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THE TRANSVERSE MOMENTUM DISTRIBUTION IN THE CENTRAL
REGION IN A PHENOMENOLOGICAL DUAL MODEL

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A B S T R A C T

A phenomenological dual amplitude is evaluated for the transverse momentum distribution at $x=0$. It shows an $\exp(-bp_{\perp})$ dependence for low $p_{\perp}$ values ($p_{\perp} \leq 1 \text{ GeV}/c$) and a subsequent flattening off for larger $p_{\perp}$. This is compared with experiments at ISR. Some analogies to the statistical bootstrap model are discussed.

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The recent ISR measurements of the single particle inclusive distribution at $x=0$ have yielded large $p_{\perp}$ cross-sections which exceed those expected from the extrapolation at low $p_{\perp}$ by several orders of magnitude. Although this flattening off has been awaited on the basis of the parton model the new results ¹⁾ are not yet conclusive whether to favour a power law parametrization $(p_{\perp})^{-6}$ as predicted by the parton models ²⁾ and similar multiperipheral considerations ³⁾. It is well consistent with data to describe the observed distribution by a superposition of $\exp(-bp_{\perp}^2)$ functions which lead to bootstrap-like solutions. The theoretical reasons for such a description will be given subsequently in a phenomenological dual model.

In the dual resonance model (DRM) the corresponding inclusive process $a+b+\bar{c} \rightarrow X$ is described by the general, unitarized B_6 amplitude. However, up to now from this amplitude only terms of the form of tree diagrams ⁴⁾⁻⁷⁾ and single loop graphs ⁸⁾ have been calculated which do not allow double pomeron exchange. These calculations have resulted in a $\exp(-4\alpha' \mu^2)$ dependence for $x=0$ where α' is the slope of the usual Regge trajectory. Here $\mu = \sqrt{m_c^2 + p_{\perp}^2}$ (the transverse mass) is used. By now a μ^2 dependence is clearly ruled out by data, and it is evident that the leading, for $s \rightarrow \infty$ non-vanishing contribution at $x=0$ must be due to double pomeron exchange.

The simplest diagram representing that mechanism is the dual, non-planar graph of Fig. 1 with two loops. For this graph the phenomenological dual model has been applied recently ⁹⁾ with a B_6 amplitude which contained modifications due to pomeron background duality. This analysis showed that the data can well be described in this dual model by choosing $\alpha' = \frac{1}{2}$ in the region of small $p_{\perp}$ (< 1.3 GeV/c). Such a result is also expected from the exact calculation of the graph in Fig. 1 ¹⁰⁾. It should also be noted that the difference between the dual amplitude and the $\exp(-bp_{\perp})$ distribution is very small in the interval 0.5, ..., 1.3 GeV/c since there is a turning point. However, for large μ the graph in Fig. 1 like each single one falls off with an $\exp(-b\mu^2)$ dependence. Therefore, we want to investigate here a series starting with the graph of Fig. 1 such that for higher μ values more than two loops take over. The importance of these diagrams in the DRM has always been stressed ^{7), 10)}.

For the higher order loop diagrams we apply again that phenomenological dual six-point amplitude making use of the Harari-Freund conjecture of pomeron background duality which is generalized without ambiguities for the considered diagrams. The amplitude is obtained from the usual B_6 amplitude of the DRM by smoothing the poles ^{7),11)} to get cut structures in all channels dual to the pomeron ones ^{*}). This way to perform the smoothing procedure by an intercept integration leads to the $\tilde{B}_6$ (the new) amplitude expressed through the DRM B_6 function in the tree approximation:

$$\tilde{B}_6 = \int_0^{\gamma_{\bar{a}c}} d\lambda_1 \int_0^{\gamma_{a\bar{c}}} d\lambda_2 \int_0^{\gamma_{\bar{b}c}} d\lambda_3 \int_0^{\gamma_{b\bar{c}}} d\lambda_4 \int_0^{\gamma_{a\bar{c}b}} d\lambda_5 \varphi_{\bar{a}c}(\lambda_1) \varphi_{a\bar{c}}(\lambda_2) \varphi_{\bar{b}c}(\lambda_3) \varphi_{b\bar{c}}(\lambda_4) \varphi_{a\bar{c}b}(\lambda_5) \cdot$$

