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RESCATTERING CORRECTIONS AND THE TRIPLE-POMERON COUPLING

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A B S T R A C T

The asymptotic consistency of an $\alpha = 1$ Pomeron pole with a non-vanishing triple-Pomeron coupling $g_p(0)$ is re-examined in connection with absorptive corrections to diffractive production. We present an absorptive multiperipheral model in which elastic rescattering among pairs of particles produced in the final state is included. In this way we calculate the contribution of enhanced many-Pomeron cuts to the inclusive distribution $d\sigma/dx$ close to $x = 1$, and show that the over-all contribution integrates to a factorized constant. We conclude on this basis that s channel iteration modifies in an important way t channel arguments and could possibly explain why $\alpha = 1$ asymptotically.

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1. - INTRODUCTION

Experimental results on pp total and differential cross-sections ¹⁾ in the ISR energy range have made definitely clear that even at these energies, the Pomeron is not simply a Regge pole at $\alpha(0) = 1$. The question then arises whether the Pomeron as a Regge pole is still a useful asymptotic concept or whether the cuts ^{2),3)} associated to it by s channel unitarity completely change the picture.

There have been various answers to this question in connection with the value of the coupling $g_P(0)$ of three Pomeron poles of zero mass. In the Gribov calculus ⁴⁾, in which $g_P(0) = 0$, the leading singularity is given by a pole plus soft cut, with a cut hierarchy (two-Pomeron cut more important than three-Pomeron cut, and so on). The same picture emerges also from two-Pomeron t channel unitarity arguments ⁵⁾. From another point of view, the condition $g_P(0) = 0$ seems to be forced by the consistency of the Pomeron pole dominance of inclusive distributions with that of elastic scattering. In fact, if $d\sigma/dx$ is at $x \approx 1$ dominated by the triple Pomeron coupling $g_P(t)$ (Fig. 1), from momentum conservation sum rules one obtains that $\sigma_{6\text{tot}} \sim (g_P(0)/\alpha') \ln \ln s$, and Pomeron pole dominance fails, unless $g_P(0) = 0$.

There are, however, some reasons why we ought to consider the possibility that $g_P(0) \neq 0$ also. Firstly, $g_P(t)$ has been found non-vanishing up to small t values ⁷⁾ ($|t| \lesssim 0.1$), with no clear-cut determination at smaller values ⁸⁾. Secondly, starting from $g_P(0) = 0$ a chain of decoupling arguments has been put forward ⁹⁾, that could end with the Pomeron decoupling from elastic scattering. Although some of the elements of the chain might be missing ¹⁰⁾, it seems desirable to exhibit models which avoid it. In this context it has been proposed ^{11),12)} that a t channel iteration with $g_P(0) \neq 0$ is the dynamical mechanism for an effective Pomeron pole with $\alpha(0) > 1$, which gives a good phenomenological description of pp total and differential cross-sections at ISR energies. In this model, however, no pole solution emerges at the end, and one has, by initial state absorption, a close saturation of the Froissart bound, and strong non-factorization at infinite energies ¹³⁾. In other models ¹⁴⁾ in which s channel iteration is predominant, the assumption of $g_P(0) \neq 0$ emphasizes absorption so much that one can end up with decreasing and perhaps asymptotically vanishing cross-sections.

After this summary of the situation, it seems inescapable to conclude that either $g_p(0) = 0$, or the Pomeron as a Regge pole can be a useful concept only at an iterative level ¹¹⁾, not asymptotically. However, the purpose of this paper is to show that an asymptotic pole dominance is not obviously ruled out by letting $g_p(0) \neq 0$. The idea is that in this case the s channel iteration is important and one cannot rely on perturbative cut calculations.

Let us in fact start with an analysis of the inclusive single particle distribution in the triple-Regge region ($x \approx 1$). The related six-point function contains, besides the triple-Pomeron graph (Fig. 1a) and its vertex or propagator modifications, the class of graphs depicted in Fig. 1b. These are usually neglected in the decoupling arguments, but are in fact negligible only if $g_p(0) = 0$. If $g_p(0) \neq 0$, they contribute to $d\sigma/dx$ singularities of the form

$$c_N \left| (1-x) \ln(1-x) \right|^{-1} \left(\frac{g_p(0)}{\alpha'} \ln \left| \ln(1-x) \right| \right)^N \quad (N = 1, 2, \dots)$$

which correspond to the behaviour $(\ln \ln s)^{N+1}$ of σ_{tot} . Our first point is therefore that if $g_p(0) \neq 0$, all these graphs (not only the triple-Pomeron) are important and have to be summed with the appropriate signs, before reaching any definite conclusion.

We then provide an explicit model in which the sum of a class of enhanced many-Pomeron graphs can be performed. Our calculation shows that the nasty $\ln \ln s$ behaviour is cancelled by the many-Pomeron cut contributions, and that $d\sigma/dx$ integrates to a constant. We then envisage the possibility that the actual asymptotic solution is still of the pole plus cut type.

Our model is basically a multiperipheral model with absorptive corrections in the final and initial states. We classify the final states according to the fireball expansion of Abarbanel, Chew and co-workers ¹⁵⁾. The basic production amplitudes are given by a chain of "fireballs" in which Pomeron poles, inelastically coupled, are exchanged. The following specifications characterize our model.

- 1) The fireball itself (events with no large rapidity gaps) builds the "inelastic" Pomeron in the absorptive part of the amplitude. Due to the success of the two-component model ¹⁶⁾, we assume that it can be fully represented by an effective multiperipheral ladder providing an output pole with $\alpha(0) = 1$. We therefore neglect long range effects (in particular, absorptive corrections) at the single fireball level.
- 2) The absorptive corrections to the basic production amplitudes are assumed to come from elastic rescattering of pairs of particles separated by large rapidity gaps, i.e., belonging to different fireballs. For instance, to first order in the inelastic coupling of the Pomeron we have singly diffractive events (Fig. 2). The leading particle can rescatter with any particle in the missing mass, and the rescattering correction is the product of S matrices in impact parameter space, according to the eikonal prescription ^{17),18)}.

