CM-P00049191

Ref.TH.1718-CERN

QUARKS AND THE HELICITY STRUCTURE OF PHOTOPRODUCTION AMPLITUDES

A.J.G. Hey ⁺) and J. Weyers

CERN - Geneva

A B S T R A C T

Algebraic properties of the transformation between constituent and current quarks are used to derive the helicity structure of photoproduction matrix elements.

⁺) Research supported by a Royal Society Fellowship.

Resonance photoproduction off nucleons is, in general, described by two independent helicity amplitudes. This helicity structure of phototransitions can be extracted from photoproduction phase-shift analyses ¹⁾ and provides very stringent tests of resonance production models. Several models have recently claimed some success in predicting the observed helicity structure of the prominently photoproduced resonances. Notable among these are the simple harmonic oscillator quark model ^{2),3)} and the quark antiquark pair creation model with vector dominance ⁴⁾ (³P₀ model). Although these models lead to different sets of predictions, at the present level of data one cannot decisively exclude either of them.

In this note we investigate the helicity structure of phototransitions implied by the algebraic properties of the transformation relating "constituent" and "current" quarks. As it turns out, both the simple harmonic oscillator quark model and the ³P₀ model correspond to specific limits of this scheme.

Let us start from the following general photoproduction matrix element

$$F_{\perp} \equiv \langle N^*, P', \lambda' | J_{\perp} | N, P, \lambda \rangle \quad (1)$$

where the N and N^* states are labelled by their momentum and helicity and $J_{\perp}$ corresponds to the transverse components of the electromagnetic current. While one-particle hadronic states seem to be successfully classified in irreducible representations of $SU(6)_{W, \text{strong}}$ ⁵⁾, the constituent quark algebra, the electromagnetic current, on the other hand, has simple transformation properties with respect to a different $SU(6)_W$ algebra, namely $SU(6)_{W, \text{currents}}$ ⁶⁾ (the current quark algebra). Recently, Melosh ⁷⁾ explicitly constructed in the free quark model a unitary transformation V relating the generators F_{α}^i of $SU(6)_{W, \text{currents}}$ to those of $SU(6)_{W, \text{strong}}$, W_{α}^i :

$$W_{\alpha}^i = V F_{\alpha}^i V^{-1} \quad (2)$$

For the so-called "good" operators ⁸⁾, which include, e.g., W_{α}^i , F_{α}^i and V , it is at least plausible that the algebraic properties of the free quark Melosh transformation, V , could be preserved in some interacting theories ⁹⁾.

For pionic transitions, this abstraction of algebraic properties from the quark model has led, with the help of PCAC, to many interesting results ¹⁰⁾. In photoproduction one measures directly the matrix elements of bad operators but current conservation may be used to relate the matrix elements of the transverse components of the electromagnetic current to those of the dipole operator $D_{\perp}$. Then we may rewrite Eq. (1) as

$$F_1 \equiv \langle N^*, p', \lambda' | J_{\perp} | N, p, \lambda \rangle = q^- \langle N^*, p', \lambda' | D_{\perp} | N, p, \lambda \rangle \Big|_{q^+ = 0; q_{\perp} = 0} \quad (3)$$

where $q = p' - p$ and $q^{\pm} = 1/\sqrt{2}(q^0 \pm q^3)$. In the nucleon rest frame, for example, $q^- = (M_{N^*}^2 - M_N^2)/(\sqrt{2} M_N)$. The dipole operator, $D_{\perp}$, which is a good operator, is given by

$$D_{\perp} = \int d^4x \delta(x^+) x_{\perp} J^+(x)$$

where, again, $x^+ = 1/\sqrt{2}(x^0 + x^3)$, $J^+ = 1/\sqrt{2}(J^0 + J^3)$.

Assuming then that there exists, in nature, a unitary operator V which transforms irreducible representations of $SU(6)_{W, \text{strong}}$ into the corresponding representations of $SU(6)_{W, \text{currents}}$ one may write

$$|\text{hadron; current}\rangle = V |\text{hadron; constituent}\rangle$$

and

$$F_1 \sim \langle N^*, p', \lambda'; \text{constituent} | V^{-1} D_{\perp} V | N, p, \lambda; \text{constituent} \rangle \quad (4)$$

In the free quark model, Melosh ⁷⁾ has suggested that $\tilde{D}_{\perp} \equiv V^{-1} D_{\perp} V$ has the following simple algebraic transformation properties:

$$\begin{aligned} \tilde{D}_{\perp} \sim & A \{ \underline{3}_S; W=0, W_z=0; \Delta L_z = \pm 1 \} + B \{ \underline{3}_S; W=1, W_z = \pm 1; \Delta L_z = 0 \} \\ & + C \{ \underline{3}_S; W=1, W_z=0; \Delta L_z = \pm 1 \} + D \{ \underline{3}_S; W=1, W_z = \mp 1; \Delta L_z = \pm 2 \} \end{aligned} \quad (5)$$

We abstract these algebraic properties from the free quark model and assume their validity for the physically measurable dipole operator.