$$\cdot B_6(-\alpha_{\bar{a}c}(0), -\alpha_{b\bar{b}}(0), -\alpha_{\bar{a}c}^c(u) + \lambda_1, -\alpha_{a\bar{c}}^c(u') + \lambda_2, -\alpha_{\bar{b}c}^c(t) + \lambda_3,$$

$$-\alpha_{b\bar{c}}^c(t') + \lambda_4, -\alpha_{a\bar{c}b}(H^2) + \lambda_5, -\alpha_{\bar{a}c}(m_c^2), -\alpha_{b\bar{b}c}(m_c^2)) \quad (1)$$

with the B_6 function of Bardakçi-Ruegg ¹²⁾ and the ordering of the arguments as given there. The Mandelstam invariants are defined in Fig. 1. The γ 's denote the length of the cuts and the functions $\varphi(\lambda)$ are weight functions. For $x=0$ the γ and $\varphi(\lambda)$ have not to be specified reducing thus the ambiguity considerably.

By now the cut trajectories $\alpha_{\bar{a}c}^c(u)$ etc., enter in our calculation. We use the result of Mandelstam ¹³⁾ and other authors

$$\alpha_{\bar{a}c}^{cut}(u) = n_u [\alpha_R(0) - 1] + 1 + \frac{\alpha_R'}{n_u} \cdot u \quad (2)$$

where n_u specifies the number of reggeons which produce the cut in the u channel ($\alpha_R(u) = \alpha_R(0) + \alpha_R' \cdot u$). This form of the cut trajectories is used in Eq. (1).

^{*}) For the graph in Fig. 1 background channels are the (1,2), (5,0), (2,3), (4,5) and the (1,3) channels. Numbers are given in Fig. 1.

In the kinematical limit where we consider Eq. (1), ($x=0$, $s \rightarrow \infty$)

$$-u = \mu \sqrt{s}, \quad -t = \mu \sqrt{s}, \quad M^2 = s - 2\mu \sqrt{s} \quad (3)$$

the intercepts can be neglected against the slope part of the trajectories.

$$\begin{aligned} \tilde{B}_c(x=0) = C \cdot B_c \left(-\alpha_{a\bar{a}}(0), -\alpha_{b\bar{b}}(0), -\frac{\alpha'_R}{n_u} \cdot u, -\frac{\alpha'_R}{n_u} \cdot u, -\frac{\alpha'_R}{n_t} \cdot t, -\frac{\alpha'_R}{n_t} \cdot t, \right. \\ \left. -\frac{\alpha'_R}{n_{M^2}} \cdot M^2, -\alpha_{a\bar{a}c}(m_c^2), -\alpha_{b\bar{b}c}(m_c^2) \right) \equiv \tilde{f}(n_u, n_u, n_t, n_t, n_{M^2}) \end{aligned} \quad (4)$$

and C denotes the loop integration factor depending on the integers n .

$$C = \int_0^{\gamma_{a\bar{c}}} d\lambda_1 \varphi_{a\bar{c}}(\lambda_1) \int_0^{\gamma_{a\bar{c}}} d\lambda_2 \varphi_{a\bar{c}}(\lambda_2) \int_0^{\gamma_{b\bar{c}}} d\lambda_3 \varphi_{b\bar{c}}(\lambda_3) \int_0^{\gamma_{b\bar{c}}} d\lambda_4 \varphi_{b\bar{c}}(\lambda_4) \int_0^{\gamma_{a\bar{c}b}} d\lambda_5 \varphi_{a\bar{c}b}(\lambda_5) \quad (5)$$