The singly diffractive case is presented in detail in Section 2, where we show that it can be treated by means of an integral equation in impact parameter, rapidity space. The form of the solution is given in Appendix A, up to the leading inverse power of $\alpha' \ln s$. A simple but physically relevant approximation is presented in Section 3. Here we give the form of $d\sigma/dx$ and σ_{tot} , which shows the cancellation mentioned before. We also generalize this treatment (average rescattering approximation) to the next term in the fireball expansion (di-triple Pomeron graph). The results obtained and possible future developments are discussed in the last section.

2. - ABSORPTIVE MULTIPERIPHERAL MODEL FOR DIFFRACTIVE PRODUCTION

A) - Definition of the model

Consider the process of single diffractive production of a large mass M , which for a given multiplicity is represented by :

$$A + B \rightarrow A + C_0 + C_1 + \dots + C_n, \quad (2.1)$$

where the particles C_i belong to the mass M state. To a first approximation this process is described by an amplitude T_n^0 in which a Pomeron pole with intercept at one and slope α' is exchanged as represented in Fig. 2. The amplitude T_n of the process (2.1) is obtained from the first approximation T_n^0 by considering rescattering corrections in the final state of particle A with any of particles C_i produced in the mass M state (cf. Introduction).

In this model the amplitude T_n can then be represented as in Fig. 3a, where the spring-like line represents the two-body S matrix. Since the subenergy s_{AC} is large, the S matrix is dominated by Pomeron pole exchange, as in Fig. 3b. These Pomeron poles are elastically exchanged, so that the amplitude considered here is in first order of the Pomeron inelastic coupling. Following an eikonal prescription ¹⁷⁾, the elastic S matrices are multiplicative in the impact parameter and rapidity representation, since the final particles are kept on the mass shell ^{*)}.

In this representation the amplitude T_n^0 is a function of the set of variables $v_i \equiv (\underline{b}_i, y_i)$, $i = 0, 1, \dots, n$, where $\underline{b}_i$ is the impact parameter of particle C_i conjugate to its transverse momentum $\underline{p}_i$; we choose the frame in which the produced particle A has $v_A = (\underline{b}_A, y_A) = (\underline{0}, 0)$. The amplitude T_n of Fig. 2 can then be written as

$$T_n(v_0, v_1, \dots, v_n) = T_n^0(v_0, v_1, \dots, v_n) \prod_{i=0}^n S_{AC}(v_i), \quad (2.2)$$

^{*)} This prescription corresponds to the sum - in the eikonal approximation - of generalized ladder exchanges between any couple of particles in the final state ¹⁷⁾ (actually, only A and C_i in this section). Note that in this way, absorption due to the incoming particle rescattering is taken into account in the S matrix $S_{AB}(\underline{B}, Y)$ (not $\sqrt{S_{AB}}$) between leading particles in the final state.

where $S_{AC}(v) = S_{AC}(\underline{b}, y)$ is the scattering matrix in the impact parameter representation of particles A and C with co-ordinate $v_A = (0, 0)$ and $v = (\underline{b}, y)$ respectively, as given in Fig. 3b. We can then write

$$S_{AC}(v) = 1 - (2\pi)^{-1} \beta_A \beta_C F(v) \quad (2.3)$$

where $F(v)$ is the Pomeron propagator function in our representation :

$$F(v) \equiv F(\underline{b}, y) = \frac{1}{y} \exp\left(-\frac{b^2}{y}\right) , \quad (2.4)$$

and $\beta_A, \beta_C \equiv \beta$ are the elastic coupling of the Pomeron with particles A and C. (For simplicity of notation in what follows, we choose units such that $4\alpha' = 1$, and we drop the index C.)

In order to complete the definition of the model, we assume that the amplitude in Fig. 2, related to the "scattering" of the Pomeron with particle B, is fully represented by an effective multiperipheral ladder with an output pole at $\alpha(0) = 1$. This prescription is meant to be a phenomenological one, but should be checked for self-consistency in our theoretical model. We write our multiperipheral ansatz in the form

$$T_n^o(v_0, v_1, \dots, v_n) = \beta_A \left(\frac{g_p(0)}{4\pi}\right)^{1/2} F(v_0) \frac{1}{\beta} \prod_{i=1}^n G^{1/2}(v_i - v_{i-1}) \beta_B^{1/2} \quad (2.5)$$

where $g_p(0)$ is the triple Pomeron coupling at zero masses, and $G(v_i - v_{i-1})$ is the impact parameter representation of our effective kernel for production of particles C that gives the inelastic Pomeron amplitude $\beta^2 F(v)$. The multiperipheral equation in the impact parameter representation becomes ^{*})

$$\beta^2 F(v) = G(v) + \int d^3v' G(v - v') \beta^2 F(v') , \quad (2.6)$$

with $d^3v = (d^2b/\pi)dy$. Equations (2.2), (2.5) and (2.6) fully define the model.

^{*}) This equation is of the convolution type and can be easily diagonalized in the variables J (angular momentum) and $\underline{k}$ (momentum transfer). By assuming the inelastic Pomeron $\alpha_o(t) = 1 + \alpha' t$, we assume $\mathcal{G}_J(\underline{k}) \simeq \beta^2 / (J-1 + \beta^2 + \alpha' k^2)$.

It is worth noting that the inclusive distribution of particle A computed from the first approximation T_n^0 is related to the six-point function discontinuity of Fig. 1a, while the distribution computed from T_n is related to a sum of graphs of the type in Fig. 1b. Note, however, that the discontinuity line in Fig. 1 crosses only the "inelastic" Pomeron (fireball) and that Pomerons in rescattering are elastically coupled. Formally the inclusive distribution of particle A in terms of its Feynman variable x and transverse momentum $\underline{p}_t$ is given by

$$(1-x) \frac{d\sigma}{dx d^2\underline{p}_t} = \sum_{n=1}^{\infty} \int \frac{d^2\underline{B}}{(2\pi)^2} e^{i\underline{B} \cdot \underline{p}_t} \frac{d^2\underline{b}_0}{\pi} \prod_{i=1}^n d^3v_i \delta(y_n - Y) \times T_n(y_0, \underline{b}_0 + \frac{\underline{B}}{2}; \dots; y_n, \underline{b}_n + \frac{\underline{B}}{2}) T_n^*(y_0, \underline{b}_0 - \frac{\underline{B}}{2}; \dots; y_n, \underline{b}_n - \frac{\underline{B}}{2}), \quad (2.7)$$

where $Y \simeq \ln s$, $Y - y_0 \simeq \ln M^2$ and $y_0 \simeq |\ln(1-x)|$. Integrating over the transverse momentum $\underline{p}_t$ one finally has

$$(1-x) \frac{d\sigma}{dx} = \frac{d\sigma}{dy_0} = \sum_{n=1}^{\infty} \int \frac{d^2\underline{b}_0}{\pi} \prod_{i=1}^n d^3v_i \delta(y_n - Y) |T_n(v_0, \dots; v_n)|^2. \quad (2.8)$$

