SPECTROSCOPY OF NUCLEI FAR FROM STABILITY**
Raymond K. Sheline
Florida State University, Tallahassee, Florida, USA

Abstract

Spherical and deformed shell energies are calculated for both the simple harmonic oscillator and the modified harmonic oscillator. When these shell effects are added to pairing and liquid drop energies, total energies are produced which suggest special stabilities for a number of nuclear regions far from the line of beta stability. The regions discussed are spherical closed shells with emphasis on 132Sn, new regions of nuclear deformation with emphasis on the neutron deficient Ra's and Th's, 80Zr, 104Zr, 208Pb, and 232Th, and coexistence with emphasis on the neutron deficient Gd's, the neutron deficient Ir's and especially the neutron deficient Hg's. The spectroscopy of very high spin states is briefly discussed in terms of decoupled bands, backbending up to spins of $\frac{1}{2}^+$ and of continuum gamma rays depopulating much higher spin states. Calculations relating the very high spin states to nuclear shapes are also presented.

1. Introduction

The study of nuclei far from stability proceeds from at least two different rationales. One method is the production of any nucleus far from stability followed by an intensive study of its properties and the properties of its decay products. In this method no emphasis is given to the choice of a particular nucleus to study. Emphasis is more nearly given to the experimental problems which allow detailed data with high statistics to be obtained. The advantage of this method is that often it produces information which would not have been predicted.

In the second method, the experiments planned are guided by theoretical expectations or semi-empirical systematics. The advantage of this method is that one is usually looking for and able to find some significant effect which could be predicted prior to experiment.

In reviewing excited state spectroscopy and structure far from beta stability, we will tend somewhat to emphasize the latter method as a way of organizing this incredibly massive topic. This paper will, therefore, I hope, have the advantage of being not only a review, but also of having predictive qualities.

This paper is based on the theoretical treatment of shell effects and the resulting shell structures in both spherical and deformed nuclei. I am very much indebted to Dr. Ingemar Ragnarsson and to Professor S.G. Nilsson for significant contributions to the calculations and interpretation of these shell effects.

2. Shell Structure in Spherical and Deformed Nuclei

Shell structure is extensively observed in modern physics and chemistry. It is prominently observed in the band structures of solids and surprisingly similar in such different systems as atoms and nuclei. By shell structure we imply a quantum mechanical level structure in which subsets of the level structure are added in concentrated or dense groupings. Between these groupings are gaps or unavailable regions. Special stability is expected when a system has filled up and, therefore, completed one of these concentrated groupings of levels. On the other hand, an abnormally low stability is expected for a system in which such a group of levels is only partially filled. These special stabilities are well known in atomic systems for the rare gases He, Ne, Ar, Kr, Xe, and Rn with completed structures of 2,10,18,36,54 and 86 electrons. A similar stability for closed shells of nucleons, first completely pointed out by Maria Mayer, occurs for spherical nuclei with "magic numbers" of 2,8,20,28,50,82 and 126 (114) neutrons or protons. Bohr and Mottelson have generalized the analysis of shell structure for spherical systems in Volume II of their book, Nuclear Structure. Figure 1, which is based on parts of Chapter 6 of the book of Bohr and Mottelson, attempts to present the conditions which must exist for shell structure. Shell structure is expected when the

$$\frac{37}{39} = a:b$$

where $a$ and $b$ are integers. The energy in Fig. 1 is shown as contour lines for the different systems. For deformed nuclei $n$ and $l$ would be replaced by $h_0$ and $n_q$. For light and medium heavy nuclei the experimentally observed shell structure is relatively close to that of the spherical oscillator having the

$$\frac{37}{39} = 2:1$$

This is shown in Fig. 1 as a solid line and brings the s and d, p and f, and, $g$ and $i$ states close together in energy favoring quadrupole shapes. On the other hand, for the heaviest nuclei, parallel spin orbit states with 3:1 ratio lie close together in energy. This shell structure which places $g$ and $j$ states close together, for example, favors $P_2$ octupole shapes. The energy contours in Fig. 1 for this type of shell structure are shown with dashed lines.

