \rightarrow 1, 1, X) + \\ + C(I^3, 0 \rightarrow -1, 1, X) + C(I^3, 0 \rightarrow 0, 1, X) \end{aligned}$$

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