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CONDITIONS ON EXCHANGE MECHANISMS FOR POLARIZATION
EFFECTS IN INCLUSIVE REACTIONS

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ABSTRACT

In the framework of the Mueller-Regge phenomenology we investigate under which conditions one could expect to observe polarization effects in the fragmentation region for inclusive reactions. On the basis of kinematical considerations and parity relations only, we find that this requires exchange of states with mixed naturalities.

1. INTRODUCTION

The importance of the three-body elastic amplitudes has been emphasized over the last few years in the framework of the Mueller formalism ¹⁾ for the study of inclusive cross-sections. These ideas of generalized optical theorem can be tested for the unpolarized as well as for the polarized inclusive distributions. As a matter of fact, if M denotes the absorptive part of the "forward" amplitude for the elastic reaction $a+b+\bar{c} \rightarrow a+b+\bar{c}$ where a , b and c are particles with definite spin, the polarized single particle cross-section for $a+b \rightarrow c+X$ is ^{*)}

$$d\sigma_p = T_r(M\rho) \quad (1)$$

where ρ is the density matrix which describes the state of polarization of a , b and c . In particular, if one does not observe any polarization, ρ is proportional to unity and we see that the measure of the unpolarized cross-section corresponds to that of the sum of the diagonal elements of M ; on the contrary, one can reach the off-diagonal part of M by measuring polarized cross-sections. This is a strong motivation for the study of any spin dependence in inclusive reactions since polarization measurements allow a direct evaluation of a new combination of amplitudes which are not present in the unpolarized cross-section. We emphasize that all current ideas, such as SRO, Mueller-Regge phenomenology, scaling, energy dependence, factorization, duality, etc., have been tested so far on unpolarized cross-sections, i.e., for non-flip amplitudes, and that it would be quite interesting to know whether they can be applied with equal success to flip amplitudes ²⁾. As a specific example, let us consider the inclusive reaction $\pi N \rightarrow \pi X$ with polarized nucleon ³⁾; as we shall see below in more detail, the corresponding "forward" amplitude M for $\pi\pi N \rightarrow \pi\pi N$ has essentially a non-spin-flip and a spin-flip part, the last one being proportional to the up-down asymmetry. It has already been shown ⁴⁾ that by kinematical arguments alone, there is no reason for the flip amplitude to vanish. However, in the fragmentation region of the incident pion, one can make a Regge analysis and it has been pointed out by Abarbanel and Gross ⁵⁾, invoking factorization, that the flip amplitude vanishes in the "forward direction" just like in two-body reactions. Their conclusion was that there should be no polarization effects in the inclusive distribution when only Regge poles are exchanged (when the final particle is not a fragment of the polarized initial particle).

*) This can be easily generalized to the case of multiparticle inclusive correlations.

In this study we shall not invoke factorization properties of Regge poles, but rather follow a very similar approach to the one used in the past for two-body amplitudes, based on analyticity properties and parity relations only. Needless to recall that the three-body amplitudes involve additional kinematical variables (for example, azimuthal angles) and our aim, in this work, is to examine precisely the dependence upon these variables. We shall show, in a simple way, that this leads to some constraints between natural and unnatural parity exchanges analogous to the well-known two-body constraint relations⁶⁾. The result of this observation is that one can get a non-zero polarization in inclusive reactions by exchanging both naturalities.

In Section 2 we shall concentrate on the reaction $\pi\pi N \rightarrow \pi\pi N$ which is the simplest example to illustrate our proof. We shall derive consistently the constraints, both in the "t channel" and in the "s channel". The generalization to any three-body reaction is straightforward, starting from partial wave analysis, and will be presented in Section 3.

2. THE REACTION $\pi\pi N \rightarrow \pi\pi N$

i) Polarized and unpolarized inclusive cross-sections for $\pi N \rightarrow \pi X$

We have seen above that these quantities are related to the forward elastic amplitudes M for the three-body reaction $\pi\pi N \rightarrow \pi\pi N$. Let us take the centre-of-mass system of the nucleon target and of one incident pion with three momenta $\vec{p}$ and $-\vec{p}$ respectively, and let us take the quantization axis along $\vec{p}$, (helicity frame). If $\vec{q}'$ denotes the momentum of the other incident pion (outgoing pion for the inclusive reaction), in the "forward direction", in the sense of the Mueller's generalized optical theorem, all particles lie in the scattering plane defined by $\vec{p}$ and $\vec{q}'$.