B) - Integral equation

The infinite sum of Eq. (2.8) can be done by introducing the matrix

$$Z_A^{(n)}(v_0, v_n) = \int \prod_{i=1}^{n-1} d^3v_i G(v_i - v_{i-1}) |S_A(v_i)|^2 G(v_n - v_{n-1}), \quad (n \geq 2) \quad (2.9)$$

with $Z_A^{(1)}(v_0, v_1) = G(v_1 - v_0)$. The matrix $Z_A^{(n)}$ is, for a given multiplicity, the distribution in the impact parameter representation of particles C_0 and C_n , when the other produced particles, $C_1, C_2, \dots, C_{n-1}$, are rescattering with a fixed centre A at $v_A = (0, 0)$.

The differential cross-section of Eq. (2.8) is then given by

$$\frac{d\sigma}{dy_0} = \frac{g_P(0) \beta_A^2 \beta_B}{4\pi \beta^2} \int \frac{d^2\underline{b}}{\pi} \frac{d^2\underline{b}_0}{\pi} F^2(v_0) |S_A(v_0)|^2 Z_A(v_0, v) |S_A(v)|^2, \quad (2.10)$$

with $v_0 = (\underline{b}_0, y_0)$ and $v = (\underline{b}, Y)$. $Z_A(v_0, v) = \sum_{n=1}^{\infty} Z_A^{(n)}(v_0, v)$ satisfies [see Eq. (2.9)] the integral equation :

$$Z_A(v_0, v) = G(v-v_0) + \int d^3v' Z_A(v_0, v') |S_A(v')|^2 G(v-v') . \quad (2.11)$$

A graphical representation in momentum space of the corresponding equation for the six-point function is given in Fig. 4. Equation (2.11) can be cast in a more convenient form by noting that G is the effective kernel that produces an output Pomeron pole [see Eq. (2.6)]. By combining the two equations, one has ^{*)} :

$$Z_A(v_0, v) = \beta^2 F(v-v_0) - \beta^2 \int d^3v' Z_A(v_0, v') H_A(v') F(v-v') , \quad (2.12)$$

with

$$H_A(v) = 1 - |S_A(v)|^2 = \frac{\beta_A \beta}{\pi} F(v) - \left(\frac{\beta_A \beta}{2\pi}\right)^2 F^2(v) . \quad (2.13)$$

Equations (2.12) and (2.13) are the formalization of our model and give Z_A in terms only of the function $F(v)$ given in Eq. (2.4). The asymptotic solution for Z_A and the corresponding distribution (2.8) are given in the next section. It may be safe at this point to observe that if the S matrix is set equal to one, the solution is simply $Z_A = \beta^2 F(v-v_0)$ and Eq. (2.8) gives, of course, the triple Pomeron formula :

$$\frac{d\sigma}{dy_0} = \frac{\beta_A^2 \beta_B g_P(0)}{32\pi \alpha'} \frac{1}{y_0} , \quad (2.14)$$

or

$$\frac{d\sigma}{dx} = \frac{\beta_A^2 \beta_B g_P(0)}{32\pi \alpha'} \frac{1}{1-x} \frac{1}{|\ln(1-x)|} .$$

^{*)} Unlike Eq. (2.6), this equation couples t channel angular momenta. It could be rewritten in a form similar to the Gribov calculus, but we prefer to use $v = (\underline{b}, y)$ because in these variables the average rescattering approximation of Section 3 is more transparent.

In the case of the fully differential cross-section of Eq. (2.7), the formulae are rather complicated but we report them for completeness :

$$\frac{d\sigma}{dy_0 d^2p_t} = \frac{\beta_A^2 \beta_B g_P(0)}{4\pi \beta^2} \int \frac{d^2B}{(2\pi)^2} e^{i\frac{B}{\beta} \cdot p_t} \frac{d^2b_0}{\pi} \frac{d^2b}{\pi} F(y_0, \underline{b}_0 + \frac{B}{2}) F(y_0, \underline{b}_0 - \frac{B}{2}) \times S(y_0, \underline{b}_0 + \frac{B}{2}) S^*(y_0, \underline{b}_0 - \frac{B}{2}) Z_A(v_0, v; \underline{B}) S_A(y, \underline{b} + \frac{B}{2}) S_A^*(y, \underline{b} - \frac{B}{2}) \quad (2.15)$$

where $Z_A(v_0, v; \underline{B})$ satisfies Eq. (2.11) with $H_A(v)$ replaced by

$$1 - S_A(y, \underline{b} + \frac{B}{2}) S_A^*(y, \underline{b} - \frac{B}{2}) .$$

Let us finally remark that, if we interpret $\underline{b}$ as a co-ordinate and y as the time, there is a formal analogy of Eq. (2.11) with a non-relativistic two-dimensional scattering off a fixed centre. The free Green's function is just the Pomeron propagator $F(v-v')$, the potential is $H_A(v)$, and the centre is at $v_A = 0$ (see Fig. 5). This analogy is useful for a possible generalization from the case here studied of single diffractive dissociation.

Consider for instance a final state in which the two leading particles are produced in the two opposite hemispheres at $x_A \simeq 1$ and $x_B \simeq -1$ ($x_{A,B}$ are Feynman variables of A and B). The basic amplitude for this process, represented in Fig. 5, has to be corrected for rescattering of particles $C_0, C_1, \dots, C_n$ with both the two leading particles A and B (and with the two leading particles between themselves). The relevant matrix in this case, $Z_{AB}(v_0, v; V)$ (see Appendix B), contains the rescattering of the produced particles in the fireball with two centres at $v_A = (0, 0)$ and $v_B = V = (\underline{b}_B, Y)$. Z_{AB} satisfies again an equation of the form of Eq. (2.12) but with

$$H_{AB}(v; V) = 1 - |S_A(v)|^2 |S_B(V-v)|^2 . \quad (2.16)$$

If one considers the leading term for large y in the potential [i.e., neglect $F^2(v)$ as compared to $F(v)$] one has

$$H_{AB}(v;V) \simeq H_A(v) + H_B(V-v) = \frac{\beta}{2\pi} (\beta_A F(v) + \beta_B F(V-v)) , \quad (2.17)$$

and Eq. (2.12) becomes analogous to the equation of scattering off two fixed centres at $v_A = 0$ and $v_B = V$.