Thus, although quadrupole type deformations are dominant in the light nuclei, it is clear that octupole deformation will ultimately be favored in the heaviest nuclei. It seems highly probable that in the appropriate regions certain super-heavy nuclei should achieve reflection asymmetric octupole
2.1. Shell Structure with the Simple Harmonic Oscillator

Geilikman found in the 1960 Kingston Conference that shapes other than spherical might be stabilized by shell structure. Similar conclusions were reached by the group at Berkeley. A very comprehensive treatment is presented in Volume II of Nuclear Structure by Bohr and Mottelson while an alternative semi-classical approach has been taken by Balan and Block and by Strutinsky. Following the treatment of Bohr and Mottelson, it is clear that shell structure should occur for non-spherical systems such that the following conditions on the eigenvalues is fulfilled:

\[ a:b:c = \frac{3e}{n_{a}} \frac{3e}{n_{b}} \frac{3e}{n_{c}} \]  \hspace{1cm} (3)

where \( a, b \) and \( c \) are integers and \( n_{a}, n_{b}, \) and \( n_{c} \) are three quantum numbers characterizing eigenstates such as the cartesian quantum numbers in the harmonic oscillator case. This condition is fulfilled when \( a:b:c \) is \( \omega x: \omega y: \omega z \) for the pure harmonic oscillator. We will mostly discuss the rotation symmetric case in which \( \omega x: \omega y = \omega z. \)

The eigenvalues for the simple harmonic oscillator are plotted against deformation in Fig. 2. The largest shell effects clearly occur for spherical nuclei. However, large shell effects are also observed for \( \omega x: \omega y = 1:2 \) (where \( \epsilon = 0.75 \)) and \( \omega x: \omega y = 2:1 \) (where \( \epsilon = 0.6 \)) where 75 and 60%, respectively, of the shell effects present in the spherical case are expected. Shell effects are also obvious in Fig. 2 for certain other deformations and are expected at still greater deformations not shown in Fig. 2 and for certain asymmetric rotors.

Fig. 1. A grid plot of the quantum numbers \( n \) vs \( i \) with various conditions for spherical shells. The various lines represent energy contours. The dashed-dotted line shows the shell structure resulting from the Coulomb force; the solid lines, the shell structure from the harmonic oscillator potential leading to quadrupole shapes; and the dashed lines, the shell structure resulting from the infinite square well scheme leading to octupole shapes.

Fig. 2. Single particle levels of the simple harmonic oscillator as a function of deformation.
2.2. Shell Effects for the More Realistic Modified Harmonic Oscillator

In Figs. 4 and 5 the modified harmonic oscillator with $l$- and $l^2$ terms is used to generate the shell effects to which pairing effects have also been added again shown as contour lines for protons and neutrons respectively. Clearly, the shell effects

Fig. 4. A contour diagram of the shell effects for the modified harmonic oscillator including pairing effects for the proton numbers indicated.
Fig. 5. A contour diagram of the shell effects for the modified harmonic oscillator including pairing effects for the neutron numbers indicated.

have been lessened but not destroyed. In addition, because axial symmetry has been maintained, the valleys and ridges remain in the contour diagrams. In Fig. 6 the same type plot is shown. Now, however, we employ the harmonic oscillator with $\gamma=30^\circ$ in which the axial symmetry is destroyed ($\omega_2^2/\omega_1^2=3:2:1$). The systematic valleys and ridges corresponding to patterns of shell effects and anti-shell effects, respectively, are destroyed because the axial symmetry no longer exists.