Therefore the general expression for the 2×2 Pauli scattering amplitude M is :

$$M = H_1 + \vec{\sigma} \cdot \vec{n} H_2 \quad (2)$$

where

$$\vec{n} = \frac{\vec{p} \times \vec{q}'}{|\vec{p}| |\vec{q}'|}$$

H_1 and H_2 are the non-spin-flip and the spin-flip amplitudes, respectively. The only up-down asymmetry Σ one can observe is along the y axis normal to the scattering plane xz , so that we define :

$$\rho_\epsilon = \frac{1 + \epsilon \Sigma_y}{2} \quad (\epsilon = \pm 1)$$

The unpolarized cross-section is then

$$d\sigma = \frac{1}{2} T_\tau M = H_1$$

and the polarized cross-section

$$d\sigma_p^\epsilon = T_\tau (M \rho_\epsilon) = H_1 + \epsilon \sin \theta' H_2$$

where θ' is the angle between $\vec{p}$ and $\vec{q}'$.

Therefore the asymmetry $\Sigma = (d\sigma_p^+ - d\sigma_p^-)/2d\sigma$ is given by

$$\Sigma = \sin \theta' H_2 / H_1 \quad (5)$$

which says that the asymmetry is proportional to the spin-flip amplitude and should vanish when $\vec{p}$ and $\vec{q}'$ are collinear (i.e., $p_\perp = 0$ for the inclusive reaction). The most general expression for $\pi\pi N \rightarrow \pi\pi N$ "forward" amplitude in terms of invariant amplitudes ⁴⁾ is

$$T = \bar{u}(p) [A + B \gamma \cdot q_1 + C \gamma \cdot q_2 + D \gamma \cdot q_1 \gamma \cdot q_2] u(p) \quad (4)$$

(p , q_1 and q_2 are, respectively, the four momenta of the nucleon and of the two pions).

In the s channel centre-of-mass of the reaction with $p = (E, \vec{p})$, $q_1 = (\omega, -\vec{p})$ and $q_2 = (\omega', \vec{q}')$, the helicity amplitudes are related to the invariant amplitudes as follows :

$$H_1 = A - \left(\frac{m^2 - E(E+\omega)}{m} \right) B - \left(\frac{pq' \cos \theta' - E\omega'}{m} \right) C + (\omega\omega' + pq' \cos \theta') D \quad (5)$$

$$H_2 = i \left(\frac{E+\omega}{m} \right) pq' D$$

where $p = |\vec{p}|$, $q' = |\vec{q}'|$ and m is the nucleon mass.

As noticed in Ref. 4), one will observe a non-zero asymmetry, provided D is non-vanishing and there is a priori no reason for that to be the case, at least from a pure kinematical point of view. Now to bring in some dynamics, we would like to consider a Mueller-Regge diagram of the type represented in Fig. 1. If one tries to calculate the contribution to D of such a diagram with the exchange of any given spin J , one is faced with a puzzling situation : there is no tensor coupling which could contribute to D . More explicitly, the contraction of the projection operator $T^{\mu_1 \mu_1 \dots \mu_J}$ on a spin J state which is completely symmetric with a $\sigma^{\mu_1 \mu_2}$ coupling at the $\bar{N}\bar{N}$ vertex gives a vanishing contribution to D *). In fact, this problem is not new, it appears in the same way for elastic nucleon-nucleon scattering, and was solved long ago by Gribov and Volkov ⁷⁾ using the constraint relations between different helicity amplitudes in the forward direction, implying simultaneous exchange of different naturalities and spins. We shall produce, in the case of inclusive reactions, analogous relations by studying first the amplitudes away from the forward direction, and by considering the forward direction on a limiting case. We shall perform this analysis both in the crossed and direct channels.

