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UNITARITY SUM RULES AND THE TWO-REGGEON CUT

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A B S T R A C T

Using (s channel) unitarity sum rules, we derive consistency requirements on the two-Reggeon cut. We find a consistent solution with the (negative) absorptive sign of the cut, and we connect its magnitude to quantities encountered in Mueller's analysis of inclusive cross-sections. Prescriptions for cuts in multiparticle processes emerge. A positive sign seems to be incompatible with the sum rules combined with simple crossing requirements.

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Interest in the two-Reggeon cut problem has been recently revived by use of new techniques aiming at a rigorous, S matrix derivation of the Reggeon calculus of Gribov and collaborators ¹⁾. The solution of this problem at the two-Pomeron cut level, could be of primary importance ²⁾ in interpreting the new experimental data on total cross-sections and inclusive spectra at the ISR and NAL ³⁾. On the other hand, cuts should almost certainly be the answer to the decoupling theorems ⁴⁾ for a leading, factorizable Pomeron pole of intercept one. It is natural, therefore, to start one's program with the two-Reggeon cut problem.

White ⁵⁾ has attacked the problem from t channel (the channel where Regge singularities are exchanged) unitarity continued to complex angular momentum and helicity. Abarbanel ⁶⁾ has used instead s channel unitarity together with the notion of "two-Reggeon irreducible" graphs.

Although at first sight these two papers seem both to agree with Gribov's form of the two-Reggeon cut, a closer look reveals that the phase (or sign) of the cut contribution is different. Whereas White agrees with Gribov et al., and with the so-called absorption sign (referred to hereafter as the negative sign), Abarbanel has the positive sign originally found by Amati, Fubini and Stanghellini ⁷⁾.

Although the majority of people is more inclined to accept the negative sign, this issue is still being debated. Abarbanel's work can be criticized along the same lines of thought as Caneschi ⁸⁾ and Dash, Fulco and Pignotti ⁹⁾, but it is hard to make these arguments, compelling ¹⁰⁾. On the other hand, White's work starts from unitarity in the physical t channel region ($t > 0$) and the continuation to the s channel ($t < 0$) could be non-trivial. Finally, some intermediate prescription in which only part of the cut has a reversed sign has been advocated recently ¹¹⁾, in connection with the rising of total cross-sections at the ISR ³⁾.

In this paper we analyze the two-Reggeon cut problem starting from unitarity sum rules ¹²⁾ in the s channel physical region (hence a consequence of s channel unitarity). This seems to be particularly convenient for further applications since the equations we use can also be used in connection with the decoupling theorems ⁴⁾. The sum rules appear to be violated by the choice of a positive cut, but can be satisfied if the negative sign is adopted. One can also see the neglect of which terms in the sum rules yields the opposite conclusion.

Admittedly the argument presented here is not very rigorous, in the sense that we shall use a little loosely the concepts of Regge and helicity poles. It should be possible however to sharpen the argument considerably, by use of the techniques of White ⁵⁾ applied to the four and six-point functions.

We shall mainly use the non-forward sum rule connecting a four-point function to a six-point function in the form

$$\text{Disc}_s T_4(a+b \rightarrow a'+b') = \sum_c \text{Disc}_{M^2} T_6(a+b+c \rightarrow a'+b'+c). \quad (1)$$

This equation is represented graphically in Figure 1. The kinematical variables we shall use are:

$$s = (p_a + p_b)^2 \rightarrow \infty, \quad M^2 = (p_a + p_b - p_c)^2$$

$$t = (p_a - p_{a'})^2, \quad t_1 = (p_a - p_c)^2, \quad t_2 = (p_c - p_{a'})^2, \quad (2)$$

and we are interested in the limit $s \rightarrow \infty$, t fixed. The $\sum_c$ in Eq. (1) implies an integration over the momentum of particle c with a weight factor $(E_c + p_c^L) s^{-\frac{1}{2}}$, where $p_c^L(E_c)$ is the longitudinal momentum (energy) of particle c in the a, b center of mass system. Thus c is in the fragmentation region of a, a' .

We are interested in projecting out from both sides of Eq. (1) the Regge cut generated by two poles at $J = \alpha_1(t)$ and $J = \alpha_2(t)$. We claim that, on the right-hand side of Eq. (1) such cut contributions can only emerge in two distinct ways:

- 1) if we exchange the α_1, α_2 cut itself between $a\bar{a}'c\bar{c}$ and $b\bar{b}'$ i.e., in Mueller's a -fragmentation vertex ¹³⁾;
- 2) from the boundary of phase space where $0 < M^2 < \epsilon s$ with $\epsilon \ll 1$.