It is sometimes difficult to visualize contour diagrams. For this reason, we now present some cross sections through the contour diagrams of the shell effects at certain specific deformations. Figure 7 presents the shell effects for spherical shapes. The dashed lines show the shell effects for the simple harmonic oscillator, whereas, the solid lines are for the modified harmonic oscillator. The damping of shell effects, shifting to higher magic numbers and the special damping in the region from 20 to 50 nucleons is particularly interesting. In Fig. 8 cross sections through the contour lines of shell effects are shown for prolate nuclei with $\epsilon=0.6$, oblate nuclei with $\epsilon=-0.75$, and asymmetric

Fig. 6. A contour diagram for the shell effects of an asymmetric nucleus where $\gamma=30^\circ$. Because of the destruction of axial symmetry, the ridges and valleys have also been destroyed.
Fig. 7. A slice through the contour diagram of shell effects for 0 deformation (spherical shape) for both the simple harmonic oscillator and the modified harmonic oscillator.

nuclei with $\epsilon = 0.866$ and $\gamma = 30^\circ$, from the top to the bottom of the figure, respectively. It is interesting to note that shell effects are considerably reduced—more so for oblate and asymmetric nuclei and that the shift in magic numbers often produces a broadening out of the magic number region in addition to an increase in the magic numbers.

2.3. Total Energy (Shell Effects, Pairing Effects Plus Liquid Drop)

It is of considerable interest, then, to add these shell effects to the liquid drop and include pairing so that one obtains a total energy diagram. This diagram along the line of beta stability is shown in Fig. 9. Particularly conspicuous in this figure, is the presence of large shell effects for spherical shapes including the out of phase closed shells for 50 closed protons and 82 closed neutrons. It is also obvious that the normal ground state deformations in the rare earth region are caused by a cooperation of an antis-shell effect for spherical shapes and the Coulomb effect of the liquid drop at large deformations. Indeed, it is of interest to follow the structure of nuclei along the line of beta stability from the regions of $\gamma = 50$ and $\gamma = 02$ to the deformed rare earth region then over two saddles, one back to spherical at 208pb, the other to a secondary valley with $\gamma = 0.4$. The secondary valley has its deepest minimum for Pt and Hg nuclei. So far there is no experimental evidence for the expected shape effects in these nuclei. We observed strong shell effects in some of the lightest nuclei (see for example $^{24}$Ne and $^{28}$Si) and again for the very heaviest nuclei at $^{242}$Pu.

In the intermediate range no really well deformed nuclei are found along the line of stability. Thus, in the intermediate nuclei, if we wish to observe shell effects for deformed nuclei, we must look in the nuclear region far from beta stability where the neutron and proton shell effects can be in phase.

Fig. 8. Slices through the contour diagrams of shell effects (beginning from the top) for deformation $\epsilon = 0.6$, $\epsilon = 0.75$, and for $\epsilon = 0.866$ (an asymmetric rotor). The simple harmonic oscillator is shown dashed; the modified harmonic oscillator with a solid line.

In Fig. 10, a schematic diagram is presented for low A, medium A and high A nuclei, respectively. For each type of nucleus, we show a shell effect involving very few levels at 0 deformation and an anti-shell effect showing many levels at deformation $\epsilon = 0.6$ on the left hand side of the figure and an anti-shell effect at 0 deformation and a shell effect at $\epsilon = 0.6$ on the right hand side of the diagram. It is
Fig. 9. A total energy contour diagram where shell effects, pairing effects, and the liquid drop have been added together. The nuclei considered lie along the line of $\beta$ stability.

Fig. 10. Interactions of liquid drop and shell effects for light, medium, and heavy nuclei. The liquid drop potential is shown dashed; the shell effects cross-hatched; and the final potential as a solid line.

clear from this diagram why one can observe strong shell effects for deformed nuclei in both the light and heavy nuclear regions. In the light nuclear region, the parabolic effect of the liquid drop shown dashed, is easily overcome by the shell effect shown with cross hatchings below the dashed liquid drop line. In the heavy region because of the fissionability of the liquid drop (again shown dashed), the liquid drop reaches a maximum at deformation $\epsilon<0.6$. The shell effect can now easily make itself felt on the potential in such a way that it produces a second minimum resulting in the fission isomers. However, for the intermediate mass region it is not clear whether shell effects will be enough to overcome the steepness of the liquid drop potential or whether it will merely serve to make a complex potential shape with slightly increased deformation like the diagram shown on the middle of the right hand side of Fig. 10. This is a broad experimental problem which stands as a challenge to us.

