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QUARK TRIPLETS IN SELF-INTERACTION AND THE LIGHT CONE STRUCTURE
OF CURRENT COMMUTATORS

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A B S T R A C T

A field theory of basic quark fields with current-current self-interaction is analyzed with respect to the leading singularity structure of current commutators on the light cone. Canonical leading singularities are found to be compatible with the four-fermion interaction whereas the structure of the bilocal operators differs essentially from the free field case.

Ref.TH.1709-CERN

25 July 1973

INTRODUCTION

Formal aspects of a field theory of permanently bound quark excitations interacting through the exchange of a "gluon" vector field have been shown to lead to the free field behaviour of current commutators on the light cone ¹⁾ as postulated by Fritzsche and Gell-Mann ²⁾. Moreover, in these treatments, the light cone behaviour of the gluon fields is assumed to be smooth enough in order not to interfere with the canonical behaviour of the currents.

We discuss the current-current (four-fermion) interaction between three coloured triplets of quark fields. As is well known from weak interaction theories involving an effective four-fermion interaction on one side, intermediate boson couplings on the other side, the algebraic structure of the two approaches is quite similar.

The current-current interaction cannot be reconciled with the free field structure of current commutators on the light cone although canonical light cone singularities can be imposed as a boundary condition. In the terminology of the quark parton model ³⁾ the scattering of a lepton current off an individual parton inside a hadron is not in general an incoherent process in the scaling limit, assuming a current-current interaction.

Sum rules derived from the local limits of the bilocal operators like the Adler sum rule ⁴⁾ and the Gross-Llewellyn Smith sum rule ⁵⁾ are valid but the local limit of derivatives of the bilocal operators differ from corresponding expressions for free fields.

Especially, the "no-gluon" sum rule ²⁾ is not valid even though there are no gluons.

The current-current interaction introduces contributions to the energy-momentum tensor with canonical dimension 6, and furthermore is non-renormalizable. These features present no obstacle to a canonical light cone singularity structure. In particular the conventional renormalization procedure is not applicable to this situation where the basic (quark) fields are constrained not to yield asymptotically free stable particles.

The paper is organized as follows. In Section 1, quantization on a plane perpendicular to a light-like ray is carried out. In Section 2, we derive the light cone structure of transverse (good-bad) current components and compare it with the corresponding quantities in the gluon model. In Section 3 we discuss the phenomenological implications of the above scheme for deep inelastic electron and neutrino scattering,

1. - EQUATIONS OF MOTION, HAMILTONIAN ON A PLANE PERPENDICULAR TO A LIGHT-LIKE RAY

We consider three triplets of fermion fields ⁶⁾

$$q_a^i(x) \quad ; \quad i = \text{red, green, blue} \\ a = u \text{ (up), } d \text{ (down), } s \text{ (strange)}$$

The equations of motion shall be derived from the Lagrangian density

$$\mathcal{L} = \frac{i}{2} \bar{q}_a^i \overleftrightarrow{\partial}_\mu q_a^i - \bar{q}_a^i M_{ab}^{ij} q_b^j \\ - \frac{g}{8} (V_{ba}^\mu V_{\mu ab} + A_{ba}^\mu A_{\mu ab}) \quad (1.1)$$

In Eq. (1.1) M denotes the mass matrix

$$M_{ab}^{ij} = \delta^{ij} \int_{ab} m_b \quad ; \quad m_u = m_d$$

V, A denote the vector and axial vector currents

$$V_{ba}^\mu = \bar{q}_b^i \gamma_\mu q_a^i \quad ; \quad A_{ba}^\mu = \bar{q}_b^i \gamma_\mu \gamma_5 q_a^i \quad (1.2)$$

and g is a coupling constant analogous to the Fermi coupling constant for weak interactions.

The above Lagrangian yields attractive forces between quark-antiquark pairs, repulsive forces between quark (antiquark) pairs ⁷⁾. The colour degree of freedom will not be relevant for the subsequent considerations. It is still explicitly treated because of the consistency of the interaction with the structure of bound state hadrons.

The equations of motion take the form

$$i \gamma^\mu \partial_\mu q_a^i - m_a q_a^i = g \left\{ \begin{aligned} & j_{ba}^+ \gamma_\mu \frac{1+\gamma_5}{2} q_b^i \\ & + j_{ba}^- \gamma_\mu \frac{1-\gamma_5}{2} q_b^i \end{aligned} \right\} \quad (1.3)$$

$$j_{ba}^{\pm \mu} = \frac{1}{4} (V_{ba}^\mu \pm A_{ba}^\mu)$$

The breaking of the chiral $SU_3 \times SU_3$ symmetry is due only to the mass terms in the Lagrangian of Eq. (1.2) as advocated by Gell-Mann, Oakes and Renner ⁸⁾

$$\partial_\mu V_{ba}^\mu = \frac{1}{i} (m_a - m_b) S_{ba}$$

$$\partial_\mu A_{ba}^\mu = i (m_a + m_b) P_{ba} \quad (1.4)$$

$$S_{ba} = \bar{q}_b^i q_a^i ; P_{ba} = \bar{q}_b^i \gamma_5 q_a^i$$

It will prove useful to express Eq. (1.3) through the right and left-handed spinor fields

$$(\tilde{u}_a^i)_\alpha \sim \frac{1-\gamma_5}{2} q_a^i ; (v_a^i)^\alpha \sim \frac{1+\gamma_5}{2} q_a^i$$

The relevant formulae in the spinor representation are given in the Appendix.

In the following we propose to study Eq. (1.3) considering the standard planes

$$x^+ = x^0 + x^3 = \text{constant}$$

and the dynamical operator

$$P^- = P^0 - P^3$$

the generator of translations along the x^+ co-ordinate.