The discontinuity in the missing mass (m) channel of Eq. (4) can be calculated in analogy to the tree case ⁴⁾⁻⁷⁾. The only difference is now its dependence on the integers n . It shows the qualitative features of the tree case. There is a finite region around the turning point beginning at $\mu \approx 0.2$ GeV/c where it is well represented by $\exp(-b\mu)$ followed by the steeper decrease which for the symmetric case $n_u = n_u$, $n_t = n_t$, comes out to be

$$\text{disc } \tilde{f} \propto \mu^{-2(\alpha_{a\bar{a}}(0) + \alpha_{b\bar{b}}(0)) - 1} \exp\left(-4\alpha'_R \frac{n_{M^2}}{n_u n_t} \mu^2\right) \quad (6)$$

which reduces to the tree case for $n_u = n_t = n_{M^2} = 1$.

To classify the different possibilities of cut combinations we discuss the three-loop diagram, Fig. 2, which then can easily be extended to more loops.

There are the possibilities of having three or four reggeons in mm ($n_{M2}=3,4$) and $n_u, n_t, n_{u'}, n_{t'}=2,3,4$. Each internal loop beyond the two we started with is associated with the introduction of a factor g^2 (the square of the three-reggeon coupling). Then the single particle distribution is given by

$$f(x=0, \mu) = \frac{A}{S} \sum_{n=3}^{\infty} g^{2(n-3)} \sum_{\substack{n_{M2}=3, \dots, n \\ n_u, n_t, n_{u'}, n_{t'} = 2, \dots, n}} \text{disc}_{M2} \tilde{f}(n_u, n_t, n_{u'}, n_{t'}, n_{M2}) \quad (7)$$

where n is the maximum number of reggeons which can be cut in mm. A is the free normalization constant which contains all the couplings of the graph of Fig. 1. As an assumption we have to postulate that the coupling g incorporates already the effects of the loop integrations such that C can be assumed to be a constant.

In order to show the behaviour of the sum in Eq. (7) we display the single terms $\text{disc}_{M2} \tilde{f}$ for $n_{M2}=3$, $n_u, n_{u'}, n_t, n_{t'}=n$ and $C=1$ and various n in Fig. 3. Within each class n the presented curves give the dominant contributions. However, the sum over the other possibilities is not negligible and changes the μ distribution. In Fig. 4 the partial sums over $n \leq l$, $l=3, 4, 5, 12$ are shown. All partial sums are normalized to the experimental value at $\mu=0.2$ GeV/c.

It is seen from Fig. 3 that the sum in Eq. (6) tends to a limiting function in such a way that for low μ values one or a few terms are sufficient. For higher μ values it has to be extended over more and higher n values. For $\mu \leq 9$ GeV/c contributions from n larger than 12 are negligible. The number of n terms necessary to reach about the limiting value for a given μ depends critically on the chosen coupling g .

The sum up to $n=12$ is shown in Fig. 5 for three different couplings together with experimental points. It is seen by comparison of Figs. 4 and 5 that the coupling constant can be determined essentially through the curvature up to 2 GeV/c. Then the behaviour for all higher μ values is completely determined. For the chosen values $g^2=0.05 \dots 0.15$ the agreement with the experimental results is rather good keeping in mind

the various approximations. The most prominent ones are the neglect of spin, the same statistical weight C given to each cut combination for diagrams with the same power of g , and the asymptotic s values ^{*}). In order to look for the behaviour of the sum in Eq. (7) without numerical calculations we perform in the following a partial summation of those graphs with $n = n_M = n_u = n_t$ over all n values. These graphs determine the distribution up to $\mu \simeq 1.6$ GeV/c as we know from the numerical analysis (compare e.g., Fig. 4).