3. - ASYMPTOTIC FORM OF $d\sigma/dx$ AND σ_{tot}

A) - Triple-Regge region

Let us consider the quantity

$$z_A(y_0, v) \equiv \frac{\beta_A^2 \beta_B g_{P(0)}}{4\pi \beta^2} \int \frac{d^2 b_0}{\pi} F^2(v_0) |S_A(v_0)|^2 Z_A(v_0, v) |S_A(v)|^2 , \quad (3.1)$$

which, according to Eq. (2.10), gives the inclusive distribution by integration over $\underline{b}$. By Eq. (2.12), neglecting all quantities of relative order $1/y$ ^{*}, the integral equation for z_A can be written in the form

$$z_A(y_0, v) \simeq \left. \frac{d\sigma}{dy_0} \right|_0 F(v) \theta(y - y_0) - \frac{\beta_A g}{4\pi \alpha'} \int dv' z_A(y_0, v') F(v') F(v - v') , \quad (3.2)$$

where $(d\sigma/dy_0)_0$ is given by the triple-Pomeron formula (2.14), and the parameter

$$g \equiv \beta^3 / 4\alpha' \quad (3.3)$$

has the meaning of "elastic" triple-Pomeron coupling. Note that, by neglecting quantities of order $1/y$ we have replaced $|S_A|^2$ in (3.1), by 1, and we have kept only the first term in the expression (2.13) of the interaction $H_A(v)$.

^{*}) This approximation is consistent because our solution [Eq. (3.12)] contains non-leading terms of order $Y^{-\epsilon}$, $0 < \epsilon < 1$.

The inhomogeneous term of Eq. (3.2) is just the triple-Pomeron contribution which gives a divergence $\sigma_{AB} \sim \ln Y$ ($Y \equiv \ln s$ from now on). By repeated iteration, we obtain terms like

$$\left. \frac{d\sigma}{dy} \right|_n \sim \left. \frac{d\sigma}{dy} \right|_0 (\ln y_0)^n (-)^n, \quad (3.4)$$

i.e., more and more singular, but with alternating sign. The latter property comes from the minus sign in front of the integral in (3.2), which, in turn, comes from the fact that $|S|^2 < 1$. We have therefore to perform the whole sum in order to ascertain the behaviour of $d\sigma/dy_0$.

The form of the complete solution of Eq. (3.2) is discussed in detail in Appendix A. Here we present a simple but physically relevant approximation which is correct for all leading terms and shows the cancellation which is the main point of this section.

The leading contribution in y , to a given order in g in the iteration of (3.2), comes from the phase space region in which

$$y \gg y' \gg y_0, \quad (3.5)$$

i.e., from strong ordering in rapidity ^{*}). In this region we can make the approximation

$$F(v-v') \simeq F(v) \theta(y-y'). \quad (3.6)$$

By substituting (3.6) into (3.2) we see that it admits a solution of the form

$$z_A(y_0, v) = \left. \frac{d\sigma}{dy_0} \right|_0 R^A(y_0, y) F(v), \quad (3.7)$$

^{*}) This is true because, due to shrinkage, the only singularities in the y' integration come from $(y'/y) \approx 0$. If the Pomeron were a fixed pole, an extra singularity at $(y'/y) \approx 1$ would appear, thus spoiling this approximation. (In that case, one has, however, the advantage that the integral equation is diagonal in the J plane.) The region (3.5) has - for the asymptopia in $\ln s$ - the same rôle as the multiperipheral strong ordering in $p_{||}$ for the asymptopia in s .

where R^A satisfies the Volterra-type integral equation

$$R^A(y_0, y) = \theta(y - y_0) - \frac{\beta_A g}{8\pi\alpha'} \int_{y_0}^y \frac{dy'}{y'} R^A(y_0, y'), \quad (3.8)$$

with the solution

$$R^A(y_0, y) = \left(\frac{y_0}{y}\right)^{\varepsilon_A} \theta(y - y_0), \quad \varepsilon_A = \frac{\beta_A g}{8\pi\alpha'}. \quad (3.9)$$

By performing the y integration on Eq. (3.7) we get the inclusive distribution

$$\frac{d\sigma}{dy_0} = \left(\frac{d\sigma}{dy_0}\right)_0 \left(\frac{y_0}{Y}\right)^{\varepsilon_A} = \frac{\beta_A^2 \beta_B g_P(0)}{32\pi\alpha' y_0} \left(\frac{y_0}{Y}\right)^{\varepsilon_A}, \quad (3.10)$$

or

$$\frac{d\sigma}{dx} = \frac{1}{1-x} \frac{\beta_A^2 \beta_B g_P(0)}{32\pi\alpha' |\ln(1-x)|} \left(\frac{|\ln(1-x)|}{\ln s}\right)^{\varepsilon_A}. \quad (3.11)$$

Note that all rescattering corrections are summarized in the factor R^A .

The inclusive distribution (3.11) has the following properties.

- (i) It reduces to the triple-Pomeron formula in the limit $\varepsilon_A \sim g \rightarrow 0$. If expanded in powers of g , it develops a series of singular terms of alternating sign of the form $(-)^{N+M} (\ln|\ln(1-x)|)^M (\ln Y)^{N-M} / |\ln(1-x)|$.
- (ii) Although the singularity at $x \approx 1$ is not reduced, the non-scaling factor $R^A(y_0, Y)$ damps off the magnitude of the peak with increasing energy. The contribution of the $x \approx 1$ region to σ_{AB} increases therefore to a constant

$$\sigma_{AB}^{(1)} = \int_1^Y \frac{d\sigma}{dy_0} dy_0 = \frac{\beta_A^2 \beta_B g_P(0)}{32\pi\alpha' \varepsilon_A} (1 - Y^{-\varepsilon_A}) \rightarrow \beta_A \beta_B \frac{g_P(0)}{4g}. \quad (3.12)$$

Note finally that (3.10) has the same singularity of a triple-Pomeron graph when y_0 (the rapidity gap) varies proportionally to Y . This fits well in a conjecture by Neff¹⁹⁾ who assumes that $d\sigma/dx$ is

dominated by the triple-Pomeron vertex in the region $\eta^Y < y_0 \lesssim Y$, where it contributes a constant to σ_{AB} , and somehow damped outside, as it happens in our model.