We shall use the following conventions for contravariant and covariant tensor components and the Lorentz invariant scalar product

$$\begin{aligned} x^\pm &= x^0 \pm x^3, & x^\perp &= (x^1, x^2) \\ x_\pm &= \frac{1}{2} x^\mp, & x_\perp &= -x^\perp \end{aligned} \quad (1.5)$$

$$x \cdot y = x^\alpha y_\alpha = \frac{1}{2} (x^+ y^- + x^- y^+) - x^\perp y^\perp$$

The components of q_a^i can be divided into canonical and dependent ("good, bad") ones according to the projection operators $(1 \pm \alpha_3)/2$

$$\begin{aligned} (c_{a-}^i, c_{a+}^i) &= (\tilde{u}_{a1}^i, v_{a1}^i) \\ (d_{a-}^i, d_{a+}^i) &= (\tilde{u}_{a2}^i, v_{a1}^i) \end{aligned} \quad (1.6)$$

The dependent components $d_\pm$ satisfy constraint equations on the plane $x^+ = \text{constant}$.

Under certain smoothness assumptions which were extensively studied in the $p_+ \rightarrow \infty$ limit of equal-time current algebra ⁹⁾, the canonical components define the light-like charges ¹⁰⁾

$$Q_{\pm b a}(x^+) = \int_{x^+ = \text{const.}} d\sigma^\mu J_{\mu b a}^\pm = \int_{x^+} dx^- d^2 x^\perp (c_{b\pm}^{*i}(x) c_{a\pm}^i(x)) \quad (1.7)$$

which, as a consequence of the canonical anticommutation rules

$$\left\{ c_{a\pm}^i(x), c_{b\pm}^{*j}(y) \right\}_{x^+=y^+} = \delta^{ij} \delta_{ab} \delta(x^--y^-) \delta^2(x^\perp - y^\perp) \quad (1.8)$$

represent the algebra of $SU_3 \times SU_3$

$$\left[Q_{\pm ba}(x^+), Q_{\pm b'a'}(x^+) \right] = \delta_{ab'} Q_{\pm ba'}(x^+) - \delta_{ba'} Q_{\pm b'a}(x^+) \quad (1.9)$$

Since we want to maintain the local properties of current algebra, Eq. (1.8) is from here on taken for granted. It is to be understood that all anti-commutators of canonical components omitted in (1.8) vanish. Then Lorentz covariance implies

$$\left\{ d_{a\pm}^i(x), d_{b\pm}^{*j}(y) \right\}_{x^-=y^-} = \delta^{ij} \delta_{ab} \delta(x^+-y^+) \delta^2(x^\perp - y^\perp) \quad (1.10)$$

The canonical energy momentum tensor

$$\mathcal{T}_{\nu\mu} = \frac{i}{2} \bar{g}_a^j \gamma_\mu \overleftrightarrow{\partial}_\nu g_a^j - g_{\nu\mu} \mathcal{L}$$

yields the momentum operators

$$\begin{aligned} P_\nu &= \int dx^- d^2x^\perp \mathcal{T}_{\nu-} \\ \mathcal{T}_{\nu-} &= \frac{i}{2} \left[c_{a+}^{*j} \overleftrightarrow{\partial}_\nu c_{a+}^j + c_{a-}^{*j} \overleftrightarrow{\partial}_\nu c_{a-}^j \right] \\ &\quad - \frac{1}{2} \delta_{\nu+} \mathcal{L} \end{aligned} \quad (1.11)$$

The momenta $P_- = \frac{1}{2} P^+, P^\perp$ conjugate to the variables on a standard plane $(x^-, x^\perp)$ do not depend explicitly on the interaction and can be expressed through the canonical components alone.

The dynamical operator P^- is obtained from the energy momentum density Θ_{+-} . We compare the expressions for Θ_{+-} for the current-current - and for the vector gluon interaction

$$\mathcal{L}^{(g)} = -e' \bar{g}_a^i \gamma^\mu G_{ij\mu} g_a^j$$

Recent considerations on renormalizable field theories with asymptotic freedom at short distances ¹¹⁾ have drawn attention to the possibility of considering the gluon fields within a non-Abelian gauge theory. If the underlying group is the SU_3 group of colour and provided that a mass gap in the sectors of Hilbert space generated by the gluon fields out of the vacuum can arise dynamically without spontaneous symmetry breakdown, it is natural to introduce a colour octet of gluon fields

$$G_{ij\mu} \quad ; \quad G_{jj\mu} = 0$$

Such a set of vector fields is on an equal footing with the set of quark fields in the sense that both types of fields do not connect physical states ¹²⁾.

The simpler case of a single vector field corresponds to

$$G_{ij\mu} = \delta_{ij} G_\mu$$

Introducing the two-component notation

$$\begin{aligned} (c_a^i)_1 &= c_{a-}^i = \tilde{u}_{a1}^i, & (c_a^i)_2 &= c_{a+}^i = v_{a1}^i \\ (d_a^i)_1 &= d_{a+}^i = v_{a2}^i, & (d_a^i)_2 &= d_{a-}^i = u_{a2}^i \end{aligned}$$

(1.12)

one obtains the energy momentum density for the current-current interaction

$$2 \mathcal{G}_{+-}^{(cc)} = \frac{1}{2} c_a^{*j} \left(m_a + i(\underline{P} \wedge \underline{\sigma})_z \right) d_a^j + h.c. \quad (1.13)$$

and for the gluon interaction

$$\begin{aligned} 2 \mathcal{G}_{+-}^{(ge)} = & \frac{1}{2} c_a^{*i} \left\{ m_a \delta_{ij} + i \left[(\underline{P} \delta_{ij} - e' \underline{G}_{ij}) \wedge \underline{\sigma} \right]_z \right\} d_a^j + h.c. \\ & + e' c_a^{*i} G_{ij} c_a^j \\ & + \text{terms not containing quark fields} \end{aligned} \quad (1.14)$$

In the derivation of Eqs. (1.13) and (1.14) use has been made of the constraint equations in order to eliminate expressions quadratic in the dependent components.

It becomes evident from Eq. (1.13) that the dynamics of the four-fermion interaction considered on a standard plane is contained entirely in the constraint equations for the dependent components since the form of $\theta_{+-}^{(cc)}$ does not depend on the interaction.

