DYNAMIC CALCULATION OF THE SPONTANEOUS FISSION HALF-LIVES FOR THE HEAVIEST NUCLEI

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Abstract

Dynamic calculations of the spontaneous fission half-lives are performed for doubly even nuclei of actinide and trans-actinide regions. No adjustable parameters are introduced. Fairly good agreement with experiment is obtained.

1. Introduction

There is a continuous progress in the synthesis of the heaviest nuclides and in the research on their decay half-lives\(^1\-^3\). In particular, much has been learned recently about the spontaneous-fission lifetimes\(^1\,^3\). This progress results in a great interest in a theoretical reproduction of the data and in an extrapolation of the theory to unknown nuclides.

In our previous paper\(^4\), the spontaneous fission half-lives have been calculated in a static way. The aim of the present research is to perform the dynamic calculations.

The calculations are performed for doubly even nuclei with \(Z=96-110\). In distinction to previous dynamic calculations\(^5\), no adjustable parameters are introduced, the non-axial deformations are taken into account and the Myers-Swiatecki droplet model\(^6\) instead of the liquid drop is used for the macroscopic part of the potential energy.

2. Description of the calculations

The spontaneous fission half-lives \(T_{sf}\) are calculated according to the formula

\[
T_{sf} = \frac{1}{n} \ln 2 \frac{1}{P},
\]

where \(n\) is the number of assaults of a

nucleus on the fission barrier per unit time. This is usually equated to the frequency of the vibration mode that may lead to fission. For the energy \(\hbar \omega = 1\text{MeV}\) of this vibration, assumed in this paper, \(\hbar \omega \approx 10^{20.38}\) sec\(^{-1}\). The probability \(P\) of penetration of nucleus through the barrier for given assault, calculated in WKB approximation, is

\[
P = \left[1 - \exp \left(-\frac{S(L_{\min})}{\hbar \omega}\right)\right]^{-1},
\]

where

\[
S(L) = 2 \sqrt{\frac{2}{A}} \sqrt{\frac{2}{\hbar \omega}} \left[V(s) - E\right] B(b) \text{ d}L
\]

with

\[
B(b) = \sum_i^a \frac{\delta_{\alpha_i}}{\delta b} \frac{\delta_{\alpha_i}}{\delta b}
\]

is the action integral calculated along a trajectory \(L\) given in the deformation space. \(V\) is the potential energy barrier along \(L\), \(b\) is the energy of the fissioning nucleus, \(\alpha_i\) and \(\alpha_j\) are the deformation parameters and \(s\) is the parameter describing the position on the trajectory \(L\). \(\delta_{\alpha_i} / \delta b\) are the mass parameters of the nucleus. By dynamical calculations of \(T_{sf}\), we understand the finding of the trajectory \(L_{\min}\) in the deformation space, which gives the minimal value of the action \(S(L)\).

The deformation space which we admit for description of fission is an important characteristics of the calculations. The space should only to describe at least two features of the shapes involved in the fission process: elongation and necking of a nucleus. However, the reflection (or left-right) asymmetry in the nuclear shape is often also important. We describe\(^7\) the elongation by the quadrupole parameter \(\varepsilon\) and necking by the hexadecapole parameter \(\varepsilon_6\). The reflection asymmetry is described by the parameter \(\varepsilon_8\), being a combination of the parameters \(\varepsilon_4\) and \(\varepsilon_6\) corresponding to
the deformations of multipolarities 3 and 5, respectively\(^2\). We consider also the non-axial quadrupole deformation \(\gamma\), found important in the calculations of the fission barriers\(^3\).

The potential energy is calculated as composed of the macroscopic part and the shell correction. For the macroscopic part, the energy of the Myers-Swiatecki droplet model\(^6\), which is an improvement of the liquid drop model, is taken. The shell correction is calculated by the Strutinsky method.

The mass parameters are obtained in the cranking approximation. After an inclusion of the pairing interaction by the BCS formalism, the formulae for them are\(^10-13\)

\[
B_{\alpha\beta} = 2\kappa^2 \sum_{\nu\nu'} \frac{\langle \nu | \hat{H} | \nu' \rangle \langle \nu' | \hat{H} | \nu \rangle}{(\varepsilon_{\nu}^\alpha + \varepsilon_{\nu}^\beta)} \times \left( u_{\nu} n_{\nu'} + u_{\nu'} n_{\nu} \right)^2 + \mathbf{p}_{\nu\nu'}^2,
\]

where \(H\) is the single-particle potential, \(u_{\nu}\) and \(n_{\nu}\) are the BCS variational parameters and \(E_{\nu}\) is the quasiparticle energy corresponding to the single-particle state \(|\nu\rangle\). The term \(\mathbf{p}_{\nu\nu'}^2\) corresponds to the coupling between the pairing interaction and the collective motion connected with fission.

The numerical calculations are performed in the following two steps:

(i) In the first step, the potential energy and the three mass parameters \(B_{33}, B_{34}, \) and \(B_{44}\) are calculated as functions of \(\varepsilon\) and \(\varepsilon_{1}\) in the following 198 grid points: \(\varepsilon = 0(0.05)1.05\) and \(\varepsilon_{1} = -0.08(0.02)0.08\), where

\[
\varepsilon_{1} = \varepsilon_{1}^* + 0.2 \varepsilon - 0.06.
\]

The grid points are shown in fig. 1. The shapes of a nucleus corresponding to the corner points are plotted for a visual illustration (dashed lines correspond to inclusion of the \(\varepsilon_{35}\) deformation).