The detailed calculations of Appendix A do not change these properties, but show that the approximate solutions (3.7) and (3.11) are to be modified in two respects :

- (i) the inclusive distribution $d\sigma/dx$ has the same form as Eq. (3.11), but ϵ_A has the value in (3.9) only in the limit $g \rightarrow 0$; it is otherwise a function of g with $0 < \epsilon_A(g) < 1$;
- (ii) at $t \neq 0$ (i.e., general impact parameter) one has, besides the pure pole term of Eq. (3.7), a series of many-Pomeron cuts accumulating at $J = 1$, whose discontinuities are less and less singular at the tip, but still contribute in the limit $t = 0$ to higher orders in g . (In other words, the solution discussed so far is a weak coupling - small g - approximation.)

B) - Average rescattering approximation and generalizations

A simple interpretation of the approximate solution (3.9) is obtained if we go back to the definition of rescattering corrections given in Section 2. According to the eikonal prescription, the probability for the singly diffractive production of Fig. 2 is to be multiplied by the rescattering factor

$$\prod_{j=0}^n |S(v_j)|^2 = \prod_{j \geq 0} |S_j|^2$$

By putting $|S_i|^2 = 1 - T_i$, we get formally

$$\prod_{j \geq 0} |S_j|^2 = 1 - \sum_{i \geq 0} T_i + \sum_{j > i \geq 0} T_i T_j - \sum_{k > j > i \geq 0} T_i T_j T_k + \dots \quad (3.13)$$

This series is a function of all co-ordinates $v_0, v_1, \dots, v_n$, and has to be integrated over phase space (at fixed y_0, Y), with a weight given by the squared production amplitude of Fig. 2. We can evaluate an average of this scattering factor if we make an approximation familiar from composite particle scattering²⁰⁾. Firstly, we assume n to be large so that we can go to the continuum limit. Typically,

$$\sum_{i < j} T_i T_j \rightarrow \int d^3 v_1 d^3 v_2 \rho(v_1, v_2) T(v_1) T(v_2) \quad . \quad (3.14)$$

$$y_0 \leq y_1 < y_2 \leq Y$$

Secondly, we neglect correlations by assuming that particle densities actually factorize

$$\rho(v_1, v_2) = \rho(v_1) \rho(v_2), \dots$$

and so on. This is equivalent to keeping only the leading term in y/y_0 , to any given order in g . It is therefore a weak coupling approximation.

It is now a simple matter to show that $\prod_{i=0}^n |S_i^A|^2$ has a finite limit for $n \rightarrow \infty$, given by

$$R^A(y_0, Y) = \exp \left[- \frac{\beta_A \beta}{\pi} \int \frac{d^2 b}{\pi} \int_{y_0}^Y dy \rho(b, y) F(b, y) \right] , \quad (3.15)$$

where $\rho(v)$ is normalized by

$$\int \frac{d^2 b}{\pi} \int_0^Y dy \rho(v) = \langle n \rangle = \beta^2 Y \quad (3.16)$$

The precise meaning of $\rho(v)$ is that of single-particle distribution at fixed $v = (\underline{b}, y)$ in the basic multiperipheral production amplitude ^{*}). For a given impact parameter and rapidity $(\underline{B}, Y) \equiv V$ of the last particle, we can calculate $\rho_V(v)$ from Eq. (2.5) and we get

$$\rho_V(v) = \beta^2 \frac{F(v) F(V-v)}{F(V)} \simeq \beta^2 F(v) \quad (3.17)$$

where the last approximation follows from the strong ordering assumption in rapidity that we have used throughout this section. Equation (3.15) then becomes

^{*}) Note that $\rho(\underline{b}, y)$ is not the Fourier transform of the measurable inclusive distribution with respect to transverse momentum. It is instead related to the non-forward six-point function by a Fourier transform with respect to the momentum transfer between C particles [cf. Eq. (2.7)]. This explains the "shrinkage" effects in (3.17) ($\langle \underline{b}^2 \rangle \sim y$).

$$R^A(y_0, Y) = \exp \left(-\varepsilon_A \int_{y_0}^Y \frac{dy}{y} \right) , \quad \varepsilon_A = \frac{\beta_A q}{8\pi\alpha'} , \quad (3.18)$$

i.e., the same result as for Eq. (3.9).

The virtue of the preceding average treatment of rescattering is that it is correct in the weak coupling limit, and can be easily extended to more complicated cases, where the integral equation approach is too involved (the obvious defect is that it neglects multiplicity dependence of rescattering around the average).

Consider now the two-particle inclusive distribution $(d^2\sigma/dx_A dx_B)$ for $x_A \approx 1$ and $x_B \approx -1$, i.e., for leading particles in opposite hemispheres (di-triple Regge region). Rescattering corrections to the basic production amplitude of Fig. 5 come from interactions of the leading particles with particles $C_0, C_1, \dots, C_N$, produced in the central region (fireball). The correction factor at fixed $y_{1,2} = -\ln(1 - |x_{A,B}|)$ can be calculated in the average rescattering approximation as follows :

$$\begin{aligned} R^{AB}(y_1, y_2, Y) &= \left\langle \prod_{Y-y_2 \geq y_i \geq y_1} |S_A(v_i - v_A)|^2 |S_B(v_B - v_i)|^2 \right\rangle \simeq \\ &\simeq \left\langle \prod_{y_i \geq y_1} |S_A(v_i - v_A)|^2 \right\rangle \left\langle \prod_{y_i \leq Y-y_2} |S_B(v_B - v_i)|^2 \right\rangle \simeq \\ &\simeq R^A(y_1, Y) R^B(y_2, Y) . \end{aligned} \quad (3.19)$$

