DYNAMICAL R-PROCESS AND NUCLEI FAR FROM THE REGION OF G-STABILITY

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Abstract

A dynamical r-process model proposed by Hillebrandt et al. 1) is discussed by putting special emphasis on the (n,γ)-laminatitivity of the results to different sets of astrophysical input parameters.

1. Introduction

Previous studies of the r-process which is known to be responsible for the formation of a bulk of heavy elements beyond iron peaks, encountered two main difficulties: Neither reliable nuclear systematics for very neutron rich nuclei nor detailed models of supernovae, the most promising astrophysical site, were available. 2) Encouraged by notable progress recently made in both fields, Hillebrandt, Takahashi and Kodama 1) performed a dynamical r-process calculation by simultaneously solving the supernova hydrodynamical equations and the nucleosynthesis equations and including the energy feedback due to β-decaying and neutrons as well as shock heating.

The present report will concentrate on the discussion of the results of ref. 1) but for different sets of input parameters in order to demonstrate that a mutual compromise between astrophysical and nuclear physics aspects has been achieved within the present uncertainties stemming from both fields.

2. Model

Here we briefly summarize the model of ref. 1).

2.1. Synthesis equations

The first equation of the nuclear network is obtained by assuming a local (n,γ)-equilibrium:

\[ \frac{d n_{Z}(A,t)}{dt} = \lambda_{Z} n_{Z}(A,t) + \frac{3}{2} \log \frac{2A^2}{9T_{9}} n_{\gamma} + \lambda_{Z-1} n_{Z-1}(t) \]

(1)

where \( n_{Z}(A) \) is the number density of nucleus with charge \( Z \) and mass number \( A \), \( n_{\gamma} \) the free neutron number density per \( cm^{-3} \), \( S_{n} \) the neutron generation energy in MeV, \( T_{9} \) the temperature in \( 10^{9} K \), and \( \lambda \) is the nuclear partition function.

The \( \beta^{-} \) decays under (n,γ)-equilibrium are described by the following time-dependent coupled equations:

\[ \frac{d n_{Z}(t)}{dt} = -\lambda_{Z} n_{Z}(t) + \lambda_{Z-1} n_{Z-1}(t) \]

(2)

with the definitions

\[ \lambda_{Z} = \sum_{A} \frac{\ln 2}{S(Z,A)} P_{\gamma}(A), \]

\[ P_{\gamma}(A) = n(Z,A)/n_{\gamma} \]

where \( T_{\beta} \) is the \( \beta^{-} \) decay half-life.

Equations (1) and (2) are assumed to be valid until the total number of neutrons has become roughly equal to the total number of nuclei ( "freezing condition"; \( n_{\gamma} \simeq n_{\gamma}^{nuc} \)). After the freezing, γ-delayed neutron emissions, α-decays, spontaneous and β-delayed fissions are taken into account but up to now the non-equilibrium (n,γ) reactions have not been included.

2.2. Hydrodynamical equations

The dynamical evolution is described by

\[ \frac{d V}{dt} = -\frac{1}{\rho} \frac{d P}{dt} - \frac{d}{dr} \left( \frac{r dr}{dt} \frac{dV}{dr} \right) \] (momentum conservation),

(3)

\[ \frac{d \rho}{dt} = \frac{d}{dr} \left( \frac{r dr}{dt} \frac{\rho}{dr} \right) \] (mass conservation),

(4)

\[ \frac{d E}{dt} - \frac{d}{dr} \left( \frac{r dr}{dt} \frac{E}{dr} \right) + \sum_{Z} \frac{E_{\gamma}(A)}{n_{\gamma}} \frac{d n_{Z}(A)}{dt} = - q_{\gamma} + q_{\beta} \]

(5)

(\( q_{\gamma} \) internal energy conservation),

\[ p = p(\theta, r, n) \]

(6)

where \( \rho \) is the density, \( p \) the pressure, \( n_{Z} \) the number density of each species \( Z \), \( E \) the internal energy density, \( \theta \) the mass density, \( M(\theta) \) the mass inside a shell of radius \( r \), \( \phi \) the gravitational constant, \( q_{\beta} \) the energy loss due to β-decays and \( q_{\gamma} \) is the energy release of nuclear reactions.