Expressions like θ_{+-} in Eq. (1.13) and (1.14) involving the dependent components within a local or multilocal product of fields have no well-defined limit on a standard plane. This fact is related to the need for renormalization. In the present framework renormalization cannot be discussed. Nevertheless it may be stated that the conventional perturbative renormalization is of no value because of the assumed lack of stable states in the sectors with triality different from zero ⁷⁾.

The constraint equations for the dependent components are obtained from (1.3)

$$\begin{aligned} 2 \left[i \partial_- d_{a+}^i - g \hat{F}_{ab(+)}^{ij} d_{b+}^j \right] &= -(\underline{P}_x - i \underline{P}_y) c_{a+}^i + m_a c_{a+}^i \\ 2 \left[i \partial_- d_{a-}^i - g \hat{F}_{ab(-)}^{ij} d_{b-}^j \right] &= (\underline{P}_x + i \underline{P}_y) c_{a-}^i + m_a c_{a-}^i \end{aligned}$$

$$\left[\left(\hat{F}_{ab}^{ij}(\pm) \right)^+ \sigma_{\mu} \right]_{\alpha\beta} = \begin{cases} (+) v_{b\alpha}^{*k} v_{a\beta}^+ \delta^{ij} + v_{c\alpha}^{*j} v_{c\beta}^i \delta_{ab} \\ (-) u_{b\beta}^{*k} u_{a\alpha}^+ \delta^{ij} + u_{c\beta}^{*j} u_{c\alpha}^i \delta_{ab} \end{cases} \quad (1.15)$$

$\hat{F}_{ab}^{ij}(\pm)$ denote two sets of chiral currents carrying non-trivial quantum numbers with respect to both the ordinary and the colour SU_3 groups. The canonical (good-good) components of $\hat{F}_{ab}^{ij}(\pm)$ depend on the canonical fermion fields only

$$\begin{aligned} \left(\hat{F}_{ab}^{ij}(\pm) \right)^+ &= 2 \left(\hat{F}_{ab}^{ij}(\pm) \right)_- = c_{b\pm}^{*k} c_{a\pm}^+ \delta^{ij} + c_{m\pm}^{*j} c_{m\pm}^i \delta_{ab} \\ &= \frac{1}{4} \left\{ \bar{q}_b^k \gamma^+ (1 \pm \gamma_5) q_a^+ \delta^{ij} \right. \\ &\quad \left. + \bar{q}_m^j \gamma^+ (1 \pm \gamma_5) q_m^i \delta_{ab} \right\} \end{aligned} \quad (1.16)$$

Unitary spin indices for the quantities

$$\hat{F}_{ab}^{ij}(\pm), \quad c_{a\pm}^i, d_{a\pm}^i, m_a, G_{ij}$$

will be suppressed whenever no confusion arises.

We separate $\hat{F}(\pm)$ into vector and axial-vector parts

$$\hat{F}(\pm) = V \pm A \quad (1.17)$$

Thus one can compare the constraint equations : for the current-current interaction, one obtains, using the notation of Eq. (1.12)

$$2 \left(i \not{\partial} - g \not{V} - g \not{A} \cdot \sigma_2 \right) c l = \left[m - i (\not{P} \cdot \sigma_2) \right] c \quad (1.18)$$

and for the vector gluon interaction

$$2(i\partial_- - e'G_-)d = \left\{ m - i\sqrt{(P_- - e'G_-)_+} \right\} c \quad (1.19)$$

The comparison of Eq. (1.13), (1.14), (1.18) and (1.19) shows a different behaviour of the current-current and the vector gluon dynamics with respect to an eventual unitary transformation between current and constituent quarks as proposed by Melosh ^{13),14)}. The idea of this transformation is to diagonalize θ_{+-} (and the constraint equations). Remembering that under the parity transformation

$$c \leftrightarrow d, \quad P \rightarrow -P, \quad \underline{\sigma} \rightarrow \underline{\sigma}$$

the rôles of canonical and dependent components are interchanged, one verifies that the following non-local substitution uniquely achieves this if interactions are ignored and if covariance of the substitution under the parity reflection is demanded ¹³⁾

$$\begin{aligned} c' &= \exp \left[-i/2 \operatorname{arctg} \left(\frac{(P \wedge \underline{\sigma})_+}{m} \right) \right] c \\ d' &= \exp \left[i/2 \operatorname{arctg} \left(\frac{(P \wedge \underline{\sigma})_+}{m} \right) \right] d \end{aligned} \quad (1.20)$$

Since $\theta_{+-}^{(cc)}$ has the same form as in the free case, (1.20) of course has the same reflect

$$\begin{aligned} 2\mathcal{G}_{+-}^{(cc)} &= \frac{1}{2} \left[c'^* m_{\perp} d' + d'^* m_{\perp} c' \right] \\ (m_{\perp}^2)_{ab}^{ij} &= \delta^{ij} \delta_{ab} (m_a^2 + P_{\perp}^2) \end{aligned} \quad (1.21)$$

$\theta_{+-}^{(gl)}$ is not diagonal after the substitution (1.20) especially since p, G do not commute. In neither case there exists an exact SU_6 symmetry ¹ even ¹ for degenerate mass parameters m_a .

