QUARK IMPRISONMENT AS THE ORIGIN OF STRONG INTERACTIONS

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ABSTRACT

We propose a formal scheme in which the only dynamical ingredients are weak and electromagnetic interactions with quarks and leptons treated on the same footing. Strong interactions are generated by the requirement that quarks do not appear physically.

It is amazing to observe how experiments seem progressively to agree with predictions of an underlying quark field theory in a quarkless world. Evidence for that comes from hadron spectroscopy and, more surprisingly, from deep inelastic lepton induced reactions. A framework that qualitatively describes all this experimental information is that of quasi-free quarks bounded in zero triality hadrons. The best picture - for both the spectrum and deep inelastic data - is that of fractionally charged quarks carrying a funny new quantum number - the colour - whose only property is to be unobservable through the requirement that physical states are colour singlets ¹⁾.

In this note, we shall attempt to give a field theoretical content to this idea. In so doing, a coloured gluon naturally appears and the theory exhibits a coloured gauge invariance.

In our approach, strong interactions appear as the consequence of the - empirical - constraint that quarks are not physical observable states. In other words, strong interactions do not represent a new dynamics, i.e., an explicit term in the action, but they appear as a manifestation of boundary conditions on weak and electromagnetic interactions.

The arguments we shall present are highly formal and we are not able, at this stage, to propose computable techniques for physical quantities.

The introduction of the colour implies that our quarks are the fundamental representation of $G \otimes SU(3)^c$ where G is the group which classifies observed hadrons and $SU(3)^c$ is the coloured group. Present experimental evidence is that G is itself an $SU(3)$ group and we will adopt this in the following. The extension to higher groups - as required by some models of weak and electromagnetic interactions which introduce charmed particles - is of course trivial.

A formalism in which boundary conditions are easy to handle is the Feynman path integral method. In a theory in which quarks and leptons interact through weak interactions, we shall write the generating functional as

$$Z(W_\mu^i) = \int_B Dq D\bar{q} \exp i \int d^4x \left[\mathcal{L}_w + \sum_{i=1}^{16} j_\mu^i W_\mu^i \right] \quad (1)$$

where $\mathcal{L}_w$ is the Lagrangian of a unified weak and electromagnetic interaction. The integration over the lepton degrees of freedom is tacitly assumed while B in Eq. (1) represents the boundary condition over the quark degrees of freedom which guarantees the lack of colour excitation. The currents j_μ^i , coupled to the external sources W_μ^i , are the observable (colour singlets) weak and electromagnetic currents which, for what regards the quark degree of freedom, are of the form :

$$j_\mu^i = \begin{cases} \bar{q} \gamma_\mu \frac{\lambda_i}{2} q & i = 1 \dots 8 \\ \bar{q} \gamma_\mu \gamma_5 \frac{\lambda_i}{2} q & i = 9 \dots 16 \end{cases} \quad (2)$$

Our aim being not the discussion of weak and electromagnetic interactions, let us consider the theory at zeroth order in them. Besides the quark kinetic term, a quark mass term $m\bar{q}q$ will appear at order $(\alpha)^0$ (α being the fine structure constant), even in a theory in which masses are provided by a spontaneously broken symmetry. It is known ²⁾ that γ_5 contributions to such a term can always be transformed away. At order $(\alpha)^0$ and avoiding writing the lepton degrees of freedom which are free at this order :

$$Z(W_\mu^i) = \int_B \mathcal{D}q \mathcal{D}\bar{q} \exp i \int \mathcal{L} [q i \not{\partial} q - m \bar{q} q + \sum_i j_\mu^i W_\mu^i] \quad (3)$$

The Green functions of the currents are - as usual - expressed as

$$\langle 0 | T(j_{\mu_1}^{i_1}(x_1) \dots j_{\mu_n}^{i_n}(x_n)) | 0 \rangle = \frac{1}{i^n Z(0)} \left(\frac{\partial^n Z}{\partial W_{\mu_1}^{i_1}(x_1) \dots \partial W_{\mu_n}^{i_n}(x_n)} \right)_{W=0} \quad (4)$$

Since all physical states can be excited from the vacuum by suitable combinations of currents, the Green functions of Eq. (3) completely determine the theory.