The corrected distribution function is therefore

$$\frac{d^2\sigma}{dy_1 dy_2} = \left. \frac{d^2\sigma}{dy_1 dy_2} \right|_0 \left(\frac{y_1}{Y} \right)^{\varepsilon_A} \left(\frac{y_2}{Y} \right)^{\varepsilon_B} \quad (3.20)$$

which also gives a constant asymptotic contribution to σ_{AB} . To leading order in $\varepsilon_A, \varepsilon_B$, we get

$$\sigma_{AB}^{(2)} \rightarrow \left(\frac{\beta_A \beta_B g_{P(0)}}{32\pi\alpha'} \right)^2 \frac{1}{\varepsilon_A \varepsilon_B} = \beta_A \beta_B \left(\frac{g_{P(0)}}{4q} \right)^2 \quad (3.21)$$

We remark that together with Eq. (3.12) this formula gives a factorized expression for the total cross-section, summarized by the residue renormalization

$$\beta_A \rightarrow \beta_A + \frac{\beta_A^2 g_P(0)}{32\pi\alpha' \varepsilon_A} = \beta_A \left(1 + \frac{g_P(0)}{4g}\right). \quad (3.22)$$

A vertex renormalization is, of course, expected if $g_P(0) = 0$, but remarkably holds also if $g_P(0) \neq 0$, at least in this weak coupling approximation.

The rigorous treatment of the di-triple Regge rescattering of Appendix B, gives the following results :

- (i) the di-triple Pomeron graph and its corrections contain a pole term which is the one expected from factorization, to any order in g ;
- (ii) the remaining contributions to $\sigma_{AB}^{(2)}$ are majorized by a constant which is g independent.

We have not ascertained the actual asymptotic behaviour of these extra contributions (whether they vanish or not). Therefore, we cannot draw a final conclusion on factorization possibly holding to any order in g ^{*)}.

4. - DISCUSSION

In this paper we have dealt with the question of whether Pomeron pole dominance at infinite energies is compatible with $g_P(0) \neq 0$. Our results on the triple and di-triple Regge regions give a partial positive answer, and show that some decoupling arguments can be circumvented by s channel iteration.

We have assumed a definite structure of the Pomeron in terms of final states. One can avoid this assumption in the Gribov calculus, but the interpretation of the results would be harder because a single Gribov graph

*) To second order in $g_P(0)$ one should also consider the more difficult problem of doubly diffractive events, and this also could give contributions to the residue renormalization. Therefore, at this stage of the calculations, the weak coupling result (3.22) is perhaps the only significant one.

contributes to several terms in the fireball expansion, according to which discontinuity is taken. A less model-dependent approach would be to use momentum conservation sum rules. There, the difficulty is that one does not know how to parametrize many-Pomeron cuts and their contribution to inclusive cross-sections.

The phenomenological implications of our model (e.g., energy dependence, correlations, etc.) depend on the evaluation of cut contributions at finite energies. A way to do that is the perturbative fireball expansion, which assumes the "inelastic" Pomeron to be an effective pole. In our approach we have used two (small) triple-Pomeron couplings : the "inelastic" one $g_P(0)$ is connected with diffractive production of large masses, and therefore with multi-Pomeron exchanges at fixed momentum transfers ; the "elastic" coupling g is connected with s channel Pomeron iteration, and is important in order to regularize the contributions of the former.

To first order in $g_P(0)$, but any order in g , we get a rising contribution to σ_{tot} from triple-Regge regions ^{*)}. If the "inelastic" Pomeron is at $\alpha_0 \simeq 1$, this implies rising cross-sections. It should be remarked that the graph of Fig. 6 which has been suggested to cancel the triple-Pomeron contribution ²¹⁾, or to overcancel it ²²⁾ is not actually relevant close to $x = 1$ ²³⁾. It belongs, in our approach, to the structure of the inelastic Pomeron. The nature of the latter is the main ambiguity left over. Our assumption that $\alpha_0 = 1$ is phenomenologically sound, but theoretically arbitrary, and it has to be re-examined if higher orders in $g_P(0)$ turn out to give a divergent behaviour. We expect, however, that these divergences, if present, are strongly softened by rescattering.

If it turns out eventually that the Pomeron (inelastic or asymptotic) is really a pole with $\alpha(0) = 1$, one still has to answer the question of why it is so. Now, if we put $g = 0$ in our model, we would have a pole at $\alpha(0) > 1$, and it is only by an infinite iteration in the s channel, at each stage of the t channel iteration that a pole at $\alpha(0) = 1$ can possibly emerge. It is therefore tempting to conjecture that $\alpha(0) = 1$ is the result of combined s and t channel iterations, while the latter would basically give $\alpha(0) > 1$ asymptotically. We believe this to be an interesting line of thought for future work.

*) The estimate of this increase does not differ from the one given in Ref. 11), to the extent that ϵ_A is small.

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APPENDIX A - MANY-POMERON STRUCTURE OF THE SOLUTION

We rewrite Eq. (3.2) in the form

$$z(y_0, v) = \frac{\beta}{y_0} F(v) \Theta(y-y_0) - \gamma \int d^3 v' z(y_0, v') F(v') F(v-v'). \quad (A.1)$$

Note that, by the scale transformation $\underline{b} \rightarrow \sqrt{\alpha} \underline{b}$, $y \rightarrow \alpha y$, $y_0 \rightarrow \alpha y_0$, we deduce

$$z(y_0; \underline{b}, y) = z(1; \underline{b}/\sqrt{y_0}, y/y_0), \quad (A.2)$$

so that we can put $y_0 = 1$ from now on.

We seek a solution of cut type that we write in the form of a spectral representation

$$z(v) = \int_1^\infty d\rho \, N(\rho, y) \rho^2 F(\rho v), \quad (\rho v \equiv (\rho \underline{b}, \rho y)). \quad (A.3)$$

By substituting in Eq. (A.1) and using the identity

$$\int d\underline{b}' F(v-v') \rho^2 F(\rho v') = F(\underline{b}, y - (1 - \frac{1}{\rho}) y'), \quad (A.4)$$

we get an integral equation for the distribution $N(\rho, y)$:

$$N(\rho, y) = \beta \delta(\rho-1) - \gamma \int_1^y \frac{dy'}{y'} \int_1^\infty \frac{d\rho' \rho'}{\rho'+1} N(\rho', y') \delta\left[\rho - \left(1 - \frac{\rho'}{\rho'+1} \frac{y'}{y}\right)^{-1}\right]. \quad (A.5)$$

Note that if we limit the y' integration to the singularity at $(y'/y) \approx 0$, we get back the strong ordering approximation of the text. In the following we discuss the exact equation (A.5).