We now return to the constraint equations (1.18). These are linear, inhomogeneous first order differential equations for the unknown dependent components. It is convenient to rewrite Eq. (1.20) in the form

$$\begin{aligned} \partial_- d_{\pm} &= (-i)g M_{\pm} d_{\pm} + I_{\pm} \\ (M_{\pm})_{ab}^{ij} &= \hat{\Gamma}_{ab(\pm)}^{ij} - \\ I_{a\pm}^j &= \frac{1}{2i} \left[m_a c_{a\pm}^j + (i\mathcal{P}_2 \mp \mathcal{P}_x) c_{a\pm}^j \right] \end{aligned} \quad (1.22)$$

The solutions of Eq. (1.22) are obtained from the fundamental system of solutions of the homogeneous equations

$$\begin{aligned} \partial_- \rho_{\pm} (x^-, y^-, x^{\perp}, x^+) &= (-ig) : M_{\pm} \rho_{\pm} (x^-, y^-, x^{\perp}, x^+) : \\ \rho_{\pm} (y^-, y^-, x^{\perp}, x^+) &= 1 ; \rho_{\pm} = (\rho_{\pm})_{ab}^{ij} \end{aligned} \quad (1.23)$$

The normal ordering in Eq. (1.23) has a meaning since spectral conditions and the canonical anti-commutation relations (1.8) allow the decomposition of the canonical components into positive and negative frequency parts.

$\rho_{\pm}$ are bilocal operators on the light cone with origin at the point

$$y = (y^-, x^{\perp}, x^+)$$

considered as functions of two four-vectors

$$x = (x^-, x^{\perp}, x^+) , \quad y = (y^-, x^{\perp}, x^+)$$

In the following, the variables $x^{\perp}, x^+$ which are parameters of Eq. (1.23) will be suppressed whenever possible. ρ is obtained from M by standard methods for linear differential equations

$$\rho_{\pm}(x^-, y^-) = \sum_n (-ig)^n \int_{y^-}^{x^-} d\xi_1^- \int_{y^-}^{\xi_1^-} d\xi_2^- \dots \int_{y^-}^{\xi_{n-1}^-} d\xi_n^-$$

$$N (M_{\pm}(\xi_1^-) M_{\pm}(\xi_2^-) \dots M_{\pm}(\xi_n^-)) \quad (1.24)$$

Matrix multiplication is understood in the definition of the products

$$M(\xi_1^-) M(\xi_2^-) \dots$$

A Lorentz covariant form can be given for $\rho_{\pm}(x, y)$ depending on two general space-time points at a light-like distance from each other. For the current-current interaction, one obtains

$$\rho_{\pm}(x, y) = N T_2 \exp \left[ig z^{\mu} \int_0^1 d\lambda \hat{T}_{\pm \mu}(y + \lambda z) \right]$$

$$= N \sum_n (-ig)^n \int_{\tau \geq \lambda_1 \geq \dots \geq \lambda_n \geq 0} d\lambda_1 \dots d\lambda_n (z \cdot \hat{T}_{\pm}(y + \lambda_1 z)) \dots (z \cdot \hat{T}_{\pm}(y + \lambda_n z)) \quad (1.25)$$

$$z = x - y$$

and for the vector gluon interaction

$$\rho_{\pm}^{(gl)}(x, y) = N^{(gl)} T_2 \exp \left[ie' z^{\mu} \int_0^1 d\lambda G_{\mu}(y + \lambda z) \right] =$$

$$= N^{(gl)} \sum_n (-ie')^n \int_{\tau \geq \lambda_1 \geq \dots \geq \lambda_n \geq 0} d\lambda_1 \dots d\lambda_n (z \cdot G(y + \lambda_1 z)) \dots (z \cdot G(y + \lambda_n z)) \quad (1.26)$$

In Eqs. (1.25) and (1.26), $N(N^{(gl)})$ denote normal ordering and T_λ the ordering of $\lambda_1, \dots, \lambda_n$ in decreasing order.

The normal ordering $N^{(gl)}$ of $(z.G)$ is not well defined due to the difficulty to restrict boson fields to a standard plane. If this difficulty is ignored ¹⁵⁾, there exists a gauge for every light-like direction z such that

$$z \cdot G_{ij} = 0$$

which implies the free field-like result

$$\rho_{\pm}^{(gl)}(x, y) = 1 + \text{gauge terms} \quad (1.27)$$

The main difference between $\rho_{\pm}$ and $\rho_{\pm}^{(gl)}$ lies in the fact that $\rho_{\pm}$ generates quark excitations and is not a singlet under the chiral $SU_3 \times SU_3$ group whereas $\rho_{\pm}^{(gl)}$ is a unit operator apart from gauge terms.

The solutions of Eq. (1.22) are determined from $\rho_{\pm}$ and $I_{\pm}$ which both depend only on the canonical components

$$d_{\pm}(x^-) = N \left\{ \int_{y^-}^{x^-} d f^- \rho_{\pm}(x^-, f^-) I_{\pm}(f^-) + \rho_{\pm}(x^-, y^-) d_{\pm}(y^-) \right\} \quad (1.28)$$

2. - CANONICAL LIGHT-CONE SINGULARITIES OF TRANSVERSE (GOOD-BAD) CURRENT COMPONENTS

We consider the current correlation functions

$$W_{\pm \mu\nu}^{ba b'a'}(q, p) = \frac{1}{4\pi} \int d^4z e^{+iqz} \langle p | [j_{\mu\pm}^{ba}(z/2), j_{\nu\pm}^{b'a'}(-z/2)] | p \rangle \quad (2.1)$$

The Bjorken limit may be considered in the following co-ordinate system

$$\begin{aligned} q^- \rightarrow \infty \quad q^+ = -m_p \xi, \quad q_\perp = 0 \\ p^+ = p^- = m_p, \quad p_\perp = 0 \quad \xi = -q^2/2(q \cdot p) \end{aligned} \quad (2.2)$$

In the above limit, one is looking at the light cone singularities of the current commutators

$$w_{\pm \mu\nu}^{ba b'a'}(x, y) = [j_{\mu\pm}^{ba}(x), j_{\nu\pm}^{b'a'}(y)] \quad (2.3)$$

in the region

$$x^+ \simeq y^+, \quad x^\perp \simeq y^\perp$$

For spin $\frac{1}{2}$ constituents only the transverse components

$$\begin{pmatrix} w_{\pm 11} & w_{\pm 12} \\ w_{\pm 21} & w_{\pm 22} \end{pmatrix}$$

contribute asymptotically ^{16),17)}. It is convenient to use the spinor basis for the transverse components of $w_{\pm}$ corresponding to circularly polarized basis currents

$$\begin{aligned} w_{LL} &= w_{11} + w_{22} + i(w_{12} - w_{21}) \\ w_{RR} &= w_{11} + w_{22} - i(w_{12} - w_{21}) \\ w_{LR} &= w_{11} - w_{22} - i(w_{12} + w_{21}) \\ w_{RL} &= w_{11} - w_{22} + i(w_{12} + w_{21}) \end{aligned} \quad (2.4)$$