Were it not for the restriction of integration to the region B, the theory would describe coloured quarks freely propagating. The restriction to B must be therefore responsible for strong interactions. We will, in fact, show that this restriction can be represented by an effective interaction term in the action of Eq. (6).

In order to find a formal expression for the integration region B, let us first remark that to avoid the appearance of coloured states it is insufficient to require the vanishing of the colour charges. Indeed, a state with neutral charge can contain opposite charges in arbitrarily separated regions. The condition must therefore imply that the coloured charges of an arbitrary volume element should vanish and, therefore, that the colour charge densities vanish. Thanks to relativistic invariance, this implies that the colour currents vanish locally. This can be physically understood by the remark that the colour currents measure the flux of colour states in space-time.

Therefore, the restriction to B in Eq. (4) requires that the integral over the functions $q(x)$ and $\bar{q}(x)$ runs only over the paths which correspond to vanishing colour currents ^{*)}. Defining the colour currents C_μ^i

$$C_\mu^i = \bar{q} \gamma_\mu \frac{\lambda_i}{2} q \quad i=1 \dots 8 \quad (5)$$

where the λ_i matrices are the same as the λ_i but operate on colour indices, we have

$$Z(W) = \int Dq D\bar{q} \delta(C_\mu^i) \exp i \int d^4x [\bar{q} i \not{\partial} q - m \bar{q} q + \sum_{i=1}^8 \int_\mu A_\mu^i W_\mu^i] \quad (6)$$

where, with the usual functional notation

$$\delta(C_\mu^i) \equiv \prod_{x, \mu, i} \delta(C_\mu^i(x))$$

Equation (6) can be written in the form

$$Z(W) = \int Dq D\bar{q} \prod_{\mu, i=1, \dots, 8} \delta(A_\mu^i) \exp i \int d^4x [\bar{q} i \not{\partial} q - m \bar{q} q + \sum_{i=1}^8 \int_\mu A_\mu^i + \sum_{i=1}^8 \int_\mu W_\mu^i] \quad (7)$$

*) Should the quark mass term in Eq. (3) vanish, the coloured symmetry group of the theory is enlarged to a chiral $SU(3) \otimes SU(3)$. The currents generated by the group would be vector and axial coloured currents and both should vanish in order to guarantee colour non-excitation. It goes without saying that the mass term is required in our world.

Equation (7) looks like the generating functional of a theory of coloured quarks coupled with a coloured gluon in which, however, the kinetic gluon term $-\frac{1}{4}F_{\mu\nu}^i F_{\mu\nu}^i$ is missing. $F_{\mu\nu}^i$ is given by

$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i - f_{ijk} A_\mu^j A_\nu^k \quad (8)$$

The action in Eq. (7) is, however, invariant under colour gauge transformations. Let us stress that the boundary condition is responsible for the colour gauge invariance of the theory. In Eq. (6), where only quarks are present, this is due to the fact that in colour gauge transformations the action changes by a quantity proportional to C_μ^i which vanishes by the delta function in the integration measure. The δ function changes by a rotation of the argument, but this gives only rise to an over-all factor which is compensated by the denominator of Eq. (4).

Due to the lack of the gluon kinetic term, the theory of Eq. (7) is not a Lagrangian field theory. It is, however, a strong coupling limit of a gauge Lagrangian field theory. Indeed, we can add to the integrand in the exponential in Eq. (7) a term

$$\lim_{g \rightarrow \infty} -\frac{1}{4g^2} F_{\mu\nu}^i F_{\mu\nu}^i$$

By rescaling the A_μ^i field, we find

$$Z(W) = \lim_{g \rightarrow \infty} \int Dq D\bar{q} \prod_{\mu,i} DA_\mu^i \exp i \int d^4x \left[\bar{q} i \not{\partial} q - m \bar{q} q - \frac{i}{4} (\partial_\mu A_\nu^i - g f_{ijk} A_\mu^j A_\nu^k)^2 \right. \\ \left. + g \sum_{i=1}^8 C^i A_\mu^i + \sum_{i=1}^{16} J_\mu^i W_\mu^i \right] \quad (9)$$

This theory, before the $g \rightarrow \infty$ limit is taken, corresponds to a gauge theory of coloured quarks and gluons in which only coloured singlet currents are coupled to external sources. Of course, before taking the $g \rightarrow \infty$ limit, coloured states are excited in pairs.