With this assumption it is straightforward, using the standard $SU(6)_W \times O(2)_{L_Z}$ classification of baryons to derive the helicity structure of phototransitions. A similar calculation has been done recently by Love and Nanopoulos ¹¹⁾ who, among other things, pointed out that the algebraic structure of Eq. (3) preserves the well-known Moorhouse selection rule ¹²⁾ for photoexcitation of resonances in the $[70, 1^-]$ multiplet. In this note, however, we examine in more detail the helicity structure predicted for the photoproduction of a resonance off neutron and proton targets. The results for all resonances belonging to the $[70, 1^-]$ and $[56, 2^+]$ multiplets of $SU(6)_{W, strong}$ are given in the Table.

For the $[70, 1^-]$ multiplet, we have two relations among the helicity amplitudes of a given resonance. These are ^{*}), for the $^2(8)_{\frac{3}{2}}$ usually identified with the $D_{13}(1545)$

$$F_+^p + 3 F_+^n = \sqrt{3} (F_-^p + 3 F_-^n) \quad (6)$$

and for the $^4(8)_{5/2}$, identified with the $D_{15}(1670)$

$$F_+^n = -\sqrt{2} F_-^n \quad (7)$$

For the $[56, 2^+]$ multiplet we have three such relations. For the $^2(8)_{5/2}$ $[F_{15}(1690)]$,

$$F_+^p + 3/2 F_+^n = \sqrt{2} (F_-^p + 3/2 F_-^n) \quad (8)$$

for the $^2(8)_{\frac{3}{2}}$ $[F_{13}(1860)]$

$$\sqrt{3} (F_+^p + 3/2 F_+^n) = F_-^p + 3/2 F_-^n \quad (9)$$

and finally for the $^4(10)_{7/2}$ $[F_{37}(1950)]$

$$F_+^{p,n} = -\sqrt{5/3} F_-^{p,n} \quad (10)$$

^{*}) $F_+^{p,n}$ are the $\lambda = \frac{1}{2}$ to $\lambda = \frac{3}{2}$ transition amplitudes off proton (p) or neutron (n) target: $F_-^{p,n}$ are the $\lambda = \frac{1}{2}$ to $\lambda = -\frac{1}{2}$ amplitudes.

It is interesting to compare our predicted helicity structure with that of the simple harmonic oscillator and 3P_0 models. We ignore for the present discussion, any explicit mass-splitting factors (such as q^-). Both of these models omit the term in the transition matrix with $\Delta L_z = \pm 2$. Furthermore, in most versions of the simple harmonic oscillator model ^{2),3)}, one parametrizes the transition by an orbital flip and a spin flip term only. Algebraically these are identical to $\{ \underline{35}, W=0, W_z=0, \Delta L_z = \pm 1 \}$ and $\{ \underline{35}, W=1, W_z = \pm 1; \Delta L_z = 0 \}$ i.e., they correspond to the first two terms in Eq. (5). Hence, in terms of our parameters the naive quark model corresponds to

$$C = 0 \quad ; \quad D = 0 \quad (11)$$

Let us note in particular that the selection rule of Copley, Karl and Obryk ²⁾ for the helicity $\frac{3}{2}$ amplitudes of the ${}^2(\underline{8})$ resonances of the $\{ \underline{56}, 2^+ \}$ is not, in general, valid in our scheme.

Similarly it is straightforward to show that the 3P_0 model is equivalent to assuming

$$A = C \quad ; \quad D = 0 \quad (12)$$

The available photoproduction data do not allow as yet to definitely exclude any model. A naïve glance at the data would seem to indicate that $C \neq 0$ is probably required. It is noteworthy that slightly better data on the prominently photoproduced resonances such as ${}^2(\underline{8})_{\frac{1}{2}}$ $S_{11}(1545)$ and ${}^2(\underline{8})_{\frac{3}{2}}$ $d_{13}(1520)$ would make stringent tests of these models possible. For higher SU(6) multiplets such as $\{ \underline{56}, 2^+ \}$ the present data are too scanty to even reliably estimate our new parameter D.

Let us conclude with a few remarks.

- 1) Naïve quark model predictions are claimed to be in reasonable agreement with experiment except for the Roper resonance. Since our model establishes no connection between the photocoupling constants of the $\{ \underline{56}, 0^+ \}$ containing the Roper and, say, the $\{ \underline{56}, 2^+ \}$ that difficulty is certainly not present in our scheme.
- 2) As in the case of $\pi N \rightarrow \pi \Delta$ scattering, the transformation between current and constituent quarks can be used to predict the signs of photoproduction amplitudes at resonance ¹³⁾.