The equation of state is given by a sum of contributions from the radiation field, the electron gas, the neutrons and the nuclei.

2.3. Initial conditions

Although no generally accepted supernova model is available at the moment, a great progress during the last years in this field has enabled us to adopt a "reasonable" model under the following restrictions. It should smoothly fit to advanced stages of stellar evolution. The peak density of the ejected material should be sufficiently high to show high degree of neutronization ( high total neutron to proton ratio N/P ). The velocity field of the ejected material should explain the observed supernova light curves and give a total luminosity of \( 10^{52} \) L\(_{\odot} \) ergs.

The density distribution is obtained by consulting Arnett's 22 Mg presupernova star calculation. 3) An adiabatic law is then used to get the temperature distribution. On this density-temperature profile a velocity field is superimposed which is similar to Wilson's calculation. 4) The initial composition is taken from a nuclear statistical equilibrium calculation by El Eid 5) for the central 1.4 Mg Ni-core and from Arnett's calculation for the matter outside.

2.4. Numerical calculation

Equations (1) to (6) have to be solved simultaneously. Several reasonable simplifications are made. For \( T_{9} \leq 25 \) the matter composition and the equation of state are taken from the nuclear statistical equilibrium with fixed N/P ratio. For \( T_{9} > 25 \) and \( n_{\gamma} \sim n_{\gamma}^{nuc} \), the actual nuclear abundances are used to calculate the equation of state and \( q_{\gamma} \) is approximated by the β-decay energy release \( q_{\beta} \).
includes the contributions of a black-body radiation and the electrons. For the matter after the freezing and at the initial densities \( 3 \times 10^7 \text{ g cm}^{-3} \), the nuclear abundances are fixed and an adiabatic expansion is assumed.

A static neutron star of mass 1 M\(_\odot\) and radius 10\(^6\) cm is assumed as an interior boundary condition and as an external gravitational field. The part out to 3\(\times 10^6\) cm is divided into 20 shells, of which the inner 8 shells containing 0.43 M\(_\odot\) (the initial Ni-core minus the neutron star) are assumed to undergo the \(r\)-process. The shock heating is taken care of by introducing an artificial viscosity term.

3. Nuclear Systematics

Since the \(r\)-process calculation needs nuclear data of some thousand neutron-rich nuclei, theoretical extrapolations should carefully be made.

3.1. Masses

The nuclear masses enter the \(r\)-process calculation mainly as the neutron separation energy \(S_N\) and the \(Q\)-value \(Q_0\). Specifically, the evaluation of \(S_N\) is very important as it approximately determines the \(r\)-process path in the N-Z plane. We have used, the mass formula recently given by van Groote et al, which has the droplet-model part plus a new semi-empirical shell correction term. In order to show the uncertainties coming from the mass formula used, we display in Fig.1 the results obtained with two different sets of parameter values. The astrophysical parameters are kept as the standard numbers (see section 4.1.). The solid curve in Fig.1 corresponds to the result of ref.1 with the parameter set giving the best r.m.s. for masses (0.57 MeV) \(\sigma_0\), and the dashed curve is obtained with the set which gives r.m.s. of 0.67 MeV but better agreements for nuclear radii and quadrupole moments.\(\sigma_0\)

3.2. \(Q\)-decay half-lives and energy release

In the dynamical \(r\)-process, the absolute values of \(Q\)-decay half-lives become very important as they determine the time scale of the synthesis which should match the astrophysical time scale. At the moment, the most reliable \(Q\)-decay half-life prediction comes from the gross theory of \(Q\)-decay.\(\sigma_0\)

We have used an approximate formula\(\sigma_0\) based on the gross theory calculation. The non-adiabatic temperature variation is considered by including the \(Q\)-decay energy release and its feedback on the hydrodynamics. The gross theory of \(Q\)-decay is used to calculate \(\sigma_0\) replacing \(\sigma_0\) in Eq.(5).