This comes from the fact that, outside of the boundary ($M^2/s > \epsilon$) and with both pionization and b fragmentation suppressed by the $(p_c^L + E_c) s^{-\frac{1}{2}}$ factor, we can always use Mueller's a -fragmentation vertex and, upon integration in the sense of the sum rule (1), we can only produce the same singularity that we exchange in the six-point function.

The sum rule (1) for the α_1, α_2 cut takes then the form of Fig. 2, where, in the region $0 < M^2/s < \epsilon$, we have used the ordinary single Regge limit (Fig. 2c), which reads, in formulae:

$$\text{Disc}_{M^2} T_{abc \rightarrow a'b'c} \xrightarrow{s/M^2 \rightarrow \infty} \sum_{i,j} \beta_{ac}^i(t_1) \beta_{a'c}^{j*}(t_2) R_i(s, t_1) R_j^*(s, t_2) F_{ij}^{bb'}(t_1, t_2, t, M^2), \quad (3)$$

where $\beta_{ac}^i(t_1)$, $\beta_{a'c}^j(t_2)$ are two ordinary two-particles-single Reggeon couplings and

$$R_i(s, t_1) = \{(\alpha_i(t_1)) s^{\alpha_i(t_1)}\}, \text{ same for } j, \quad (4)$$

with $\{(\alpha_i) = (e^{-i\pi\alpha_i} \pm \tau_i) / \sin \pi\alpha_i$, the usual signature factors. Of course there can be cut-type contributions in the t_1, t_2 channels and we simply keep them in mind for sake of saving space. $F_{ij}^{bb'}(t_1, t_2, t, M^2)$ is by definition the M^2 absorptive part of the $R_i(t_1) + b \rightarrow R_j(t_2) + b'$, particle-Reggeon scattering amplitude. When we integrate Eq. (3) in the region $0 < M^2 < \epsilon s$ and over t_1, t_2 we are led to consider the quantity

$$\Phi_{ij}^{bb'}(t_1, t_2, t, K) = \int_0^K F_{ij}^{bb'}(t_1, t_2, t, M^2) dM^2 \quad (5)$$

for $K \rightarrow \infty$ ($K \sim \epsilon s$). In general, one expects:

$$\Phi_{ij}^{bb'}(t_1, t_2, t, K) \xrightarrow{K \rightarrow \infty} N_{ij}^{bb'}(t_1, t_2, t) + \sum_e K^{\beta_e} N_{ij}^{\ell, bb'}(t_1, t_2, t), \quad (6)$$

where $N_{ij}^{bb'}(t_1, t_2, t)$ is (proportional to) the fixed pole residue ¹⁴⁾, and β_e are moving powers, typically of the triple Regge form

$$\beta_e = \alpha_k(t) - \alpha_i(t_1) - \alpha_j(t_2) + 1,$$

with $\alpha_k(t)$ the third pole exchanged in the t channel. The sum rule gives

$$-i \text{Disc } T_4 = \frac{1}{2(2\pi)^3} \int \frac{dt_1 dt_2 \Theta(-\lambda)}{s \sqrt{-\lambda(t, t_1, t_2)}} \beta_{ac}^i(t_1) R_i(s, t_1) \beta_{a'c}^{j*}(t_2) R_j^*(s, t_2) \Phi_{ij}^{bb'}(t_1, t_2, t, \epsilon s), \quad (7)$$

where, as usual, $\lambda = t_1^2 + t_2^2 + t^2 - 2t_1 t_2 - 2t t_1 - 2t t_2$.

We see immediately that the only contribution to the (α_i, α_j) cut comes from the $N_{ij}^{bb'}(t_1, t_2, t)$ piece in Eq. (6). Hence, we get

$$-i \text{Disc } T_4 \Big|_{\alpha_i, \alpha_j \text{ cut}} = \frac{1}{2(2\pi)^3} \int \frac{dt_1 dt_2 \Theta(-\lambda)}{S \sqrt{-\lambda(t, t_1, t_2)}} \beta'_{ac}(t_1) R_1(s, t_1) \beta_{ac}^{2*}(t_2) R_2^*(s, t_2) N_{12}^{bb'}(t_1, t_2, t). \quad (8)$$

We note that Eq. (8) contains the fixed pole residue linearly and not quadratically [as usual, see Refs. 1), 5), 6)] and that the sign is the "positive" one [in the sense of the complex conjugation of $R_2(s)$]. Of course, we have still to add the contribution of Fig. 2b and we first argue that this naturally brings up the square of $N_{12}^{bb'}(t_1, t_2, t)$.