ii) Crossed channel helicity amplitudes and kinematical constraints

The general expression for the scattering amplitude for $\pi\pi N \rightarrow \pi\pi N$ away from the forward direction is analogous to (4), except that we now have to consider an additional four-vector, say p' the momentum of the second nucleon. In the centre-of-mass system of the crossed channel $\bar{N}\bar{N} \rightarrow 4\pi$ we have :

$$F_{\lambda\lambda} = \bar{u}_{\lambda}(p') [A + B \gamma \cdot q_1 + C \gamma \cdot q_2 + D \gamma \cdot q_1 \gamma \cdot q_2] u(p) \quad (6)$$

and we choose the frame of reference such that $p = (E, \vec{p})$, $p' = (-E, \vec{p})$, $q_1 = (\omega, \vec{q})$ and $q_2 = (\omega', \vec{q}')$, with $\vec{p}$ along the positive z axis, $\vec{q}$ in the xz plane with polar angle θ_t and $\vec{q}'$ defined by the polar angles θ'_t and φ , according to Fig. 2. After a straightforward calculation, we obtain the following four helicity amplitudes

$$F_{++}^t(\varphi) = \frac{p}{m} A + q \cos \theta_t B + q' \cos \theta'_t C + \left[\frac{\omega \omega'}{m} p + \frac{E}{m} (\omega q' \cos \theta'_t - \omega' q \cos \theta_t) \right. \\ \left. - \frac{p q q'}{m} (\cos \theta_t \cos \theta'_t + \sin \theta_t \sin \theta'_t e^{i\varphi}) \right] D \quad (7.a)$$

*) We are grateful to D. Wray for pointing out to us this result.

$$F_{+-}^t(\varphi) = \frac{qE}{m} \sin\theta_t B + \frac{q'E}{m} \sin\theta'_t e^{-i\varphi} C + [\omega q' \sin\theta'_t e^{-i\varphi} - \omega' q \sin\theta_t] D \quad (7.b)$$

$$F_{--}^t(\varphi) = - F_{++}^t(-\varphi) \quad (7.c)$$

$$F_{-+}^t(\varphi) = F_{+-}^t(-\varphi) \quad (7.d)$$

In two-body reactions where one does not have to consider the dependence upon the azimuthal angle φ , Eqs. (7.c) and (7.d) are just the usual parity relations which reduce from four to two the number of independent amplitudes. Here parity conservation implies weaker conditions like symmetry properties with respect to reflection in the scattering plane (xz). From relations (7) one can easily deduce the expressions for the invariant amplitudes in terms of the helicity amplitudes and in particular for B, C and D, we obtain

$$D = im (F_{++}^t(\varphi) + F_{--}^t(\varphi)) / (2 p q q' \sin\varphi \sin\theta'_t \sin\theta_t) \quad (8.a)$$

$$C = \frac{im}{2Eq' \sin\theta'_t \sin\varphi} \left[(F_{+-}^t(\varphi) - F_{-+}^t(\varphi)) - \frac{m\omega}{pq \sin\theta_t} (F_{++}^t(\varphi) + F_{--}^t(\varphi)) \right] \quad (8.b)$$

$$B = \frac{m}{2Eq \sin\theta_t} \left[(F_{+-}^t(\varphi) + F_{-+}^t(\varphi)) - i \omega t \varphi (F_{+-}^t(\varphi) - F_{-+}^t(\varphi)) + \frac{im\omega'}{pq' \sin\theta'_t \sin\varphi} (F_{++}^t(\varphi) + F_{--}^t(\varphi)) \right] \quad (8.c)$$

In the "forward direction" ($p=p'$), E goes to zero and we see that in order to avoid kinematical singularities in B and C, the helicity amplitudes should fulfil two constraint relations.

Now if one exchanges a state with definite naturality σ ($\sigma = \pm 1$) in the crossed channel, one can show (see Appendix) that

$$F_{\lambda'\lambda}^t(\varphi) \simeq -\sigma (-)^{\lambda-\lambda'} F_{-\lambda'-\lambda}^t(\varphi) \quad (9)$$

Therefore, if $\sigma = +1$

$$F_{++}^t(\varphi) \simeq - F_{--}^t(\varphi) \quad (10)$$

$$F_{+-}^t(\varphi) \simeq F_{-+}^t(\varphi)$$

which means that D and C are zero from Eqs. (8.a) and (8.b), i.e., the asymmetry Σ is also zero [see Eqs. (3) and (5)]. In the forward direction, Eq. (8.c) implies that $F_{+-}^t(\varphi)$ must also vanish or, in other words, that the constraint relations are satisfied by evasion. In any case, the non-spin-flip amplitudes do not vanish so that natural parity exchange alone does contribute to the unpolarized cross-section.