Then, the effect of the reflection symmetry deformation \(\varepsilon_{35}\) on the potential energy is added in the following way

\[
\Delta V_{\text{conv}}(\varepsilon, \varepsilon_{1}) = V(\varepsilon, \varepsilon_{1}) - V(\varepsilon_{35}, \varepsilon_{1} - \varepsilon_{35}^*),
\]

where \(\varepsilon_{35}\) and \(\varepsilon_{35}^*\) are the combinations of \(\varepsilon\), \(\varepsilon_{1}\) and \(\varepsilon_{3},\varepsilon_{5}\) deformations, respectively, define in ref.\(^3\). Thus, the deformations \(\varepsilon_{1}\) and \(\varepsilon_{5}\) are accounted only approximately and only in the potential energy. For given \(\varepsilon\), the energy is reduced for all \(\varepsilon_{1}\) by the same amount as obtained for \(\varepsilon_{1}\) corresponding to the bottom of the potential energy valley.

Fig. 1. Grid points for which the potential energy and the mass parameters are calculated.

The minimal value \(S(\varepsilon, \varepsilon_{1})\) is obtained by minimization of the action integral, eq. (2a), with respect to all possible trajectories crossing all 198 grid points.

(ii) In the second step, the dynamical correction to the action integral due to the \(\varepsilon_{35}\) deformation is calculated. This is obtained as a difference between the minimal action in the \(\varepsilon, \gamma\) plane, \(S(\varepsilon_{35}, \gamma)\), and the action along the \(\gamma = 0\) trajectory, i.e.

\[
\Delta S = S(\varepsilon_{35}, \gamma_{\min}) - S(\varepsilon_{35}, \gamma = 0).
\]

To get this, the potential energy and the mass parameters \(B_{33}, B_{34}, B_{44}\) are calculated in the 50 grid points: \(\varepsilon = 0(0.1)1.0, \gamma = 0(10^\circ)60^\circ\). In the macroscopic part of the potential energy, the hexadecapole degree of freedom is taken into account.

The total action is

\[
S = S(\varepsilon_{35}, \gamma_{\min}) + \Delta S.
\]

In all the calculations, the modified harmonic oscillator potential is used with the \(A = 242\) parameters corresponding to

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the actinide nuclei. The variant of the pairing strength independent of deformation is taken.

3. Results and discussion

3.1. Results

Fig. 2. Contour maps of the potential energy and mass parameters for $^{258}$Fm.

The energy is counted from the first minimum (0), the mass parameters from the minimal value (Min.) specified at each plot. Deformation corresponding to the minimum is indicated by a cross. Static and dynamic fission trajectories are shown.

This can be directly seen in fig. 3. The parameter is smaller, on the average, along the dynamic trajectory than along the static one.

The half-lives are presented in fig. 4. Most of the experimental points are cited after ref. 4). Few of them, the most recent ones, are taken from ref. 1). We can see that for almost all nuclides the discrepancy between the theory and experiment is less than 2 orders. The agreement is about the same as in our previous paper 4), where the lifetimes were calculated statically. However, in that paper we had one adjustable parameter while there is no one in the present paper. In particular, we find that in the dynamic calculations there is no need for a renormalization of mass parameters, while the renormalization was needed in the static calculations 4).
3.2. Change in the half-life systematics

It is seen in fig. 4 that the systematics of the experimental half-lives changes when one goes from $Z=102$ to $Z=104$. The maximum of $T_{sf}$ at the neutron number $N=152$ disappears and $T_{sf}$ increases continuously with increasing $N$. In the theoretical lifetimes this change appears slightly too early what can be probably easily corrected by a small change in the droplet parameters which are adjusted to lighter nuclei.

3.3. Role of the droplet model

The role of the droplet model in reproducing the lifetimes appears important. With the liquid drop (14), we cannot even reproduce the decrease of $T_{sf}$ for $N>152$ for $Fm$ and $No$ isotopes. Such reproduction needs a value of the symmetry coefficient $\kappa_2$ in the surface term larger than the

Fig. 4. Logarithms of the spontaneous fission half-lives.
value 1.78 obtained in ref. 14). In fact, for good reproduction of T_{\text{ef}} two parameters of the surface term were fitted to T_{\text{ef}} for each element in the previous dynamic calculations 5).

3.4. Role of the $\gamma$ deformation degree of freedom

Fig. 5 gives the potential energy of $^{244}\text{Fm}$ as function of the deformations $\alpha$ and $\gamma$. We can see from the static trajectory to fission that the $\gamma$ degree of freedom reduces the static fission barrier by about 1 MeV. However, if the dynamics is considered, the trajectory is much closer to the axially symmetric shape $\gamma=0$. As such picture is rather typical for nuclei considered, we can say that the dynamics reduces the effect of the $\gamma$ deformation on the half-lives considerably.

4. Conclusions

The following conclusions may be drawn from our research:

(i) Dynamic calculations with no adjustable parameters reproduce the spontaneous fission half-lives of doubly even nuclei with Z=96-104 fairly well.

(ii) Use of the droplet model instead of the drop model is important. A small refit of the droplet parameters would probably still improve the agreement.

(iii) Dynamics reduces the half-lives by about 1.5 - 2.0 orders with respect to the static half-lives, for nuclei considered.

(iv) Dynamics reduces the effect of the $\gamma$ deformation on the half-lives. It prefers the fission trajectories close to the axially symmetric ones, for nuclei considered.

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