2.4. Magic Numbers and a Road Map for Exploration

Recognizing that there is some doubt about magic numbers for deformed nuclei in the middle mass region because of the counteractive liquid drop and shell effects, we, nonetheless, list in Table 1 the magic numbers given by the shell effects alone for various quadrupole and asymmetric shapes. This table summarizes the results of Figs. 3 through 8.

Using Table 1, we have constructed a nuclear periodic table (Fig. 11) in which the magic numbers for spherical nuclei and for $\epsilon=0.6$ prolate nuclei are shown as
Table 1. Closed shell magic numbers for nuclei with various quadrupole deformations and shapes. (See e.g. Fig. 2) The numbers correspond to calculations for the simple harmonic oscillator. When $j^1$- or $j^1$-terms are employed the numbers in parentheses result. The last column gives the "magic numbers" for an asymmetric rotor.

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>-1.2</th>
<th>-0.75</th>
<th>0</th>
<th>0.6</th>
<th>0.86</th>
<th>1.0</th>
<th>0.87 ($\gamma=30^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_4$</td>
<td>1:3</td>
<td>1:2</td>
<td>1:1</td>
<td>2:1</td>
<td>3:1</td>
<td>4:1</td>
<td>3:2:1</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>20</td>
<td>10</td>
<td>6</td>
<td>8</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>26</td>
<td>40 (50)</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>44</td>
<td>70 (82)</td>
<td>28-32</td>
<td>18</td>
<td>14</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>112 (114, 126)</td>
<td>40 (40-44)</td>
<td>24</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>168 (184)</td>
<td>60-64</td>
<td>36</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>182</td>
<td>100</td>
<td></td>
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</tr>
</tbody>
</table>

vertical and horizontal lines. The area of interest, of course, is only the region enclosed by the neutron and proton drip lines. Where stable octupole shaped ground states may possibly be observed. Other regions are indicated symbolically by a single nucleus, usually represented by a double closed shell. Many of these regions are so exotic that it may take some time to reach them. Other regions, which we will discuss in some detail next, are already being explored.

Figure 11 is a kind of road map of the interesting places we hope to visit in the years ahead. We have shown the region of the super heavy nuclei and also the region where stable octupole shaped ground states may possibly be observed. Other regions are indicated symbolically by a single nucleus, usually represented by a double closed shell. Many of these regions are so exotic that it may take some time to reach them. Other regions, which we will discuss in some detail next, are already being explored.

![Fig. 11. The nuclear periodic table showing stable nuclei. Spherical closed shells are shown as heavy lines; deformed closed shells ($\chi=0.6$) as light lines; regions of special interest are marked with the isotope or in some cases with the structural effect expected.](image-url)
3. Spherical Closed Shells Far From Stability

Much of the success of structural studies in low energy nuclear physics originated with the clear understanding of closed shells in spherical nuclei. The existence of double closed shells along the line of stability at $^4$He, $^{160}$Ca and $^{208}$Pb is a cornerstone of this knowledge. However, because neutron and proton orbitals are out of phase with each other over much of the region of the nuclear periodic table, many of the interesting double closed shell spherical nuclei lie off the line of beta stability. Indeed, many of these nuclei lie far off the line of beta stability (although still within the region where nuclei are heavy particle stable) that they have not been studied and indeed represent an interesting challenge for the future.