The theory is manifestly invariant under the colour gauge group and the infinite volume element which cancels in the ratio of Eq. (4) can be extracted with the usual Fadeev-Popov method ³⁾. This, of course, can also be done directly in Eq. (7).

For finite g the theory will produce not only coloured quarks, but also coloured gluons. For large g the relevant values of the functions $A(x)$ and $C(x)$ decrease as $1/\sqrt{g}$ (as seen from stationary phase arguments) thanks to the non-Abelian character of the coloured group. In the $g \rightarrow \infty$ limit we recover the fact that the gluon degree of freedom disappears and that in the quark sector only the coloured singlet agglomerates can be excited. The $g \rightarrow \infty$ limit gives rise, in a quark gluon scheme, to dense Feynman diagrams which have been argued to produce only composite particles with spectrum and interactions typical of dual models ⁴⁾.

In our scheme, single quarks cannot be observed, even locally. One may therefore wonder how quarks can accomplish the task of explaining deep inelastic scaling which is commonly associated with single quark free propagation on the light cone. Indeed, scaling is connected to the light cone singularity of a product of two observable currents j_μ^i . The bilocal operators, whose matrix elements are measured in deep inelastic experiments, are coloured gauge invariant quantities. In the formalism of Eq. (1), where only quark degrees of freedom are present, it is a hard task to construct such an object. The equivalent expression of Eq. (2) is more suited to that purpose : before integrating over A_μ^i the action is the one of quarks in an external coloured field. By applying techniques similar to those of Ref. 5), the bilocal $O_\mu^i(x,y)$ can be shown to have the form :

$$O_\mu^i(x,y) = \bar{q}(y) L_z e^{-i \int_x^y \sum_{i=1}^8 \chi_i A_\mu^i dz_\mu} \partial_\mu \frac{L_z}{2} q(x) \quad (10)$$

where L_z represents a line ordering. This line ordering arises from the non-Abelian character of our coloured gauge group and is due to the fact that the matrices $\sum_i A_\mu^i \chi_i$ at sufficient space-time points do not commute. Using the techniques introduced by Feynman ⁶⁾, this non-commutativity can be handled by introducing a formal dependence of $\chi_i(z)$ on the space-time point and ordering all expressions so obtained.

Moreover, at fixed z the different $\chi_i(z)$ do not commute and therefore we cannot still reduce the sum over i in the exponent of Eq. (10) to a product of exponentials, which would be needed for our subsequent arguments. To do that we must apply the Feynman tricks again, and give to the $\chi_i(z)$ an extra dependence on a variable, call it s , with respect to which all our subsequent expressions are to be ordered. We can therefore write

$$Q_{\mu}^i(x,y) = \bar{q}(y) L_{z,s} \prod_i e^{-i \int_0^y ds \int_x^y dz \chi_i(z,s) A_{\mu}^i(z)} \delta_{\mu} \frac{1}{z} q(x) \quad (11)$$

In order to discuss matrix elements of the bilocal operator, let us consider a generic Green function

$$\frac{1}{Z(0)} \int \mathcal{D}q \mathcal{D}\bar{q} \prod_{\mu,i} \mathcal{D}A_{\mu}^i e^{i \int dx (\bar{q} i \not{\partial} q - m \bar{q} q)} \Gamma Q_{\mu}^i(x,y) \quad (12)$$

where Γ is a product of observable currents j_{μ}^i .