One can see this by iteration, connecting first $\text{Disc } T_6$ to $\sum_d \text{Disc } T_8$, picking up the contributions from the boundary of phase space (Fig. 3a) and so on. This again selects $N_{12}^{bb'}(t_1, t_2, t)$ as indicated by the cross. By iteration one gets (Fig. 3):

$$-i \text{Disc } T_4 \Big|_{\text{cut}} = \frac{1}{2(2\pi)^3} \int \frac{dt_1 dt_2 \Theta(-\lambda)}{S \sqrt{-\lambda(t, t_1, t_2)}} R_1(s, t_1) R_2^*(s, t_2) N_{12}^{bb'}(t_1, t_2, t) N_{12}^{aa'}(t_1, t_2, t). \quad (9)$$

Equation (9) can be proved directly to be consistent if we use

$$-i \text{Disc } T_6 \Big|_{\text{cut}} = \frac{1}{2(2\pi)^3} \int \frac{dt_1 dt_2 \Theta(-\lambda)}{S \sqrt{-\lambda(t, t_1, t_2)}} R_1(s, t_1) R_2^*(s, t_2) N_{12}^{bb'}(t_1, t_2, t) N_{12}^{aa'cc}(t_1, t_2, t), \quad (10)$$

where $N_{12}^{aa'cc}(t_1, t_2, t, \dots)$ (shown in Fig. 4), being itself a Mueller-type discontinuity, satisfies the unitarity sum rule:

$$N_{12}^{aa'}(t_1, t_2, t) = \sum_c \left(\beta'_{ac}(t_1) \beta_{ac}^{2*}(t_2) + N_{12}^{aa'cc}(t_1, t_2, t, \dots) \right). \quad (11)$$

The pole term $\beta_{ac}^1 \beta_{a'c}^{2*}$ is of course provided by the contribution of Eq. (8) coming from the boundary of phase space.

If we stop at this point our result, Eq. (9), looks identical to Abarbanel's ⁶⁾. We must ask, however, whether we have taken into account all the contributions to the six-point function cut (Fig. 2b). We shall argue that there are two contributions that we have neglected, those

shown in Fig. 5b, c, in which the intermediate states building up the fixed pole are not exchanged in the channel $R_i + a + \bar{c} \rightarrow R_j + a' + \bar{c}$, (Fig. 5a), but in the channels $R_i + a \rightarrow R_j + c + \bar{c} + a'$, $R_i + a + c + \bar{c} \rightarrow R_j + a'$. We are not too sure here about the exact definition of these fixed poles in a rigorous theory of complex angular momentum singularities, but we can see at least these diagrams in a field theory diagrammatic sense as taking care of rescattering of the initial and final state in the production amplitudes whose overlap function leads to the four-point function absorptive part.

The fact that we have to introduce these extra terms together with that of Fig. 5a seems to be an inescapable consequence of crossing. Indeed, if we consider, in a $2 \rightarrow 4$ process, the s channel discontinuity governed by the cut, the analog of Fig. 5a becomes Fig. 5b, with one of the incoming lines turning into an outgoing one. Crossing tells us that such a contribution should also be there for the $3 \rightarrow 3$ process.

It is quite obvious that these extra contributions are enough to upset the agreement previously found. Indeed, when we integrate the M^2 discontinuity of Fig. 5b, we can again use the sum rules to yield the term indicated in Fig. 6, which clearly has a tendency to reverse the sign of the cut. Although we have not proven uniqueness, we have found that a consistent solution of the sum rules is that shown in Fig. 7. In that figure we present the general solution for the $n \rightarrow n$ Mueller type (but in general non-forward) discontinuity, where all particles except two (particles, b, b') are in the same fragmentation region. For simplicity we have not drawn in the figure the lower part of the diagram, which is the same for all terms (the two Reggeons coupled to b, b' through the fixed pole).

In checking the consistency of our prescription, we have to insist that the diagrams of Fig. 7 are simple S matrix products of two Regge amplitudes with real, on-shell intermediate states. Similarly, the real, as well as the imaginary, part of T_4 get cut contributions from real intermediate states only. This seems at first sight quite strange for diagrams such as that of Fig. 5b, c, where such real intermediate states are in the s channel and not in the M^2 channel where the discontinuity is taken, but it is in fact consistent with the absorptive model¹⁶⁾, with the result of White⁵⁾ and with Gribov's Reggeon calculus¹⁾. A little attention has to be paid therefore to the fact that such an S matrix product is not real-analytic, hence the continuation from $s+i\epsilon$ to $s-i\epsilon$ has to be done by hand¹⁷⁾. This explains the minus signs which appear in Fig. 7.