$^{28g}$, $^{78n}$ and $^{100}$Sn are examples of double closed shell nuclei not yet studied which should ultimately be available for study. By the time of the Second International Conference on the Properties of Nuclei far from the Region of Beta Stability, the nucleus $^{56}$Ni, also an example of a double closed shell spherical nucleus, had been extensively studied. Since the time of that conference the study of $^{140}$Sm has begun in earnest. Using fission products as a source, and often using isotope separators, a number of nuclei in the vicinity of $^{140}$Sm have been studied. Perhaps, the most striking evidence for the double closed shell nature of $^{140}$Sm is shown in Fig. 12. The systematics of 3- octupole shapes in the Sm isotopes and in the N=82 isotones is fairly convincing evidence for the tentative assignment of the first excited state in $^{140}$Sm at 4041 keV. This work was done by Kerek and a Stockholm group utilizing a 0.12 second $^{140}$Sm activity which populates the 3- octupole vibration in $^{140}$Sm. Kerek and a Stockholm group have also studied the one-particle one-hole nucleus $^{132}$Sn following the decay of $^{132}$Sn. In all, five positive parity states have been observed in $^{132}$Sn, the lowest three of which can be assigned to the configuration $2g_{7/2}^2 \times (2d_{5/2})$.

The configurations are precisely those expected from the systematics of nuclei near the line of stability and show a very interesting comparison with these nuclei. Holm and the Szeged group have studied $^{132}$Sn and $^{134}$Sn. Recently, Wolf and Chifetiz have even been able to study the magnetic moment of the 6- state in the nucleus with two protons beyond the $^{132}$Sn double closed shell nucleus. This study is being reported at this conference and again supports the assumption that the $^{132}$Sn region can be understood in terms of the spectroscopy of nuclei closer to the beta line of stability.

When one considers how remote the $^{132}$Sn region seemed at the time of the last conference in 1970 and what an interesting and important field of research it has proven to be, it gives us hope that some of the other regions of double closed shell nuclei which now seem so remote may be available sooner than we now think.

Fig. 12. The 3- states in the Z=50 Sn isotopes and in the N=82 isotones, drawn in such a way as to emphasize the assignment of the tentative octupole state in $^{140}$Sm.

4. New Regions of Nuclear Deformation

As we have seen in Section 2, normal quadrupole nuclear deformation may arise either from the combined effects of anti shell effects and the liquid drop as occurs in the normal rare earth and actinide regions or as a result of special shell effects for very specific deformations. We will begin with considerations of the more normal kinds of deformation and then go into deformed shell effects.

4.1. Normal Nuclear Deformation with $\epsilon = 0.25 - 0.3$

It has long been recognized that those regions of the nuclear periodic table not close to closed shells could be expected to involve a normal deformation with $\epsilon = 0.25 - 0.3$. Indeed, this rationale led to the observation of the new region of nuclear deformation in the neutron deficient Ba isotopes where both neutron and proton numbers lie in the intermediate range between 50 and 82. We have now reached heavy ion energies and techniques which should allow us to observe the normal deformation region where both neutrons and protons lie in the intermediate range between 82 and 126. Probably, very neutron deficient Ra or Th nuclei can now be studied by in-beam gamma ray spectroscopy to prove the deformation by observing rotational bands. This would be particularly simple in even-even nuclei. Those nuclei which are particularly amenable to study are $^{202}$Ra, $^{204}$Ra, $^{206}$Ra,
204\text{h}, 206\text{h}, and 208\text{h}. These nuclei are of considerable interest as the prototypes of this new normal region of deformation, but they may also have a bonus since they lie very close to the region where one expects an $\epsilon=0.6$ deformation because of a deformed shell effect which we will consider in the next section.

4.2. Deformed Closed Shells Far From Stability

One of the most interesting experimental problems in the years ahead will be the attempt to discover deformed nuclei resulting from special closed shells and double closed shells far from stability. As pointed out in Section 2, whereas there is little doubt about the existence of these effects in the lightest and heaviest nuclei, there is doubt about the ability of the shell effects to overcome the liquid drop effects in the middle mass region. In this section we will discuss nuclei where large shell effects are expected without concern about the competition with the liquid drop effects which still must be experimentally determined. Figure 11 points out a number of the more interesting regions by labeling a nucleus where the deformed shell effect is most expected. In the lightest region these nuclei include $^{14}\text{Be}$, $^{26}\text{Ne}$, $^{26}\text{Ca}$, and $^{44}\text{Ca}$. Most of these nuclei have not even been observed, but in the years ahead probably most of them will be observed. In either their ground states or in an excited state, one should observe a rotational band with a very small value of $\hbar^2/2I$ corresponding to a value of $\epsilon=0.6$.