R(L) stand for right (left) handed circular polarization respectively. Dropping for the moment the unitary spin indices, one obtains

$$\begin{aligned} \begin{pmatrix} w_{LL} & w_{LR} \\ w_{RL} & w_{RR} \end{pmatrix} &= \begin{pmatrix} [c_+^* d_+, d_+^* c_+] & [c_+^* d_+, c_+^* d_+] \\ [d_+^* c_+, d_+^* c_+] & [d_+^* c_+, c_+^* d_+] \end{pmatrix} \\ \begin{pmatrix} w_{-LL} & w_{-LR} \\ w_{-RL} & w_{-RR} \end{pmatrix} &= \begin{pmatrix} [d_-^* c_-, c_-^* d_-] & [d_-^* c_-, d_-^* c_-] \\ [c_-^* d_-, c_-^* d_-] & [c_-^* d_-, d_-^* c_-] \end{pmatrix} \end{aligned} \quad (2.5)$$

According to Eqs. (1.8) and (1.9), the leading light cone singularity in the region

$$x^+ \simeq y^+, \quad x^- \simeq y^-$$

is given by the canonical singularity of the dependent components for

$$x^- = y^-$$

Equation (1.28) formally shows that this remains true for $x^- \neq y^-$. Thus it follows immediately

$$\begin{aligned} w_{\pm RL} = w_{\pm LR} = 0 &\Rightarrow w_{\pm 11} = w_{\pm 22} = w_{\pm 1} \\ w_{\pm 12} &= -w_{\pm 21} = i w_{\pm 3} \end{aligned}$$

$$\begin{aligned} (RR \rightarrow R) \quad w_{\pm R} &= 2(w_{\pm 1} + w_{\pm 3}) \\ (LL \rightarrow L) \quad w_{\pm L} &= 2(w_{\pm 1} - w_{\pm 3}) \end{aligned} \quad (2.6)$$

The light cone expansion of $w_{\pm R, L}$ is determined from the corresponding expansion of the anticommutators

$$\{d_{\pm}(x), d_{\pm}^*(y)\}_{x^+, x^- \rightarrow y^+, y^-}$$

The canonical anticommutation relations (1.10) together with Eq. (1.28) yield the leading light cone singularity

$$\{d_{\pm}(x), d_{\pm}^*(y)\}_{x^+, x^- \rightarrow y^+, y^-} \sim \rho_{\pm}(x, y) \delta(x^+ - y^+) \delta^2(x^- - y^-) \quad (2.7)$$

The essential difference between the light cone structure of commutators within the current-current interaction and the free field-like case lies in the deviation of the operators

$$\rho_{\pm}(x, y)$$

from unity and not in a possible deviation from the canonical scaling law.

Up to this point no divergences have appeared. They do enter the scene for current commutators which, to leading order on the light cone are of the form

$$\begin{aligned} &\delta(x^+ - y^+) \delta^2(x^- - y^-) N(c^*(x) \rho(x, y)) c(y) \quad \text{or} \\ &\delta(x^+ - y^+) \delta^2(x^- - y^-) c^*(x) N(\rho^*(y, x) c(y)) \end{aligned} \quad (2.8)$$

The above terms contain divergences which originate from an incorrect short distance expansion of the transverse current components

$$\left\{ c_{\pm}^*(x) d_{\pm}(y) \right\}_{x \rightarrow y}, \quad \left\{ d_{\pm}^*(x) c_{\pm}(y) \right\}_{x \rightarrow y}$$

This problem is related to the renormalization of the four-fermion interaction but does not influence the leading light cone structure :

$$\begin{aligned} W_{\pm R(L)}^{ab/b'a'} &\sim B_{\pm R(L)}^{ab/b'a'}(x,y) \delta(x^+-y^+) \delta^2(x^\perp - y^\perp) \\ B_{+L}^{ab/b'a'} &= : c_{a+}^{*i}(x) \rho_{+bb'}^{ij}(x,y) c_{a'+}^j(y) : \\ B_{+R}^{ab/b'a'} &= - : c_{b'+}^{*i}(y) \rho_{+a'a}^{ij}(y,x) c_{b+}^j(x) : \\ B_{-L}^{ab/b'a'} &= - : c_{b'-}^{*i}(y) \rho_{-a'a}^{ij}(y,x) c_{b-}^j(x) : \\ B_{-R}^{ab/b'a'} &= : c_{a-}^{*i}(x) \rho_{-bb'}^{ij}(x,y) c_{a-}^j(y) : \end{aligned} \quad (2.9)$$

We note that the current-current interaction yields a definite prediction [Eq. (2.9)] despite the fact that it is non-renormalizable by virtue of the condition that light-like charges exist. The bilocal operators in Eq. (2.9) are not diagonal in the quark quantum numbers or in the terminology of the quark parton model the currents do not scatter incoherently off individual partons in the deep inelastic limit in as far as the operators $\rho_{\pm}$ are not diagonal in the $SU_3 \times SU_3$ quantum numbers

$$\rho_{\pm a a'}^{ij}(x,y) \neq \delta_{aa'} \rho_{\pm}^{ij}$$

The bilocal operators which satisfy the light cone algebra according to Fritzsche and Gell-Mann²⁾ are obtained in substituting for $\rho_{\pm aa'}^{ij}$ in Eq. (2.9) the free field expression

$$\rho_{\pm}^{(free) ij} = \int ij \delta_{aa'} \quad (2.10)$$

$B_{\pm R, L}$ represent canonical components of chiral vectors

$$B_{\pm R, L} = \left[\left(B_{V R, L} \right)^{\mu} \pm \left(B_{A R, L} \right)^{\mu} \right] \delta^{\mu+} \quad (2.11)$$

We end this section by noting the short (light-like) distance expansion of $\rho_{\pm}$

$$\rho_{\pm}^{ij}(x, y) \underset{\substack{x \rightarrow y \\ (x-y)^2=0}}{\sim} \int ij \delta_{ab} - i g z^{\mu} \tilde{F}_{ab(\pm)}^{\mu} (R) \quad (2.12)$$

$$R = \frac{1}{2}(x+y), \quad z = x-y$$

The chiral vectors $\tilde{F}_{(\pm)}$ are given by Eq. (1.16).