In order to perform the $\mathcal{D}A_{\mu}^i$ integration in Eq. (12), it is convenient to introduce light cone variables in the frame defined by $x=0$, $y_{-} \equiv y_0 - y_3 = y_T = 0$. The integration can be formally performed and yields :

$$\begin{aligned} & \frac{1}{Z(0)} \int \mathcal{D}q \mathcal{D}\bar{q} \prod_{\mu,i} \mathcal{D}A_{\mu}^i e^{i \int dx (\bar{q} i \not{\partial} q - m \bar{q} q)} \Gamma \\ & \cdot \bar{q}(y) \prod_{i,\mu, z \in [0,y]} \delta(C_{-}^i) \delta(C_{+}^i) L_{z,s} \delta(C_{+}^i - \int_0^y dz \int_x^y ds \chi_i(z,s)) q(0) \cdot \\ & \cdot \prod_{i,\mu, z \in [0,y]} \delta(C_{\mu}^i(z)) \end{aligned} \quad (13)$$

The line ordered δ function in Eq. (13) is a formal notation for the exponential in Eq. (12). If the group were Abelian, the χ_i should be replaced by unity in Eq. (13), the line ordering would not be necessary and the δ function would be a usual functional δ function. Anyhow, for what regards the colour indices, the line order represents an infinite product of matrices - all ordered along the light cone segment $[0,y]$ - sandwiched between $\bar{q}(y)$ and $q(0)$.

We find therefore that, owing to the $\bar{q}(y)$ and $q(0)$ in Eq. (13), we generate a coloured current, along the light cone segment between them, which has a constant value. From the physical point of view, this implies that having two quarks which are unobservable objects, localized in different space-time points, the theory provides with a polarization current which neutralizes their effect as far as colour is concerned. Indeed, the presence of that coloured current is due to gauge invariance. To understand this fact

let us perform a coloured gauge transformation on the quark field. Besides transforming the two quark functions $\bar{q}(y)$ and $q(0)$, the currents C in the arguments of the δ function and the kinetic term in the exponential are also transformed.

The line ordered product of δ functions of transformed quantities can be recast as products of the original ones, using the Feynman techniques ⁶⁾. This gives rise to an over-all transformation that cancels the q and $\bar{q}$ variations and, besides, a line ordered exponential factor that cancels exactly the variation of the kinetic piece. This form of compensation reminds the one characteristic of a polarization current. In this sense, we can interpret the bi-local as two localized quarks. However, the initial condition of a local vanishing of colour currents imposes that together with the two quarks we unavoidably find a polarization current between them. That the polarization current is localized on the light cone segment instead of being everywhere spread, is a consequence of the integration over A_μ^i which, on the other hand, guarantees quark imprisonment. This is suggestive of a string interpretation which is, however, different from the one suggested by Fritzsche and Gell-Mann ⁷⁾. Indeed, their line element is that of the A_μ^i field in the expression of Eq. (10), while ours is related to the polarization current between the quarks and is a consequence of their not coming out. It is amusing that both here, as well as in the dense Feynman diagram analogy, the connection with dual models is enforced by the boundary condition that quarks are not present. Stimulated by this analogy, we are presently investigating the possibility of using dual theoretical techniques in order to analyze the spectrum in our approach. We expect that, as in dual models, the dynamics of hadron events and the spectrum of states should appear together.

A feature which, anyhow, we expect to be valid is that the structure of final states should be essentially similar in hadron and lepton induced events. This is because in our approach strong interactions are not a new dynamical item, the only fundamental dynamical interactions being the weak and electromagnetic ones.

To conclude, we want to stress that the picture that emerges from this still very formal theory seems to us very appealing. It represents a change of philosophy : instead of looking for hadrodynamical mechanisms to prevent quarks to come out, we propose that it is the latter which produces the first.

The only basically dynamical ingredient of the theory would be the unified weak and electromagnetic interactions involving quarks and leptons on the same grounds. The only difference among them is that leptons are observed and quarks are not. This unobservability is, in our scheme, the origin of strong interactions.

R E F E R E N C E S

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