3.3. \(Q\)-delayed neutrons

The inclusion of delayed neutron emissions after the freezing is known\(\sigma_0\) to eliminate the strange even-odd effect, which appears in the abundance curve calculated at the freezing but not in the observed abundance curve. The cascade calculation has been performed in the same way as in ref.\(\sigma_0\). As an example, the frozen abundances for \(A220\) are plotted in Fig.1 by crosses.

3.4. Partition function

The nuclear partition function \(\varphi\) appearing in Eq.(1) is defined by \(\varphi = (2\pi \hbar \omega)^{\frac{3}{2}} e^{-\frac{\hbar \omega}{kT}}\), where \(\hbar\) and \(\omega\) are the excitation energy and the spin of the \(1\)-th nuclear state and \(k\) and \(T\) are the Boltzmann constant and the temperature, respectively. It should be noted that \(\varphi\) strongly reflects the pairing, shell and deformation effects. An approximate formula for \(\varphi\) is given in ref\(\sigma_0\), which we have used with slight changes for the magic-number and deformation effects to be consistent with the mass formulae used.

3.5. Nuclear data in the heaviest mass region

The spontaneous, neutron-induced and \(\alpha\)-delayed fissions and the alpha-decays are included in the dynamical \(r\)-process calculation by using the method of ref\(\sigma_0\). It turned out that within the dynamical model the synthesis will very probably terminate by an exhaustion of free neutrons rather than by neutron-induced (or spontaneous) fissions as formerly thought. Thus the fission (spontaneous, \(\alpha\)-delayed) processes enter the calculation only after the freezing. The relative abundances of the heaviest elements at 5\times 10\(^9\) years after the event are plotted with crosses in Fig.2 together with those shortly after the \(r\)-process and those of the present day's observations.

4. Astrophysical Parameters

The supernova model discussed in section 2 left us with four parameters, namely, the initial temperature at the neutron star surface \(T_{\text{max}}\), the maximum velocity in the outgoing shock wave \(v_{\text{max}}\), the onset temperature of the \(r\)-process \(T_{\text{onset}}\), and the maximum total neutron to proton ratio \((N/P)_{\text{max}}\).

4.1. Standard values

Considering the restrictions mentioned in section 2,3. Hillenbrand et al.\(\sigma_0\) obtained the standard values for those four parameters:

\[
\begin{align*}
T_{\text{max}} &= 1.2 \times 10^{10} \text{ K}, \\
v_{\text{max}} &= 5 \times 10^{6} \text{ cm/sec}, \\
T_{\text{onset}} &= 5 \times 10^{9} \text{ K}, \\
(N/P)_{\text{max}} &= 7. 
\end{align*}
\]

The calculated abundance curve is reproduced in Fig.2 by solid line.

4.2. Uncertainties

Among the standard values (7), \(T_{\text{max}}\) agrees with Wilson's calculation, and \(T_{\text{onset}}\) is suggested by a nuclear network calculation of Thielemann and Hillenbrand (10) as a temperature at which the composition of matter starts to deviate from that of nuclear statistical equilibrium. As small changes of these parameters do not change the calculated abundance curves significantly, we have fixed them as the standard numbers. In order to see the uncertainties stemming from the astrophysical parameter values, we show in Fig.2 the results with two different sets of \(v_{\text{max}}\) and \((N/P)_{\text{max}}\), namely, \(v_{\text{max}} = 4 \times 10^{6} \text{ cm/sec}, (N/P)_{\text{max}} = 6\), and \(v_{\text{max}} = 5 \times 10^{5} \text{ cm/sec}, (N/P)_{\text{max}} = 8\), together with the standard ones.