3. - STRUCTURE FUNCTIONS FOR DEEP INELASTIC CURRENT HADRON SCATTERING

The structure functions for deep inelastic lepton hadron scattering are determined from the (spin averaged) matrix elements

$$\langle p | B_{\pm R, L} \left(\frac{z}{2}, -\frac{z}{2} \right) | p \rangle = \frac{p^+}{2} f_{\pm R, L}(p, z) \quad (3.1)$$

$$p \cdot z = \frac{1}{2} m_p z^-$$

Since there is only a vector, i.e., p which has a non-vanishing canonical component $p^+ = m_p$ it follows

$$\begin{aligned} f_{+L}^{ab/b'a'}(\xi) &= f_{-R}^{ab/b'a'}(\xi) = f^{ab/b'a'}(\xi) \\ f_{+R}^{ab/b'a'}(\xi) &= f_{-L}^{ab/b'a'}(\xi) = -f^{b'a'/ab}(-\xi) \end{aligned} \quad (3.2)$$

Equation (3.1) corresponds to the normalization

$$\langle P | B_{\pm R, L}(\frac{1}{2}, -\frac{1}{2}) | P \rangle = \frac{1}{2} P_{\mu} f_{\pm R, L}(P^2) + z_{\mu} (\dots) \quad (3.3)$$

The structure functions in the scaling limit are related to the Fourier transforms

$$F^{ab/b'a'}(\alpha) = \frac{1}{2\pi} \int d\xi e^{-i\alpha \xi} f^{ab/b'a'}(\xi) \quad (3.4)$$

The reflection $\alpha \rightarrow -\alpha$ corresponds to the replacement of quark by anti-quark excitations. Thus we define

$$\bar{F}^{a'b'/ba}(\alpha) = -F^{ab/b'a'}(-\alpha) \quad (3.5)$$

Normalizing the currents $j_{\pm ba} \gamma_{\mu}$ in Eq. (2.1) to the standard form

$$\bar{q}_b' \gamma_{\mu} (1 \pm \gamma_5) q_a' = 4 j_{\pm ba} \gamma_{\mu}$$

one obtains in the scaling limit

$$\begin{aligned} W_{+L} &\rightarrow 4 F^{ab/b'a'}(\xi) \\ W_{+R} &\rightarrow 4 \bar{F}^{ba/a'b'}(\xi) = -4 F^{b'a'/ab}(\xi) \end{aligned} \quad (3.6)$$

$$W_{-L} \rightarrow W_{+R}$$

$$W_{-R} \rightarrow W_{+L}$$

$$F_1 = -g^2/2(q \cdot p)$$

For vector currents normalized to

$$\bar{q}_b \gamma_\mu q_a$$

one obtains

$$W_{V,L} \rightarrow \frac{1}{4} (W_{+L} + W_{-L}) = F^{ab/b'a'}(\xi) + \bar{F}^{ba/a'b'}(\xi) \quad (3.7)$$

$$W_{V,R} \rightarrow W_{V,L}$$

The conventional structure functions

$$F_1 = \lim_{bj} W_1 \quad F_3 = \lim_{bj} g_P/2 \quad W_3$$

are related to the scaling limits of $W_{\pm R, L}$ in the following way

$$W_{\pm L} \rightarrow 2 (F_{1\pm} - F_{3\pm})$$

$$W_{\pm R} \rightarrow 2 (F_{1\pm} + F_{3\pm}) \quad (3.8)$$

$$W_{V,L(R)} \rightarrow 2 F_{1\pm} \quad F_{1+} = F_{1-} = F_1, \quad F_{3+} = -F_{3-} = F_3$$

If the operators $\int_{\pm}(x,y)$ were diagonal in quark quantum numbers

$$\int_{\pm}^{ij} = \int_{ab} \rho_{\pm}^{ij}$$

one would obtain

$$F^{ab/b'a'}(\xi) = \int_{bb'} F^{aa'}(\xi) \quad (3.9)$$

With the ansatz (3.9) the structure functions of the parton model emerge.
A particular consequence of Eq. (3.9) is the local relation due to
Llewellyn Smith¹⁸⁾

$$F_3^{\nu p} - F_3^{\bar{\nu} p} = 6(F_1^{ep} - F_1^{en}) \quad (3.10)$$

which connects neutrino and electroproduction structure functions.

In the approximation of vanishing Cabibbo angle, we obtain the following set of structure functions :

$$\begin{aligned} F_1^{\nu p} - F_3^{\nu p} &= 2 F \frac{d u / u d}{d x} \left(\frac{x}{2} \right) \\ F_1^{\nu p} + F_3^{\nu p} &= -2 F \frac{u d / d u}{d x} \left(\frac{x}{2} \right) = 2 \bar{F} \frac{u d / d u}{d x} \left(\frac{x}{2} \right) \\ F_1^{\bar{\nu} p} - F_3^{\bar{\nu} p} &= 2 F \frac{u d / d u}{d x} \left(\frac{x}{2} \right) \\ F_1^{\bar{\nu} p} + F_3^{\bar{\nu} p} &= -2 \bar{F} \frac{d u / u d}{d x} \left(\frac{x}{2} \right) = 2 \bar{F} \frac{d u / u d}{d x} \left(\frac{x}{2} \right) \\ 2 F_1^{ep} &= \sum_{i,j=\bar{u},d,s} Q_i^{(p)} Q_j^{(p)} \left[F^{ij/ij} \left(\frac{x}{2} \right) - F^{ij/\bar{i}\bar{j}} \left(\frac{x}{2} \right) \right] \\ 2 F_1^{en} &= \sum_{i,j=\bar{u},d,s} Q_i^{(n)} Q_j^{(n)} \left[F^{ij/ij} \left(\frac{x}{2} \right) - F^{ij/\bar{i}\bar{j}} \left(\frac{x}{2} \right) \right] \quad (3.11) \end{aligned}$$