For $\sigma = -1$

$$F_{++}^t(\varphi) \simeq F_{--}^t(\varphi) \quad (11)$$

$$F_{+-}^t(\varphi) \simeq - F_{-+}^t(\varphi)$$

For $E \rightarrow 0$, inspection of (8.b) and (8.c) together with (11) shows that all four helicity amplitudes must vanish. This means that unnatural parity exchange can contribute neither to the unpolarized cross-section nor to the polarized one, nor to the asymmetry Σ . This analysis shows that whenever one insists on having a definite naturality exchanged in the cross channel, for natural parity exchange the asymmetry is zero and for unnatural parity both the asymmetry and the cross-section are zero.

iii) Direct channel helicity amplitudes and kinematical constraints

In the centre-of-mass system of the s channel, we have

$$F_{\lambda'\lambda} = \bar{u}_{\lambda'}(p') \left[A + B \gamma \cdot q_1 + C \gamma \cdot q_2 + D \gamma \cdot q_1 \gamma \cdot q_2 \right] u_{\lambda}(p) \quad (12)$$

and we choose the frame of reference such that $p = (E, \vec{p})$, $q_1 = (\omega, -\vec{p})$, $q_2 = (\omega', \vec{q}')$ and $p' = (E', \vec{p}')$, with $\vec{p}'$ in the xz plane, the polar angles for $\vec{q}'$ and $\vec{p}'$ being defined in Fig. 3. We obtain in this case

$$F_{++}^s = (h_1 + h_2 p p') \cos \frac{\theta_2}{2} + q' (h_4 p' + h_3 p) \left(\cos \frac{\theta_2}{2} \cos \theta'_s + \sin \frac{\theta_2}{2} \sin \theta'_s e^{i\psi} \right) \quad (13.a)$$

$$F_{+-}^s = (h_1 - h_2 p p') \sin \frac{\theta_2}{2} + q' (h_4 p' - h_3 p) \left(\cos \frac{\theta_2}{2} \sin \theta'_s e^{-i\psi} - \sin \frac{\theta_2}{2} \cos \theta'_s \right)$$

$$\begin{aligned} F_{++}^s(\varphi) &= F_{--}^s(-\varphi) \\ F_{+-}^s(\varphi) &= -F_{-+}^s(-\varphi) \end{aligned} \quad (13.b)$$

where

$$\begin{aligned} h_1 &= \frac{\sqrt{(E+m)(E'+m)}}{2m} \left[A - (m-\omega-E)B + \omega'C + (\omega'(\omega-E+m) + 2pq'\cos\theta'_s) D \right] \\ h_2 &= \frac{1}{2m\sqrt{(E+m)(E'+m)}} \left[-A + (E+\omega+m)B + \omega'C + (\omega'(E-\omega+m) - 2pq'\cos\theta'_s) D \right] \\ h_3 &= -\frac{\sqrt{(E'+m)}}{2m\sqrt{(E+m)}} \left[C + (\omega+E+m) D \right] \\ h_4 &= \frac{\sqrt{(E+m)}}{2m\sqrt{(E'+m)}} \left[-C + (\omega+E-m) D \right] \end{aligned} \quad (14)$$

Now if one exchanges a state with definite naturality σ ($\sigma = \pm 1$) in the crossed channel one can show (see Appendix) that

$$F_{\lambda'\lambda}^s(\varphi) = \sigma (-)^{\lambda'-\lambda} F_{-\lambda'-\lambda}^s(\varphi) \quad (15)$$

In case $\sigma = +1$, Eq. (15) leads to the following two conditions :

$$\begin{aligned} (h_3 p + h_4 p') e^{i\varphi} &= (h_3 p + h_4 p') e^{-i\varphi} \\ (h_4 p' - h_3 p) e^{-i\varphi} &= (h_4 p' - h_3 p) e^{i\varphi} \end{aligned}$$

from which we conclude that both h_3 and h_4 are zero. This means that C and D vanish, i.e., the asymmetry Σ is zero.

For $\sigma = -1$ one will be easily convinced that all helicity amplitudes must vanish in the forward direction $\theta_s = 0$, and therefore that both the polarized and unpolarized cross-sections vanish.

It is clear that the only way to get a non-zero asymmetry is to exchange a state with mixed naturalities, or in the old terminology, to satisfy the kinematical constraints by conspiracy.