Finally, we have been perhaps too naive in taking the M^2 discontinuities of the six-point function (Fig. 7) in that we do not seem to allow a cut through both Reggeons. This appears to be justified for cuts involving two dual Reggeons¹⁸⁾ whose s channel structure (Fig. 8) is given by simple poles. On the other hand, we should remember that, when M^2 is continued around its singularities, other energies get also continued this same way. This brings contributions to $\text{Disc}_M^2 T_6$ which look like those resulting from "cutting both Reggeons".

In Mandelstam's diagram¹⁹⁾ for the two-Reggeon cut, there is indeed a discontinuity associated with "cutting both Reggeons" which is needed in order to restore the absorptive sign of the cut and the usual ratio of real to imaginary parts. For diagrams where the two Reggeons cannot be cut simultaneously, (e.g., the dual diagram of Fig. 8), the counting is slightly different from that in the Mandelstam diagram²⁰⁾ and such that the same final answer emerges. Hence, we think that our argument and our results should be valid irrespectively of the above complications.

In conclusion, we have obtained expressions for the two-Reggeon cut contribution to Mueller's type absorptive parts in $n \rightarrow n$ processes, in such a way to satisfy unitarity sum rules. The sign of the cut follows the absorptive model sign and agrees with Gribov and with White. The discrepancy with Abarbanel is to be attributed to Abarbanel's neglect of contributions such as those of Fig. 5b, c. Our approach deals directly with inclusive type discontinuities and relates the magnitude of the cut to inclusive cross-sections in various interesting cases. We hope, finally, that, in this approach, it will be possible to reconsider the problems of Pomeron coupling (and decoupling) without a complete neglect of its cuts.

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FIGURE CAPTIONS

- Figure 1 : Simplest, non-forward unitarity sum rule.
- Figure 2 : Projection of the sum rule on the α_1, α_2 cut
- a) cut term in the four-point function;
 - b) cut contribution to the fragmentation region of the six-point function;
 - c) boundary of phase space for the six-point function.
- Figure 3 : Iterative solution of the equation in Fig. 2. The cross means that the fixed pole contribution is to be picked up.
- Figure 4 : Unitarity sum rule for the fixed pole residue.
- Figure 5 : Possible contributions to the cut in the six-point function:
- a) contributions included in Ref. 6);
 - b), c) additional contributions neglected in Ref. 6).
- Figure 6 : A contribution to the sum rule coming from the diagram of Fig. 5b.
- Figure 7 : A solution for $\text{Disc } T_4$, $\text{Disc } T_6$, $\text{Disc } T_8$ consistent with unitarity sum rules. Figures should be completed by coupling the two Reggeons to b, b' as in Fig. 5 (i.e., through the fixed pole).
- The $\pm$ sign near the Reggeons indicate on which side of the branch cuts one has to continue the various energies. The dotted line indicates that a discontinuity is taken in the corresponding variable.
- Figure 8 : The simplest dual diagram giving a two-Reggeon cut.

FIG.1

FIG.2

FIG.3

$$\begin{array}{c} a \\ \nearrow \\ \bullet \\ \nwarrow \alpha_1 \end{array} \times \begin{array}{c} \bullet \\ \nearrow \alpha_2 \\ \nwarrow a' \end{array} = \sum_c \left[\begin{array}{c} a \\ \nearrow \\ \bullet \\ \nwarrow \alpha_1 \end{array} \xrightarrow{c} \begin{array}{c} \bullet \\ \nearrow a' \\ \nwarrow \alpha_2 \end{array} + \begin{array}{c} a \\ \nearrow \\ \bullet \\ \nwarrow \alpha_1 \end{array} \xrightarrow{c} \begin{array}{c} \bullet \\ \nearrow c \\ \nwarrow \alpha_2 \end{array} \xrightarrow{c} \begin{array}{c} \bullet \\ \nearrow a' \\ \nwarrow \alpha_2 \end{array} \right]$$

FIG.4

FIG.5

FIG. 6

$$\begin{aligned}
-2i \operatorname{Disc}_s T_4 &= \text{Diagram 1} + \text{Diagram 2} \\
-2i \operatorname{Disc}_{M^2} T_6 &= \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} \\
&\quad - \text{Diagram 6} + \text{Diagram 7} \\
-2i \operatorname{Disc}_{M^2} T_8 &= \text{Diagram 8} + \text{Diagram 9} \\
&\quad + \text{Diagram 10} + \text{Diagram 11} \\
&\quad - \text{Diagram 12} - \text{Diagram 13} \\
&\quad + \text{Diagram 14} + \text{Diagram 15}
\end{aligned}$$

The diagrams represent various Feynman-like topologies with external lines labeled a, a', c, d and internal lines with wavy lines and crosses. Signs (+, -) are indicated near the wavy lines.

FIG.7

FIG. 8