In the intermediate mass region, prolate deformed shell effects might be expected for $^{64}\text{Ge}$, $^{80}\text{Zr}$, $^{104}\text{Zr}$, $^{128}\text{Cd}$, and $^{180}\text{Cd}$. None of these nuclei have been observed. However, in the case of $^{80}\text{Zr}$ and $^{104}\text{Zr}$, we are approaching the study of these nuclei.

In Fig. 13, a contour diagram of the $2^+$ first excited states is presented$^{18}$ for nuclei approaching $80\text{Zr}$ and $104\text{Zr}$. It is obvious from these contours that we seem to be approaching minima precisely where they are expected in $^{80}\text{Zr}$ and $^{104}\text{Zr}$. The best experimental estimate of the value of $\epsilon$ for $^{104}\text{Zr}$ is $\epsilon=0.4$ instead of the expected 0.6. This may be an indication that in this intermediate mass region the effect of the liquid drop is to reduce the expected deformation of the shell effect.

Except for the fission isomers in the heavy mass region, we might expect double closed shells for $^{202}\text{Rn}$ and $^{232}\text{Rn}$. Their position on Fig. 11 indicates that they are highly neutron deficient and highly neutron excess species, respectively. Probably both of these nuclei are accessible with today's techniques.

4.3. Coexistence of Different Shapes in the Same Nucleus

Two of the best established cases of coexistence of different shapes in the same nucleus are $^{16}_6\text{O}$ and the fission isomers. In both of these cases the effect owes its existence to shell effects. In $^{16}_6\text{O}$ we have a double closed shell spherical ground state and an asymmetric rotor with $\hbar^2/2I=3.21$. The magic number of neutrons and protons for each shape is 8 (see Table 1). The existence of the second minimum in the fission isomers is due to the shell effect for neutron numbers from 142 to 150 (see Table 1). The recently observed$^{17,18}$ existence of approximately spherical and prolate deformed shapes for the $11/2^+\{505\}$ band in $^{150}\text{Cd}$ may also be due to shell effects in nuclei. Thus, for example, $^{150}\text{Cd}$ is a doubled closed shell nucleus. It should, however, be noted that none of the bands observed has a deformation even approaching $\epsilon=0.6$ in $^{150}\text{Cd}$.

![Contour plot of the experimental energy of the lowest 2+ state given in keV as a function of N and Z. Stable isotopes are indicated by solid black squares. It suggests deep minima in the contours for 80Zr and 104Zr, marked in the figure with x's.](image)
A very exciting example of coexistence has been found in the neutron deficient Ir isotopes. It has been particularly carefully studied in the case of $^{187}$Ir and $^{189}$Ir by the Polish-Biala group\(^1\) and by the Dresden group\(^2\). Certainly some of the most interesting and exciting spectroscopy has occurred in the transition region of the Os, Ir, Pt, Au and Hg nuclei. The observation of large numbers of decoupled bands by the Daly group at Purdue and the Michigan State group, and the discovery of the very low lying $0^+$ states in the neutron deficient Pt isotopes by the Orsay group\(^3\) are only two examples.