We have recovered exactly the same results in the direct and crossed channels. The analysis in the direct channel, however, suggests a method to generalize the study to any three-body reaction as we shall see in the next section.

3. GENERAL DERIVATION

We now intend to derive the results obtained in the preceding paragraph in a more direct and more general way. In analogy with what has been done in the past for two-body amplitudes, we expand in partial waves the 3 to 3 amplitude $abc \rightarrow a'b'c'$, in order to make explicit the dependence of helicity amplitudes on angular variables, then go to the "forward direction" of the process. We shall work in the over-all s channel centre-of-mass system $\vec{p} + \vec{q}_1 + \vec{q}_2 = 0$, with p, q_1, q_2 being, respectively, the momenta of particles a, b and c . We first expand in partial waves the (bc) state in its rest system $\vec{Q} = \vec{q}_1 + \vec{q}_2 = 0$

$$|\vec{Q}=0; \lambda_1 \lambda_2\rangle = \sum_{s\Lambda} D_{\Lambda \lambda_1 - \lambda_2}^s(\varphi_q, \theta_q) |\vec{Q}=0; s\Lambda \lambda_1 \lambda_2\rangle \quad (16)$$

In this expression λ_1 and λ_2 are the helicities of particles b and c and φ_q, θ_q the polar angles of, say, particle b in a reference frame (x, y, z) with z axis along $-\vec{p}$. To go to the over-all s channel centre-of-mass system, we must apply a boost along $-\vec{p}$, and obtain for the (bc) state :

$$|\vec{Q}=-\vec{p}; \lambda_1 \lambda_2\rangle = \sum_{s\Lambda} D_{\Lambda \lambda_1 - \lambda_2}^s(\varphi_q, \theta_q) |\vec{Q}=-\vec{p}; s\Lambda \lambda_1 \lambda_2\rangle \quad (17)$$

Then we couple the bc system to particle a , and expand again in partial waves, with total angular momentum J and magnetic quantum number M , as follows :

$$|\lambda_1 \lambda_2 \lambda_3\rangle = \sum_{JMS\Lambda} D_{M \lambda - \Lambda}^J(\bar{\varphi}, \bar{\theta}) D_{\Lambda \lambda_1 - \lambda_2}^s(\varphi_q, \theta_q) |JMS\Lambda \lambda_1 \lambda_2\rangle \quad (18)$$

In this last expression λ is the helicity of particle a and $\bar{\varphi}, \bar{\theta}$ are the polar angles of $\vec{p}$ with respect to the fixed frame of reference XYZ . Finally we compute the helicity amplitude for the process $abc \rightarrow a'b'c'$ with the obvious notation :

$$F_{\lambda'_1 \lambda'_2 \lambda'_3 \lambda_1 \lambda_2 \lambda} = \sum_{\substack{JMS\Lambda \\ J'M'S'\Lambda'}} D_{M \lambda - \Lambda}^J(\bar{\varphi}, \bar{\theta}) D_{\Lambda \lambda_1 - \lambda_2}^s(\varphi_q, \theta_q) D_{M' \lambda'_1 - \Lambda'}^{J'*}(\bar{\varphi}', \bar{\theta}') D_{\Lambda' \lambda'_1 - \lambda'_2}^{S'*}(\varphi'_q, \theta'_q) \langle J'M'S'\Lambda' \lambda'_1 \lambda'_2 | T | JMS\Lambda \lambda_1 \lambda_2 \rangle \quad (19)$$

[Nowhere have we made explicit the energy dependence. We assume $|\vec{p}| = |\vec{p}'|$, $|\vec{q}_1| = |\vec{q}'_1|$, $|\vec{q}_2| = |\vec{q}'_2|$.] To simplify the preceding expression, we choose the fixed frame of reference XYZ such that $\vec{p}$ is along Z ($\vec{\varphi} = \vec{\theta} = 0$) and $\vec{p}'$ is in the XZ plane ($\vec{\varphi}' = 0$) and call $\theta_s = \bar{\theta}'$ the scattering angle between $\vec{p}$ and $\vec{p}'$ (see Fig. 4). From angular momentum conservation, we have $J = J'$ and $M = M'$, and Eq. (19) reduces to the following expression :