$$Q_{\bar{u}}^{(p)} = Q_d^{(n)} = \frac{2}{3} \quad Q_d^{(p)} = Q_{\bar{u}}^{(n)} = -\frac{1}{3}$$

$$Q_s^{(p)} = Q_s^{(n)} = -\frac{1}{3}$$

We have dropped a label p characterizing the proton distribution functions

$$F_{(p)}^{ab/b'a'}(\xi)$$

in Eq. (3.11). The coherent contributions to electroproduction are represented by the structure functions

$$F^{ii/jj}(\xi) \quad i \neq j$$

It is instructive to consider F_1^{en} in the diagonal form

$$2 F_1^{ep} = \sum_{m=u', d', s'} \left(Q_m^{(p)}(\xi) \right)^2 d_p^m(\xi)$$

$$2 F_1^{en} = \sum_{m=u', d', s'} \left(Q_m^{(n)}(\xi) \right)^2 d_p^m(\xi)$$

upon diagonalizing the matrix

$$\begin{aligned} & \frac{1}{2} \left\{ F_{(p)}^{ii/jj}(\xi) + F_{(p)}^{jj/ii}(\xi) + (\xi \rightarrow -\xi) \right\} = \\ & = \int_{m,i}^{(p)}(\xi) d_p^m(\xi) \int_{m,j}^{(p)}(\xi) \\ & \quad S^T \cdot S = \mathbb{I} \end{aligned}$$

and introducing the effective charges

$$\begin{aligned} Q_m^{(p)}(\xi) &= \int_{m,j}^{(p)}(\xi) Q_j^{(p)} \\ Q_m^{(n)}(\xi) &= \int_{m,j}^{(p)}(\xi) Q_j^{(n)} \end{aligned}$$

which in general depend on ξ . The above effective charges may deviate substantially from the respective quark charges which immediately explains why the inequality

$$\frac{1}{4} \leq F_1^{en}/F_1^{ep} \leq 4$$

must not be valid in the present model ¹⁹⁾. We note that there is no trace in general of the Llewellyn Smith relation (3.10)

Sum Rules

Sum rules can be derived from the expansion

$$f_{(P)}^{ab/b'a'}(p^2) \sim_{p^2 \rightarrow 0} \delta_{bb'} Q_{aa'}^{(P)} + i(p^2) \left\{ \delta_{bb'} \gamma_{aa'}^{(P)} + \delta_{bb'} \chi_{aa'/cc}^{(P)} - \chi_{ab/b'a'}^{(P)} \right\} \quad (3.12)$$

The coefficients $Q_{aa'}^{(p)}$, $\gamma_{aa'}^{(p)}$, $\chi_{ab/b'a'}^{(p)} = \chi_{b'a'/ab}^{(p)}$ characterize the local quantities

$$\begin{aligned} \langle P / \bar{q}_a^i(0) \gamma_\mu q_a^i(0) / P \rangle &= 2P_\mu Q_{aa'}^{(P)} \\ \langle P / \frac{i}{2} \bar{q}_a^i(0) \gamma_\mu \overleftrightarrow{\partial}_\nu q_a^i(0) / P \rangle &= 2P_\mu P_\nu \gamma_{aa'}^{(P)} + \text{trace terms} \\ \frac{g}{4} \langle P / N \{ \bar{q}_a^i(0) \gamma_\mu (1 \pm \gamma_5) q_a^i(0) \bar{q}_b^k(0) \gamma_\nu (1 \pm \gamma_5) q_b^k(0) \} / P \rangle & \\ = 2P_\mu P_\nu \chi_{ab/b'a'}^{(P)} + \text{trace terms} & \end{aligned} \quad (3.13)$$

Again if ρ_{ab}^{ij} were diagonal

$$\rho_{ab}^{ij} = \delta_{ab} \rho_{ab}^{ij}$$

one would recover the free field result

$$f_{(P)}^{(free) ab/b'a'}(p^2) \sim_{p^2 \rightarrow 0} \delta_{bb'} Q_{aa'}^{(P)} + i(p^2) \delta_{bb'} \gamma_{aa'}^{(P)} \quad (3.14)$$

The trace $\mathcal{T}_{aa}^{(p)}$ is given by the matrix element of the energy momentum tensor and therefore is equal to 1 independently of the state p . This yields together with the assumed validity of (3.14), the so-called "no-gluon" sum rule ²⁾

$$\int_0^1 d\xi \cdot \xi \left[9(\overline{F}_1^{ep} + \overline{F}_1^{eq}) - \frac{3}{2}(\overline{F}_1^{vp} + \overline{F}_1^{\bar{v}p}) \right] = 1 \quad (3.15)$$

Instead one finds, using Eq. (3.12)

$$\begin{aligned} & \int_0^1 d\xi \cdot \xi \left[9(\overline{F}_1^{ep} + \overline{F}_1^{eq}) - \frac{3}{2}(\overline{F}_1^{vp} + \overline{F}_1^{\bar{v}p}) \right] = \\ & = 1 + 3 \left(\chi_{\bar{u}\bar{u}/ss}^{(p)} + \chi_{d\bar{d}/ss}^{(p)} \right) + 6 \chi_{\bar{u}\bar{u}/dd}^{(p)} \\ & \quad - \frac{3}{2} \left[\chi_{\bar{u}\bar{u}/\bar{u}\bar{u}}^{(p)} - 2 \chi_{\bar{u}\bar{d}/d\bar{u}}^{(p)} + \chi_{d\bar{d}/d\bar{d}}^{(p)} \right] \end{aligned} \quad (3.16)$$