5. Summary

Here we summarize the main features of the dynamical \(r\)-process model proposed by Hillenbrand et al.\(\sigma_0\):

(1) The observed \(r\)-process abundance curve can be reproduced fairly well by assuming only one supernova event. This is a great contrast to the former classical static or dynamical models. But to Schramm's\(\sigma_0\) where the general feature of the observed curve was reproduced by one event under
Fig. 1 r-process abundance curves calculated with two different sets of mass formula parameters. The astrophysical input parameters are fixed as (7). --- : set given in ref. 8b), --- : set given in ref. 8a), ● : observed r-process abundances taken from ref. 16), x : frozen abundances in the innermost shell which exclusively synthesizes the heaviest nuclei with $A \geq 200$. (Note that for the outer shells synthesizing the lighter elements the effect of delayed neutrons has already been included to escape a complexity of the picture.)

Fig. 2 Calculated r-process abundance curves for different sets of the astrophysical parameters. --- : standard set (7), --- : $v_{\text{max}} = 4 \times 10^3$ cm/sec instead of $5 \times 10^3$ cm/sec, ----- : $(\beta_\text{2max}) = 8$ instead of 7. Also plotted ( by x) are the calculated relative abundances of $^{208}$Pb, $^{208}$Bi, $^{232}$Th, $^{235}$U, $^{239}$Pu at $5 \times 10^9$ years after the event. The observed abundances are again plotted by dots ●.
the assumption of a continuous seed-nuclear supply by $\alpha$, $\beta$- and $\gamma$-reactions.

(2) The time scale is of an order of 1 sec as we expect from the $\beta$-decay systematics. This solves a difficulty of mismatching time scales from astrophysical and nuclear physics aspects as seen in Cameron et al. who had to assume unreasonably short $\beta$-decay half-lives.

(3) The inclusion of $\beta$-decay energy release and its feedback on the hydrodynamics keeps the temperature higher compared with the case of adiabatic expansion, leading to a temperature-density relationship favourable to getting the right $r$-process path. Goto discussed this non-adiabaticity but overestimated the effect, in neglecting the feedback on hydrodynamics.

(4) The calculated abundance curves are fairly insensitive to the astrophysical input parameters if varied within reasonable limits.

(5) The inclusion of delayed neutron emissions after the freezing eliminates the even-odd effects sufficiently. Another smoothing effect will come from the non-equilibrium freeze-out as discussed by Schnaerr, not included in our calculation.

(6) The $r$-process terminates by an exhaustion of free neutrons rather than by neutron-induced fissions as previously thought. The maximum $z$ synthesized within the dynamical model discussed here is at most 20, depending on the astrophysical parameter values. None in the following talk proposes that the $r$-process might terminate by $\beta$-delayed fissions. It seems, however, that his conclusion comes merely from an inadequate assumption on the $\beta$-strength functions.

(7) The residual deviation of the calculated abundance curve from the observed one (especially, the shifts of the peaks) suggests possible improvements of the nuclear systematics as well as the supernova models. Judging from item (4), the systematics (in particular, the mass formula) should carefully be reinvestigated. It should be warned that the droplet-model based mass formulas very often give rather poor results for the neutron separation energies which are one of the crucial quantities in the $r$-process calculation. It will be worthwhile to note here that sometimes certain inaccuracies come from the numerical method of solving the dynamical equations. For example, we have found that the better agreement concerning the $\approx 1.95$ peak with $\Gamma = 1.9 \times 10^5$ cm/sec instead of the standard number $5 \times 10^5$ cm/sec (see, Fig.2) stems from the too large width assumed for the innermost shell.

(8) A nice way to check the $r$-process calculation can be done by making use of the radioactive elements in conjunction with the age of the Galaxy. Adopting the method of Schnaerr and Wasserburg, we have found that the relative $r$-process abundances calculated with the standard numbers (for $32^N$, $32_{\beta}$, $32_{\gamma}$, $32_{\alpha}$, and $32_{\beta}$-Pu give a reasonable age of the Galaxy. The details will be published elsewhere.

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