Certainly one of the most interesting examples of coexistence involves the study of neutron deficient Hg isotopes. This problem was precipitated by the discovery\(^2\) of anomalous optical isotope shifts in the light Hg isotopes. It has been followed by a number of experimental and theoretical studies. Measurement of the spins and magnetic moments of the odd-$A$ Hg isotopes with $A=180$-185 suggests that they may be the strongly deformed Nilsson state $1/2^+$\[^{521}\]. On the other hand, the systematics of the $2^+$ states in the neutron deficient Hg isotopes implies a vibrational rather than a rotational character. The CERN ISOLDE group has also studied the Hg isotopes in a number of different ways, especially using alpha decay\[^{33}\]. The Berkeley group\(^{44}\) studied the neutron deficient Hg isotopes using in-beam gamma spectroscopy while a Chalk River group\[^{55}\] studied the lifetimes of the states in $^{184}$Hg. Decay scheme spectroscopy of the Tl isotopes by the UNISOR group\[^{46}\] have made possible the study of some additional states in $^{186}$Hg and $^{188}$Hg including a $0^+$ excited state and all low spin states of the rotational band in $^{188}$Hg. The present status of this study suggests that there is a shift from a generally vibrational spectrum to rotational spectrum. This shift occurs at the $2^+$, $4^+$, and $6^+$ states in $^{184,186,188}$Hg, respectively. Furthermore, a change in the lifetimes of the states occurs in just the place expected in a transition from vibrational to rotational spectra. Most of the systematics for the Hg isotopes from $A=184$ to $A=190$ is summarized in the backbending plot shown in Fig. 14\[^{26}\]. One notices that the backbending begins earlier in each Hg isotope as the neutron number decreases.

A number of different theoretical papers have attempted to explain this phenomenon\[^{27-32}\]. There seems now to be a general agreement that we are dealing with a potential energy surface with two minima and a reasonably classic example of coexistence. Usually the calculated positions of the two minima differ by only a few hundred keV and involve small oblate deformations and relatively large prolate deformations (\(\epsilon=0.27\)). Although it is probably just a coincidence, it is interesting to point out that the 104 neutrons in $^{184}$Hg represent a deformed closed shell for an oblate structure, whereas 80 protons is close to a deformed prolate closed shell.

![Fig. 14. A backbending plot of the yrast states for the even Hg isotopes with A=184-190.](image)

**5. The Spectroscopy of High Spin States in the Yrast Band**

Since the time of the last Conference on Nuclei far from Stability, one of the most popular areas of research has been the study of very high spin states. For the most part, these studies have involved in-beam gamma-ray spectroscopy following (proton, deuteron, alpha or heavy ion, xn) reactions although some decay scheme studies have also contributed. Because these nuclear reactions generally produce neutron deficient species, this topic is particularly appropriate for this conference. Three of the main topics of high spin states have been the study of decoupled bands, the study of the distortion of normal rotational bands, usually called backbending and the study of very high spin states using continuum gamma rays and calculations.

### 5.1. Decoupled Bands

Many interesting examples of decoupled bands have been contributed to this conference. Experimentally decoupled bands in odd $A$ nuclei are characterized by the coupling of the last odd nucleon to the "band members" of the neighboring even-even cores giving a single maximum spin for each member of the core. The explanation in terms of rotation-aligned bands of Stephens and Simon\[^{33}\] has been quite successful and this success has led to extensions for triaxial nuclei\[^{34}\] and successful weak coupling models\[^{35}\]. It is clear that this area of research will continue to be fruitful as we try to learn the relationships between the bands and the cores on which they are built. Such an understanding should prove particularly valuable in the cases of coexisting decoupled bands of different deformation in the same nucleus. Here particles and holes in different cores probably come into play.

### 5.2. Backbending Bands

Since the discovery by Johnson and the Stockholm group\[^{36}\] of the anomaly in energy structure of rotational bands in the vicinity of $12^+$ to $16^+$ in the rare earth nuclei a considerable effort by Lieder and the KFA group, Berkeley groups, the Michigan State group, John Rasmussen and a Yale group, a Brookhaven group, and many other groups, has
uncovered many other examples of backbending and extended the observation of the yrast bands of even nuclei to spins of 22 or 24. Interestingly enough, it now appears that, despite considerable effort, it will be very difficult with in-beam methods to go above these spins.