Sum rules are obtained from the identities

$$\begin{aligned} & \int_{-1}^{+1} d\xi \cdot \overline{F}_{(p)}^{ab/b'a'}(\xi) = \delta_{bb'} Q_{aa'}^{(p)} \\ & \int_{-1}^{+1} d\xi \cdot \xi \cdot \overline{F}_{(p)}^{ab/b'a'}(\xi) = \delta_{bb'} \mathcal{V}_{aa'}^{(p)} \\ & \quad + \delta_{bb'} \chi_{aa'/cc}^{(p)} - \chi_{ab/b'a'}^{(p)} \end{aligned} \quad (3.17)$$

The zero moment sum rules are unaffected by the current correlation parameters $\chi_{ab/b'a'}^{(p)}$ and are then identical to corresponding sum rules in the quark parton model. Thus the Adler sum rule ⁴⁾ and the Gross-Llewellyn Smith sum rule remain valid in the present model

$$\begin{aligned} & \int_0^1 d\xi (\overline{F}_1^{\bar{v}p} - \overline{F}_1^{vp}) = 1 \\ & \int_0^1 d\xi (\overline{F}_3^{\bar{v}p} + \overline{F}_3^{vp}) = -3 \end{aligned} \quad (3.18)$$

One easily verifies that there are no more sum rules for the observable structure functions which do not involve the unknowns $\chi^{(p)}$. E.g., the first moment of Eq. (3.10) becomes

$$\int_0^1 dx \, x \left[F_3^{\nu p} - F_3^{\bar{\nu} p} - 6(F_1^{ep} - F_1^{en}) \right] =$$

$$= \chi_{\bar{u}u/\bar{u}u}^{(p)} - \chi_{d\bar{d}/d\bar{d}}^{(p)} - 2(\chi_{\bar{u}u/ss}^{(p)} - \chi_{d\bar{d}/ss}^{(p)}) \quad (3.19)$$

CONCLUSIONS

It was shown that within a model quark field theory with four-fermion current-current interaction, the light cone singularities of (transverse) current commutators can be reconciled with the canonical ones only if the algebraic structure of free (quark) field commutators is modified.

The resulting bilocal coefficient of the leading singularity contains definite exotic contributions in the current-current annihilation channel. The incoherent addition of single quark scatterings cannot be maintained. The scattered quark parton cannot leave a given hadron without definite quantum numbers flowing back in such a way that incoherence is lost.

Even though there are no gluons, the "no-gluon" sum rule do not hold. This is another consequence of the modified bilocal operators. Neutrino and electroproduction scaling functions are not related in any way.

For neutrino scattering, two sum rules remain valid :

$$\int d\xi (\mathcal{F}_2^{\nu p} - \mathcal{F}_1^{\nu p}) = 1 \quad \text{Adler sum rule}$$
$$\int d\xi (\mathcal{F}_3^{\nu p} + \mathcal{F}_3^{\bar{\nu} p}) = -3 \quad \text{Gross, Penellyn Smith sum rule}$$

The sum rule derived by Bjorken ²⁰⁾ for deep inelastic scattering of polarized electrons off polarized nucleons also remains valid.

ACKNOWLEDGEMENTS

It is a pleasure to thank H. Leutwyler, J. Stern and Y. Georgelin for discussions in the early stage of this work, and Y. Zarmi for his advice and a careful reading of the manuscript.

APPENDIX : THE SPINOR REPRESENTATION

We use the following representation of the γ matrices

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_k = -\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} \quad k=1,2,3$$

$\sigma_{1,2,3}$: Pauli matrices

$$\gamma_5 = i \gamma_0 \gamma_1 \gamma_2 \gamma_3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

to define the left and right-handed fields

$$\frac{1+\gamma_5}{2} q_a^i = \begin{pmatrix} 0 \\ v_a^i \end{pmatrix}, \quad \frac{1-\gamma_5}{2} q_a^i = \begin{pmatrix} u_a^i \\ 0 \end{pmatrix} \quad (A.1)$$

$\alpha, \alpha' = 1, 2$

A vector or axial vector has the spinor representations :

$$\begin{aligned} f^\mu &\rightarrow f_{\alpha\beta} = (f^\mu \sigma_\mu)_{\alpha\beta} \\ f^{\delta\delta'} &= (f_\mu \sigma_\mu)^{\delta\delta'} \end{aligned} \quad (A.2)$$

In the spinor representation, the chiral currents formed by the quark fields

$$4(j_{ba}^{\pm})_\mu = V_{\mu ba}^{\pm} \pm A_{\mu ba}^{\pm} = \bar{q}_b^r \gamma_\mu (1 \pm \gamma_5) q_a^s$$

become factorized

$$\begin{aligned} (j_{ba}^{\pm})^{\delta\delta'} &= v_b^{\delta r} v_a^{s \delta'} \\ (j_{ba}^{\pm})_{\alpha\beta} &= \tilde{u}_b^{\delta r} \tilde{u}_a^{s \alpha} \end{aligned} \quad (A.3)$$

The currents generating charges of the chiral $SU_3 \times SU_3$ group correspond to traces of $j^{\pm rs}$ for the colour quantum number

$$j_{ba\pm}^i = j_{ba\pm}^{rr} \quad (A.4)$$

The equations of motion (1.3) take the form

$$\begin{aligned} p^{\beta\alpha} \tilde{u}_{a\alpha}^i - m_a v_a^{i\beta} &= g (j_{ba-})^{\beta\alpha} \tilde{u}_{b\alpha}^i \\ p_{\alpha\beta} v_a^{i\beta} - m_a \tilde{u}_{a\alpha}^i &= g (j_{ba+})_{\alpha\beta} v_b^{i\beta} \end{aligned} \quad (A.5)$$

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