Since we are interested in $y \gg 1$, we perform a Mellin transform ^{*)} on Eq. (A.5), as follows :

$$N_\lambda(\rho) \equiv \int_1^\infty dy \, y^{-\lambda-1} N(\rho, y) = \frac{\beta}{\lambda} \delta(\rho-1) - \gamma \int_1^\infty d\rho' K_\lambda(\rho, \rho') N_\lambda(\rho'), \quad (A.6)$$

^{*)} Note that we are seeking leading powers in $\ln s$, not in s .

and after an integration by parts,

$$(N_\lambda, f) = \frac{\beta}{\lambda} f(1) - \frac{\gamma}{\lambda} \int_1^\infty d\rho' N_\lambda(\rho') \left(1 + \frac{1}{\rho'}\right)^{\lambda-1} \\ \times \left[\left(\frac{\rho'}{\rho'+1}\right)^\lambda f(\rho'+1) + \int_{1/\rho'+1}^1 dx (1-x)^\lambda \frac{\partial}{\partial x} f\left(\frac{1}{x}\right) \right] \quad (\text{A.13})$$

which has a meaning also for $-1 \leq \text{Re } \lambda \leq 0$.

It is clear that the residue of (N_λ, f) at the $\lambda = 0$ pole vanishes if (A.10) is satisfied. We then look for a pole singularity of $N_\lambda(\rho)$ in $\text{Re } \lambda < 0$:

$$N_\lambda(\rho) \simeq n(\rho) / (\lambda + \varepsilon) \quad , \quad \varepsilon > 0 \quad , \quad (\text{A.14})$$

so that the residue distribution $n(\rho)$ satisfies the homogeneous integral equation

$$(n, f) = \frac{\gamma}{\varepsilon} \int_1^\infty d\rho' n(\rho') \left(1 + \frac{1}{\rho'}\right)^{-1-\varepsilon} \left[\left(1 + \frac{1}{\rho'}\right)^\varepsilon f(\rho'+1) + \int_{1/\rho'+1}^1 dx (1-x)^{-\varepsilon} \frac{\partial}{\partial x} f\left(\frac{1}{x}\right) \right] \quad (\text{A.15})$$

Keeping in mind this continuation procedure, we can practically estimate the eigenvalues $\mu(\lambda)$ of Eq. (A.8), and consequently ε by the Fredholm series in $\text{Re } \lambda > 0$. In the trace approximation, one has

$$\mu(\lambda) \simeq - \int_1^\infty \frac{d\rho}{\rho^2} \left(1 - \frac{1}{\rho^2}\right)^{\lambda-1} = - \frac{\sqrt{\pi}}{2} \frac{\Gamma(\lambda)}{\Gamma(\lambda+1/2)} \underset{\lambda \rightarrow 0}{\simeq} - \frac{1}{2\lambda} \quad (\text{A.16})$$

This function is negative definite in $\text{Re } \lambda \geq 0$, vanishing like $1/\sqrt{\lambda}$ as $\lambda \rightarrow +\infty$, and has poles at $\lambda = 0, -1, -2, \dots$ with negative residues, as in Fig. 7. Further terms in the Fredholm series contribute to the poles $\lambda = -1, \lambda = -2, \dots$ but do not change the general analytic structure. Since the leading singularity $\lambda = -\varepsilon$ satisfies the equation

$$\mu(-\varepsilon) = 1/\gamma \quad (\text{A.17})$$

we easily conclude that $\varepsilon \rightarrow \gamma/2$ as $\gamma \rightarrow 0$, and in general $0 < \varepsilon < 1$, and $\varepsilon < \gamma/2$.

Let us now remark that, by Eq. (A.8), $N_\lambda(\rho)$ has a singularity $(\rho - N)^{\lambda + N - 2}/(\lambda + \varepsilon)$ close to $\rho = N > 1$. By a Fourier-Laplace transform we then find

$$\tilde{z}(J, \underline{k}) = \int \frac{d\lambda d\rho N_\lambda(\rho) \Gamma(\lambda+1)}{(\gamma-1 + \underline{k}^2/\rho)^{\lambda+1}} \underset{J-1 \rightarrow -\frac{\underline{k}^2}{N}}{\simeq} (\gamma-1 + \frac{\underline{k}^2}{N})^{N-1} \left(\frac{N}{\underline{k}^2}\right)^{N-\varepsilon} \quad (\text{A.18})$$

We can see from this formula that many-Pomeron cuts have weaker and weaker tip discontinuities at fixed $\underline{k}^2 = |t| \neq 0$, but conspire together to build a $(J-1)\varepsilon^{-1}$ singularity in $\tilde{z}(J, 0)$. The corresponding cut structure in the elastic scattering amplitude $\tilde{a}(J, \underline{k})$ is found by integration over y_0 .

At fixed $t = -\underline{k}^2$ we find

$$\begin{aligned} \tilde{a}(J, \underline{k}) &= \int_1^\infty d\rho \int_1^\infty d\gamma \int_1^\gamma \frac{dy}{y} N(\rho, y) e^{-\alpha' \frac{\underline{k}^2}{\rho} \gamma} e^{-(J-1)\gamma} \simeq \\ &\simeq \int_1^\infty \frac{d\rho n(\rho)}{(\gamma-1 + \alpha' \frac{\underline{k}^2}{\rho})} \frac{1}{\varepsilon} \left[1 - \Gamma(1-\varepsilon) (\gamma-1 + \alpha' \frac{\underline{k}^2}{\rho})^\varepsilon \right] \end{aligned} \quad (\text{A.19})$$

while the $\underline{k} \neq 0$ behaviour depends on detailed properties of the distribution $n(\rho)$ (that we have not studied), it is clear that for $\underline{k} = 0$ one has a pole, combined with a not too hard cut, of type $(J-1)\varepsilon^{-1}$ at the tip.