$$F_{\lambda'_1 \lambda'_2 \lambda'_1 \lambda'_2 \lambda} = \sum_{J S \Lambda S' \Lambda'} d_{\lambda - \Lambda \lambda' - \Lambda'}^J(\theta_s) D_{\Lambda \lambda_1 \lambda_2}^S(\varphi_q, \theta_q) D_{\Lambda' \lambda'_1 \lambda'_2}^{S' *}(\varphi'_q, \theta'_q) \quad (20)$$

$$< S' \Lambda' \lambda'_1 \lambda'_2 | T^J | S \Lambda \lambda_1 \lambda_2 >$$

This expression can be understood as the partial wave expansion of a quasi-two-body reaction $a_\lambda + (bc)_\Lambda \rightarrow a'_{\lambda'} + (b'c')_{\Lambda'}$ with states (bc) and $(b'c')$ having, respectively, spin and helicity $(S\Lambda)$ and $(S'\Lambda')$. The decay of each of these states into particles b and c (respectively, b' and c') is described by the angular dependence upon θ_q, φ_q (respectively, θ'_q, φ'_q).

We now go to the "forward direction" limit. Let us remark first that the (xyz) frame of reference in the (bc) rest frame is specified only by its z axis which is along $\vec{Q} = -\vec{p}$. We can take the y axis as being the Y axis which is orthogonal to $\vec{Q}$ anyhow. Similarly, this can be done for the (x', y', z') frame associated with the $(b'c')$ system, so that in the forward limit $\theta_s \rightarrow 0$ all three systems (X, Y, Z) , (x, y, z) and (x', y', z') finally coincide and $\varphi_q = \varphi'_q = \varphi$. In this limit ($\theta_s = 0$) only terms such that $\lambda - \Lambda = \lambda' - \Lambda'$ are non-vanishing in the partial wave expansion (20) (a well-known result of angular momentum conservation for two-body reactions), so that the φ dependence of the helicity amplitudes factorizes out. Explicitly, one obtains :

$$F_{\lambda'_1 \lambda'_2 \lambda'_1 \lambda'_2 \lambda} \simeq e^{i\varphi(\lambda - \lambda' + \lambda_1 - \lambda_2 - \lambda'_1 + \lambda'_2)} \quad (21)$$

$$F_{\lambda'_1 \lambda'_2 - \lambda'_1 \lambda_1 \lambda_2 - \lambda} \simeq e^{i\varphi(\lambda' - \lambda + \lambda_1 - \lambda_2 - \lambda'_1 + \lambda'_2)} \quad (22)$$

When only one type of naturality is exchanged in the crossed channel both amplitudes given in (21) and (22) should be equal up to a sign. A necessary condition is

$$\lambda = \lambda' \quad (23)$$

and consequently

$$\Lambda = \Lambda' \quad (24)$$

The conclusion is therefore that only amplitudes which conserve helicity at the aa' vertex survive in the "forward direction" for well-defined natural-ity exchanged in the crossed channel. At the same time helicity should be conserved at the $(bc), (b'c')$ vertex ($\Lambda = \Lambda'$). In that case the inclusive cross-section is independent of the initial polarization. On the other hand, when two states with opposite naturality are simultaneously exchanged and are linked by some constraint [which actually can be derived from Eq. (20)], there is no longer any reason for any amplitude to vanish, and one will observe polarization effects. In any case, the helicity amplitude can be rewritten as

$$F_{\lambda'_1 \lambda'_2 \lambda' \lambda_1 \lambda_2 \lambda} = \sum_{\Lambda \Lambda'} \mathcal{F}_{\Lambda' \lambda' \Lambda \lambda}(\lambda'_1, \lambda'_2, \lambda_1, \lambda_2, \theta_s, q) \delta_{(\Lambda' - \lambda') - (\Lambda - \lambda)} \quad (25)$$

i.e., F is given as a sum over all possible helicity non-spin-flip amplitudes for the pseudo two-body reaction $a+(bc) \rightarrow a'+(b'c')$ where the subsystems (bc) and $(b'c')$ have, respectively, helicity Λ and Λ' . Notice that the upper blob in the Mueller-Regge diagram of Fig. 1, which describes the fragmentation of b into c is therefore clearly not given by the same function for the unpolarized cross-section ($\Lambda = \Lambda'$) and for polarization measurements ($\Lambda - \Lambda' = \lambda - \lambda' \neq 0$).