By inserting Eq. (6) into Eq. (7) one has

$$f(0, \mu) \propto \sum_{n=3}^{\infty} e^{-n \cdot \gamma} \left(\frac{n}{\beta}\right)^{\frac{p}{2}+1} e^{-\frac{\beta}{n}} \quad (8)$$

where $\beta = 4 \alpha_R' \mu^2$, $\gamma = -2 \ln g$, $p = -2(\alpha_{a\bar{a}}(0) + \alpha_{b\bar{b}}(0)) - 3$. The sum can be replaced by a continuous integration, since the terms with $n = n_M = 2$, $n_u \neq n_u$, $n_t \neq n_t$, do provide the necessary background. The sum is also started at $n=0$ since these additional contributions are negligible for $\mu > 0.2$ GeV/c. Then one has the integral ($\alpha_{a\bar{a}}(0) = \alpha_{b\bar{b}}(0) = 1$):

$$f(0, \mu) \propto \frac{1}{(\beta)^{2.5}} \int_0^{\infty} dx x^{5/2} e^{-\beta/x} e^{-\gamma \cdot x} = \frac{2}{\beta^{2.5}} \left(\frac{\beta}{\gamma}\right)^{7/4} K_{7/2}(\sqrt{4 \cdot \beta \cdot \gamma}) \quad (9)$$

which reads

$$f(0, \mu) \propto (4 \mu \sqrt{\alpha_R'})^{-3/2} K_{7/2}(4 \mu \sqrt{\alpha_R' \cdot \gamma}) \quad (10)$$

^{*}) For non-asymptotic s values one however sees that the linear part of the distribution in the logarithmic plot extends to higher μ values since the drop-off of the exponentials in the B_6 amplitude, Eq. (1) is increased by the intercepts.

if one reverses the various definitions. Now using the approximate representation of the K functions for large arguments

$$K_{7/2}(4\mu\sqrt{\alpha'_R \gamma}) \approx \sqrt{\frac{\pi}{8\mu\sqrt{\alpha'_R \gamma}}} \exp(-4\mu\sqrt{\alpha'_R \gamma}) \quad (11)$$

one realizes the linear dependence of the e function on μ . As compared to the thermodynamical bootstrap the model predicts the same dependence on μ in the exponential. However, the power of $(4\mu\sqrt{\alpha'_R})$ in Eq. (10) comes out to be different ¹⁴⁾. This difference has the reason that the parametrization of disc B_6 by Eq. (6) does not work for small and intermediate μ . The μ singularity $p = -2(\alpha_{aa}(0) + \alpha_{bb}(0)) - 3$ is too strong and has to be weakened for non-asymptotic μ values. Setting, however, $p = -2$ gives complete similarity to the bootstrap solution, even in the index of the K function.

The partial sum over special index combinations which was discussed above represents the total sum only for μ values up to 2 GeV/c. For higher μ it represents a lower bound. In that region the class with $n \neq n_{M2}$ and the difference $n - n_{M2}$ as large as possible gives the leading contribution. We are not yet able to present a closed form of the total summation of Eq. (7) to estimate the effect of the rise beyond the linear part and to determine the asymptotic μ behaviour.

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 :

The diagram shows the non-planar, two-loop graph which is identified with double-pomeron exchange.

Figure 2 :

The three-loop graph with the different possibilities to choose the n integers in the missing mass and u channel.

Figure 3 :

The distribution $\text{disc}_{M^2} \tilde{f}$ [Eq. (6)] for special diagrams with $n_{M^2}=3$, $n_u=n_t=n_{u^*}=n_{t^*}=n$ for various n . These graphs provide the leading contribution to the summation.

Figure 4 :

The partial summation of Eq. (7) for $n \leq \ell$ with $\ell=3, 4, 5, 12$. The shaded area for $\ell=12$ denotes a possible numerical error in the calculation of the double integrals. For this plot the coupling constant g^2 is 0.10.

Figure 5 :

The inclusive distribution $f(0, \mu)$ in mb for various couplings $g^2=0.05, 0.10, 0.15$. For comparison experimental points for π^+ at $\sqrt{s}=44.4$ GeV from M. Jacob, Batavia report (Fig. 2a) and the linear curve which fits for small μ are plotted.

Fig. 1

Fig. 2

Fig. 3

Fig.4

Fig.5