- 3) If one is willing to abstract algebraic properties of "bad" operators from the free quark model, it is clear that our predictions can be extended to electroproduction: instead of going through the dipole operator prescription, one may then abstract algebraic properties directly for the transverse components of the electromagnetic current. Although this procedure seems very unreliable to us it leads, in this particular case, to an identical algebraic structure. However, it is worth pointing out that contrary to naïve quark models ³⁾ such a scheme does not predict violent changes in the helicity structure of nucleon resonance electroproduction as a function of q^2 , the virtual photon mass squared.
- 4) For pionic transitions, the 3P_0 model ⁴⁾ was shown ¹⁰⁾ to be equivalent, up to mass-splitting factors, to a model based on PCAC and on the algebraic properties of the transformed axial charge $\tilde{F}^{i5} \equiv V^{-1} F^{i5} V$. This equivalence is clearly lost for vector transitions.

It is a pleasure to acknowledge interesting conversations on the subject of this note with S.P. de Alwis, J. Mandula and J. Rosner.

R E F E R E N C E S

- 1) R. Walker, Proc. of Fourth International Symposium on Electron and Photon Interactions at High Energies (Liverpool, 1969), ed., by D.W. Braben and R.E. Rand ;
A. Donnachie, Proc. of 1971 International Symposium on Electron and Photon Interactions at High Energies (Cornell, 1971), ed., by N.B. Mistry;
R.G. Moorhouse and H. Oberlack, Phys.Letters 43B, 44 (1973).
- 2) D. Faiman and A. Hendry, Phys.Rev. 173, 1720 (1968); 180, 1572 (1969);
L.A. Copley, G. Karl and E. Obryk, Phys.Letters 29B, 117 (1969);
Nuclear Phys. B13, 303 (1969);
R.P. Feynman, M. Kislinger and F. Ravndal, Phys.Rev. D3, 2706 (1971).
- 3) F.E. Close and F. Gilman, Phys.Letters 38B, 541 (1972);
F.E. Close, F. Gilman and I. Karliner, Phys.Rev. D6, 2533 (1972).
- 4) J.L. Rosner and W.P. Petersen, Phys.Rev. D7, 747 (1973).
- 5) H.J. Lipkin and S. Meshkov, Phys.Rev.Letters 14, 670 (1965); Phys.Rev. 143, 1269 (1966);
K.J. Barnes, P. Carruthers and F. von Hippel, Phys.Rev.Letters 14, 82 (1965).
- 6) R. Dashen and M. Gell-Mann, Phys.Letters 17, 142, 145 (1965).
- 7) H.J. Melosh, PhD thesis (1973), California Institute of Technology, unpublished.
- 8) In the light-cone formulation, good operators depend only on the projection $q_+ \equiv \frac{1}{2}(1 + \alpha_3)q$ of the quark field while bad operators involve $q_- \equiv \frac{1}{2}(1 - \alpha_3)q$ as well. In general q_- is related to q_+ by a constraint equation which depends on the dynamics. See, e.g., Ref. 9) and earlier references quoted therein.
- 9) See e.g.,
E. Eichten, J. Willensen and F. Feinberg, SLAC PUB-1258 (1973);
S.P. de Alwis and J. Stern, CERN preprint TH.1679 (1973).
- 10) F. Gilman and M. Kugler, Phys.Rev.Letters 30, 518 (1973);
A.J.G. Hey and J. Weyers, Phys.Letters 44B, 263 (1973);
F. Gilman, M. Kugler and S. Meshkov, Phys.Letters (to be published).
A.J.G. Hey, J. Rosner and J. Weyers, Nuclear Phys. (to be published).

- 11) A. Love and D.V. Nanopoulos, University of Sussex preprint (1973).
- 12) R.G. Moorhouse, Phys.Rev.Letters 16, 772 (1966).
- 13) While this note was being written we received a preprint by F. Gilman and I. Karliner (SLAC-PUB-1271) where these predictions are worked out in the special case $C=0$. We thank Drs. Gilman and Karliner for sending us a preprint of their paper prior to publication.