4. CONCLUSIONS

We have shown in this paper under which conditions one may expect to observe polarization effects for inclusive reactions in the fragmentation region, assuming that the dynamical properties of the scattering amplitudes can be deduced from the Mueller-Regge phenomenology, in particular from the single Regge limit. We found that for the inclusive reaction $a+b \rightarrow c+X$, with c in the fragmentation region of b , the cross-section will be independent of the polarization state of particle a when the dynamics of the reaction can be represented by the Mueller diagram depicted in Fig. 1 with

exchange of states, in the crossed channel, with well-defined naturality. Polarization effects can be only observed when a state with mixed naturality is exchanged, implying either a kind of conspiracy mechanism or cut contributions⁸⁾. Our conclusions are quite similar to those of Abarbanel and Gross⁵⁾ with the important difference that we do not make use explicitly of factorization properties of Regge poles.

Clearly at high energy, when the Pomeron dominates the amplitudes, all polarization effects should disappear, due to its definite naturality, even if it does not conserve helicity. This is equivalent to the S.R.O. hypothesis that there are no long range correlations, including spin correlations. More generally, there should be no polarization effects in those reactions which are claimed to be exotic, whatever is the criterion used to define this class of reactions, due to the absence of secondary Regge exchanges.

Let us make clear once more that all the preceding remarks assume that c is a fragment of b and not a fragment of the polarized particle a . If this were the case, there would be no reason not to observe polarization effects even with only Pomeron exchange (short range effects). Actually, polarization has been observed in particle a fragmentation region for $\nu p \rightarrow \pi^- X$ ⁹⁾ and $K^- p \rightarrow \Lambda^+ X$ ¹⁰⁾.

It is easy to be convinced that all our arguments can be easily extended to correlation measurements, given in the Mueller-Regge language by the discontinuity of a $4 \rightarrow 4$ amplitude, corresponding to Fig. 1 with six legs in the upper blob. Correlations will be independent of the polarization of particle a when the exchange states have definite naturality.

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A P P E N D I X

NATURALITY CONSERVING AMPLITUDES

In this Appendix we generalize to $3 \rightarrow 3$ amplitudes the notion of naturality conserving amplitudes. We shall only sketch the proof as the derivation is just a direct and straightforward translation of the well-known results for two-body amplitudes.

i) In the crossed channel $a\bar{a} \rightarrow (bc) + (\bar{b}\bar{c})$ we consider the over-all centre-of-mass system $\vec{p}_a = \vec{p} = -\vec{p}_{\bar{a}}$ and $\vec{Q} = \vec{q}_b + \vec{q}_c = -(\vec{q}_{\bar{b}} + \vec{q}_{\bar{c}})$. We expand in partial waves the $|bc\rangle$ and $|\bar{b}\bar{c}\rangle$ systems, and with obvious notation, we write

$$\begin{aligned} |bc\rangle &= \sum_{S\Lambda} D_{\Lambda \lambda_1 - \lambda_2}^S(\varphi_q, \theta_q) |S\Lambda \lambda_1 \lambda_2\rangle \\ |\bar{b}\bar{c}\rangle &= \sum_{S'\Lambda'} D_{\Lambda' \lambda'_1 - \lambda'_2}^{S'}(\varphi'_q, \theta'_q) |S'\Lambda' \lambda'_1 \lambda'_2\rangle \end{aligned} \quad (A.1)$$

and deduce the partial wave expansion for reaction $a\bar{a} \rightarrow (bc) + (\bar{b}\bar{c})$

$$\begin{aligned} F_{\lambda\lambda' \lambda_1\lambda_2 \lambda'_1\lambda'_2} &= \sum_{JS\Lambda S'\Lambda'} d_{\lambda-\lambda' \Lambda-\Lambda'}^J(\theta) D_{\Lambda \lambda_1 - \lambda_2}^{S*}(\varphi_q, \theta_q) D_{\Lambda' \lambda'_1 - \lambda'_2}^{S'*}(\varphi'_q, \theta'_q) \chi \\ \chi &< \lambda\lambda' | T^J | S\Lambda \lambda_1 \lambda_2; S'\Lambda' \lambda'_1 \lambda'_2 \rangle \end{aligned} \quad (A.2)$$

where λ and λ' are, respectively, the helicity of particles a and $\bar{a}$, and θ the scattering angle between $\vec{p}$ and $\vec{Q}$.