Comparison of two different methods of plotting these yrast bands is shown in Fig. 15. In the lower half of the figure the usual method is presented where energy is plotted vs \( I(I+1) \). In the upper half of the figure the corresponding plot of \( \frac{2\hbar^2}{I} \) vs \( (\hbar \omega)^2 \) is presented.

It should be completely clear from Fig. 15 that any band which has higher moment of inertia than the ground state band and which occurs low enough in the spectrum will ultimately overtake the ground state band and cause backbending. The degree of mixing as shown in Fig. 15 is the determining factor in whether the anomaly in rotational spacings will actually backbend or not.

A large number of bands which could produce backbending have been suggested by theorists. These include (1) Coriolis-anti-pairing (CAP) bands suggested by Mottelson and Valatin\(^2\) which considerably predated the observation of backbending, (2) decoupled bands of Stephens and Simon\(^3\), coexistent bands of higher deformation \( \phi \) and \( \gamma \) \( \approx 30^\circ \) of Mottelson\(^4\) and Volkov\(^5\). It now appears probable that at least in the rare earth region the major contribution to backbending comes from the decoupled bands of Stephens and Simon although it is quite possible that there is some contribution in the rare earth region for the rigid non superfluid bands of Mottelson and Valatin\(^6\).

5.3. Very High Spin States in Nuclei

The advent of relatively high energy heavy ion accelerators now makes it possible to find a compound nucleus resulting from fusion reactions. Although this makes the study of high spin states possible, the earliest reports on these experiments, as for example from the Berkeley group,\(^7\) suggest that only gross continuum gamma rays and their multiplicities are available in these experiments since it is impossible to resolve individual gamma-ray lines involving spins above \( \approx 24 \). Although this is still a very valuable kind of spectroscopy, it lacks the detail and therefore, much of the systemsatics of the gamma ray cascades with which we are all familiar.

It is, therefore, with considerable interest that we follow the suggestion of Bohr and Mottelson\(^8\) that yrast traps (isomeric states of very high spin) may exist. Recently, Fassbinder and the KFA group\(^9\) and S.G. Nilsson and the Lund group\(^4\) have studied the mechanism for the formation of these traps in greater detail. It should be noted that the Fassbinder calculation suggests that the existence of these yrast traps will be restricted to a few nuclei. This may explain why the Berkeley group has thus far been unable to observe them experimentally. It is, of course, completely possible that if they exist, one would expect to be able to observe the entire cascade originating from the yrast trap. It is an exciting possibility for all of us.

In the meantime, theorists have been busy calculating the shapes of very high spin states. Among the active groups is the Neergaard Fashkevich group\(^5\) at Dubna and the S.G. Nilsson group\(^4\) of Lund, both of whom use the combination of liquid drop and shell effects without pairing. An example of these calculations (from the S.G. Nilsson group) for a group of neutron deficient "rare earths" is presented in \( \gamma \)-\( \delta \) polar coordinate diagrams in Fig. 16. Microscopic calculations have been carried out by the Fassbinder group\(^7\) and Hartree Fock calculations by an MIT-Saclay collaboration\(^8\). The deformed energy surfaces are summarized in Fig. 16 for each spin by giving the minimum for that spin. In general, when one begins with a prolate nucleus in its \( 0^+ \) ground state, it increases or decreases its deformation with increasing spin moving approximately along the prolate axis, may have or may not move into the \( \gamma \) plane, in some cases to an oblate nucleus (as expected for the liquid drop) and at the
highest spins tends again to move toward a prolate structure prior to fission.

6. Conclusion

Recognizing that most nuclei lie well off the line of beta stability, it is clear that the spectroscopy of these nuclei has an illustrious future.

The road map of shell effects will probably not predict the future of our spectroscopy any more successfully than previous attempts. If it serves to stimulate experiment and theory, it will have been successful. One does not have to be a medium to predict that the spectroscopy and structure of high spin states will occupy an important place in our future.

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REFERENCES


45) R. Neergård and V.V. Pashkevich, to be published.


47) A. Faessler, R.H. Hilton and K.R. Sandhya Devi, to be published.