SU(6)	$2S+1 [\underline{\text{SU}(3)}]_J$	F_-^p	F_+^p	F_-^n	F_+^n
	$^2\bar{8}_{1/2}$	$\frac{1}{6}A + \frac{1}{6}B - \frac{1}{6}C$	-	$-\frac{1}{6}A - \frac{1}{18}B + \frac{1}{18}C$	-
	$^2\bar{8}_{3/2}$	$\frac{1}{6}\sqrt{\frac{2}{3}}A - \frac{1}{3}\sqrt{\frac{2}{3}}B - \frac{1}{6}\sqrt{\frac{2}{3}}C$	$\frac{1}{2}\sqrt{\frac{1}{6}}A + \frac{1}{2}\sqrt{\frac{1}{6}}C$	$-\frac{1}{6}\sqrt{\frac{1}{2}}A + \frac{1}{9}\sqrt{\frac{1}{2}}B + \frac{1}{18}\sqrt{\frac{1}{2}}C$	$-\frac{1}{2}\sqrt{\frac{1}{6}}A - \frac{1}{6}\sqrt{\frac{1}{6}}C$
	$^2\bar{10}_{1/2}$	$\frac{1}{6}A - \frac{1}{18}B + \frac{1}{18}C$	-		
$[\underline{70}, 1^-]$	$^2\bar{10}_{3/2}$	$\frac{1}{6}\sqrt{\frac{2}{3}}A + \frac{1}{9}\sqrt{\frac{2}{3}}B + \frac{1}{18}\sqrt{\frac{2}{3}}C$	$\frac{1}{2}\sqrt{\frac{1}{6}}A - \frac{1}{6}\sqrt{\frac{1}{6}}C$		
	$^4\bar{8}_{1/2}$	0	-	$\frac{1}{18}B - \frac{1}{18}C$	-
	$^4\bar{8}_{3/2}$	0	0	$\frac{1}{18}\sqrt{\frac{1}{5}}B - \frac{2}{9}\sqrt{\frac{1}{5}}C$	$\frac{1}{6}\sqrt{\frac{1}{5}}B - \frac{1}{3}\sqrt{\frac{1}{15}}C$
	$^4\bar{8}_{5/2}$	0	0	$-\frac{1}{6}\sqrt{\frac{1}{5}}B - \frac{1}{6}\sqrt{\frac{1}{5}}C$	$\frac{1}{6}\sqrt{\frac{1}{5}}B + \frac{1}{6}\sqrt{\frac{1}{5}}C$
SU(6)	$2S+1 [\underline{\text{SU}(3)}]_J$	F_-^p	F_+^p	F_-^n	F_+^n
	$^2\bar{8}_{3/2}$	$-\frac{2}{15}\sqrt{\frac{2}{3}}A' - \frac{2}{15}\sqrt{\frac{2}{3}}B' + \frac{2}{15}\sqrt{\frac{2}{3}}C'$	$-\frac{1}{15}\sqrt{2}A' - \frac{1}{15}\sqrt{2}C' - \frac{4}{15}D'$	$+\frac{4}{45}\sqrt{2}B' - \frac{4}{45}\sqrt{\frac{5}{2}}C'$	$+\frac{2}{45}\sqrt{2}C' + \frac{8}{45}D'$
	$^2\bar{8}_{5/2}$	$-\frac{2}{15}A' + \frac{2}{15}\sqrt{3}B' + \frac{2}{15}C'$	$-\frac{2}{15}\sqrt{2}A' - \frac{2}{15}\sqrt{2}C' + \frac{2}{15}D'$	$-\frac{4}{45}\sqrt{3}B' - \frac{4}{45}C'$	$+\frac{4}{45}\sqrt{2}C' - \frac{4}{45}D'$
$[\underline{56}, 2^+]$	$^4\bar{10}_{1/2}$	$+\frac{4}{45}\sqrt{\frac{2}{3}}B' - \frac{4}{45}\sqrt{\frac{2}{3}}C' - \frac{4}{15}\sqrt{\frac{1}{3}}D'$	-		
	$^4\bar{10}_{3/2}$	$-\frac{2}{45}\sqrt{2}B' - \frac{4}{45}\sqrt{3}D'$	$-\frac{2}{15}\sqrt{\frac{5}{3}}B' + \frac{4}{45}\sqrt{2}C' + \frac{4}{45}D'$		
	$^4\bar{10}_{5/2}$	$-\frac{2}{45}\sqrt{\frac{6}{7}}B' + \frac{2}{9}\sqrt{\frac{6}{7}}C' - \frac{4}{15}\sqrt{\frac{1}{7}}D'$	$-\frac{4}{15}\sqrt{\frac{5}{7}}B' + \frac{4}{45}\sqrt{\frac{1}{7}}C' - \frac{8}{45}\sqrt{\frac{2}{7}}D'$		
	$^4\bar{10}_{7/2}$	$+\frac{4}{15}\sqrt{\frac{6}{7}}B' + \frac{8}{45}\sqrt{\frac{6}{7}}C' - \frac{2}{45}\sqrt{\frac{6}{7}}D'$	$-\frac{4}{9}\sqrt{\frac{3}{35}}B' - \frac{8}{9}\sqrt{\frac{1}{35}}C' + \frac{2}{9}\sqrt{\frac{2}{35}}D'$		

$F_{\pm}^p = F_{\pm}^n$ for excitation of 10 resonances.

PHOTOPRODUCTION HELICITY STRUCTURE FOR $[\underline{70}, 1^-]$ and $[\underline{56}, 2^+]$ RESONANCES