APPENDIX B - INTEGRAL EQUATION FOR THE DI-TRIPLE REGGE REGION

As anticipated in Section 2 B), this problem is analogous to a two-dimensional scattering off two fixed centres, at $v = (\underline{Q}, 0)$ and $v = V = (\underline{B}, Y)$ respectively. Consider the operator function $Z_{AB}(v_1, v; V)$ which satisfies the integral equation

$$Z_{AB}(v_1, v; V) = \beta^2 F(v-v_1) - \beta^2 \int d^3 v' Z_{AB}(v_1, v'; V) H_{AB}(v', V) F(v'-v), \quad (B.1)$$

where H_{AB} is given in Eqs. (2.16), (2.17). The double inclusive distribution is obtained from Z_{AB} by integration over $\underline{b}_1, \underline{b}_2$ (which is obvious), and over $\underline{B}$ (which sets to zero the over-all momentum transfer) :

$$\frac{d^2 \sigma}{dy_1 dy_2} = \left(\frac{\beta_A \beta_B}{4\pi \beta} \right)^2 \int \frac{d^2 b_1}{\pi} \frac{d^2 b_2}{\pi} \frac{d^2 B}{\pi} |S_A(v_1)|^2 F^2(v_1) Z_{AB}(v_1, V-v_2; V) |S_B(v_2)|^2 F^2(v_2). \quad (B.2)$$

By neglecting terms of relative order $1/Y$, and introducing the notation

$$\begin{aligned} \bar{Z}_{AB}(v_1, v_2; V) &\equiv Z_{AB}(v_1, V-v_2; V), \quad H_i^V(v) \equiv H_i(V-v), \\ T(v_1, v_2) &\equiv \beta^2 F(v_1-v_2), \quad \bar{T}(v_1, v_2; V) \equiv \beta^2 F(V-v_1-v_2), \end{aligned} \quad (B.3)$$

we can rewrite Eq. (B.1) for $\bar{Z}_{AB}$ as

$$\begin{aligned} \bar{Z}_{AB} &= \bar{T} - \bar{Z}_{AB} (H_A^V + H_B) T = \\ &= \bar{T} - T^t (H_A + H_B^V) \bar{Z}_{AB} \end{aligned} \quad (B.4)$$

After some algebra, one can recast the set of Eqs. (B.4) in the form

$$\bar{Z}_{AB} = (1 + T^t H_A)^{-1} \left[\bar{T} + T^t H_B^V \bar{Z}_{AB} H_A^V T \right] (1 + H_B T)^{-1}, \quad (B.5)$$

where T^t denotes the transposed matrix, and we have singled out the inhomogeneous term

$$\bar{Z}_0 = (1 + T^t H_A)^{-1} \bar{T} (1 + H_B T)^{-1}. \quad (B.6)$$

Since the expression for $\bar{Z}_0$ contains $\underline{B}$ only in $\bar{T}$, the $\underline{B}$ integration in Eq. (B.2) is trivial, and the resulting contribution to the inclusive distribution is

$$\frac{d^2\sigma}{dy_1 dy_2} \simeq \frac{1}{\beta_A \beta_B} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} \Theta(Y - y_1 - y_2). \quad (B.7)$$

This in turn contributes to σ_{AB} the factorized asymptotic piece expected, to second order in $g_p(0)$, from the single particle distribution [cf. Eq. (3.20) in the weak coupling limit].

The remaining contributions to $\bar{Z}$ from (B.5) can be majorized by noting that, since $|S_{ij}|^2 < 1$, one has

$$\bar{Z} < \bar{T}$$

and therefore

$$\bar{Z} - \bar{Z}_0 < (1 + T^t H_A)^{-1} T^t H_B^v \bar{T} H_A^v T (1 + H_B T)^{-1}. \quad (B.8)$$

The corresponding bound for the correction $\Delta\sigma_{AB}$ comes from integrating (B.2) over y_1, y_2 . Noting that

$$T (1 + H_B T)^{-1} F^2 = \int dy_0 z_B(y_0, v)$$

is the regularized triple-Pomeron term, i.e., asymptotically proportional to $F(v)$, we get

$$\begin{aligned} \Delta\sigma_{AB} &< \text{const.} \int d^3v_1 d^3v_2 d^2B F(v_1) F(V-v_1) F(V-v_1-v_2) F(V-v_2) F(v_2) = \\ &= \text{const.} \int d^2Q_1 dy_1 d^2Q_2 dy_2 e^{-Q_1^2(Y-y_1+y_2)} e^{-Q_2^2(Y+y_1-y_2)} e^{-(Q_1+Q_2)^2(Y-y_1-y_2)} \\ &= \text{const} \int \frac{dy_1 dy_2 \Theta(Y-y_1) \Theta(Y-y_2) \Theta(Y-y_1-y_2)}{3Y^2 - 2Y(y_1+y_2) - (y_1-y_2)^2} \lesssim \text{const.} \end{aligned} \quad (B.9)$$

A check of the powers of β in (B.8) shows that this constant is of order $g_P(0)^2$, but is independent of β , or g .

It should be remarked that by a majorization like the one in (B.8) one loses operators like $(1+H_B T)^{-1}$ which usually make the asymptotic behaviour more convergent. Therefore, a more refined analysis of (B.5) is needed in order to see whether $\Delta \sigma_{AB}$ vanishes or not asymptotically.

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FIGURE CAPTIONS

- Figure 1 (a) Triple-Pomeron graph.
(b) A class of multi-Pomeron cut corrections.
Discontinuity lines are dotted.
- Figure 2 Basic amplitude for diffractive production of a single "fireball" of particles $C_0, C_1, \dots, C_n$ of momenta $p_0, p_1, \dots, p_n$, and over-all mass M .
- Figure 3 (a) Amplitude for single diffractive production with rescattering corrections in the final (and initial) state.
(b) The spring-like line represents the S matrix, which is dominated by a Pomeron exchange.
- Figure 4 Momentum space representation of the integral equation for the six-point function, analogous to Eq. (2.11). The dotted lines represent multiperipheral propagators.
- Figure 5 Basic production amplitude for the process $A+B \rightarrow A+B+C_0+\dots+C_n$, in the region $x_A \simeq 1$ and $x_B \simeq -1$, with $x_{A,B}$ scaling variables for the leading particles.
- Figure 6 Graph contributing to σ_{tot} , but not to $d\sigma/dx$ in the region $x \simeq 1$.
- Figure 7 Graphic solution of the implicit eigenvalue equation (A.17).

FIG.1

FIG.2

FIG. 3

FIG. 4

FIG. 5

FIG.6

FIG.7