We now consider the naturality conserving partial wave amplitudes, corresponding to naturality $\sigma = \pm 1$ in the $a\bar{a}$ channel

$$T^{J\pm} = \langle \lambda\lambda' | T^J | S\Lambda \lambda_1 \lambda_2; S'\Lambda' \lambda'_1 \lambda'_2 \rangle \pm \eta \bar{\eta} (-1)^{s-s'} \langle -\lambda - \lambda' | T^J | S\Lambda \lambda_1 \lambda_2; S'\Lambda' \lambda'_1 \lambda'_2 \rangle \quad (A.3)$$

where η and $\bar{\eta}$, s and s' are, respectively, the parity and spin of particles a and $\bar{a}$. Finally, using the fact that for large value of the missing mass M^2 , (i.e., energy of the abc system) $\cos\theta$ is also large, we derive that

$$d_{\lambda-\lambda', \Lambda-\Lambda'}^J(\theta) = (-)^{\lambda-\lambda'} d_{-\lambda+\lambda', \Lambda-\Lambda'}^J(\theta) + O(1/M^2) \quad (A.4)$$

It is now clear how to obtain the asymptotic relation (for $M^2 \rightarrow \infty$) defining the naturality conserving amplitudes. One finds :

$$F_{\lambda\lambda', \lambda_1\lambda_2, \lambda'_1\lambda'_2}^{\pm t} = F_{\lambda\lambda', \lambda_1\lambda_2, \lambda'_1\lambda'_2}^t \pm \gamma\bar{\gamma} (-)^{s-s'} (-)^{\lambda-\lambda'} F_{-\lambda-\lambda', \lambda_1\lambda_2, \lambda'_1\lambda'_2}^t \quad (A.5)$$

ii) Similarly, one can obtain naturality conserving helicity amplitudes in the over-all centre-of-mass system of the direct channel $abc \rightarrow a'b'c'$, using the factorization properties of the crossing matrix

$$F_{\mu'\mu'_1\mu'_2, \mu\mu_1\mu_2}^s = \sum_{\lambda_i \lambda'_i} D^{s'_1}(X'_1)^{\lambda'_1}_{\mu'_1} D^{s'_2}(X'_2)^{\lambda'_2}_{\mu'_2} d^{s'}(X_a)^{\lambda'}_{\mu'} \quad (A.6)$$

$$D^{s_1}(X_1)^{\lambda_1}_{\mu_1} D^{s_2}(X_2)^{\lambda_2}_{\mu_2} d^{s'}(X_a)^{\lambda'}_{\mu'} F_{\lambda\lambda', \lambda_1\lambda_2, \lambda'_1\lambda'_2}^t$$

It is an easy exercise from the symmetry relation $d^s(\chi)^\lambda_\mu = (-)^{\lambda-\mu} d^s(\chi)^{-\lambda}_{-\mu}$ to obtain from (A.5) and (A.6) the naturality conserving amplitudes in the direct channel

$$F_{\mu'\mu'_1\mu'_2, \mu\mu_1\mu_2}^{\pm s} = F_{\mu'\mu'_1\mu'_2, \mu\mu_1\mu_2}^s \pm \gamma\gamma' (-)^{s-s'} (-)^{\mu'-\mu} F_{-\mu'\mu'_1\mu'_2, -\mu\mu_1\mu_2}^s \quad (A.7)$$

Notice that definitions (A.5) or (A.7) are valid in any other frame of reference which do not change the orientation of $\vec{p}_a$ and $\vec{p}_{a'}$.

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FIGURE CAPTIONS

Figure 1 : Mueller-Regge diagram for the reaction $a(p)+b(q_1)+c(q_2) \rightarrow a'(p')+b'(q'_1)+c'(q'_2)$.

Figure 2 : Centre-of-mass frame of reference in the crossed channel $a\bar{a} \rightarrow (bc) + (\overline{bc})$.

Figure 3 : Centre-of-mass frame of reference in the direct channel $(\vec{p} = -\vec{q}_1)$.

Figure 4 : Over-all centre-of-mass frame of reference in the direct channel $(\vec{p} = -\vec{Q} = -\vec{q}_1 - \vec{q}_2)$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4