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MASS CORRECTIONS TO SCALING IN DEEP INELASTIC PROCESSESR. Barbieri, J. Ellis, M.K. Gaillard ^{*)} and G.G. Ross

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A B S T R A C T

We study subasymptotic hadron target and quark-parton mass corrections to scaling in deep inelastic scattering, ignoring interactions. The results can be summarized using a modified scaling variable common to parton, light-cone and short distance operator product expansion formalisms, but with model-dependent spectral conditions. The analysis is expected to break down near the kinematic boundaries because of the bound state nature of hadrons. Related effects probably also dominate mass corrections due to very light quarks, but the analysis should be applicable to the production of new heavy quarks in neutrino production. Experimental deviations from scaling in deep inelastic electroproduction do not seem to be describable in terms of mass corrections alone, suggesting that interaction effects may be important at large momentum transfers as suggested by the renormalization group.

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1. INTRODUCTION

Scaling in deep inelastic leptonproduction ¹⁾ or in e^+e^- annihilation is equivalent to the absence of scale parameters with the dimensions of masses, as in the quark-parton model ²⁾ for example. Hadronic matter abounds in mass scales, and these have important threshold effects at low energies. In renormalizable field theories there are also deviations from scaling at high energies due to the persistent effects of scales associated with the interactions and expressible in terms of the renormalization group ³⁾. Experimentally, scaling is not exact, and we want to know what mass scales are doing the dirty work. A possibly incomplete list of candidate masses includes

- i) the target hadron mass ^{*});
- ii) the masses of hadronic constituents, assumed to be quark-partons;
- iii) an interaction mass scale μ at which the strong interaction coupling constant has a fixed value, e.g., $g^2/4\pi = 1$ in coloured quark-vector gluon theory;
- iv) possible non-perturbative mass scales expressing the "size" of hadrons. Examples include the pion decay constant f_π , the Regge trajectory slope α' or the average transverse momentum $\langle p_T \rangle$ typical of hadronic processes.

In disentangling these effects we first note that the non-perturbative scales seem to be typically $O(m_\pi$ to 300 MeV), reflecting a hadron "size" of ~ 1 fermi. Thus they are probably less important at moderate energies than the scale-breaking effects of target nucleon and heavy (e.g., charm) quark masses. On the other hand, they may not be negligible compared with light (up, down or strange) quark masses. In order to see the possible effects of an interaction mass scale μ , it is therefore desirable ⁴⁾ to compute as completely as possible the effects of target and quark-parton masses, completely neglecting parton interactions. For consistency with approximate scaling it seems necessary that strong interactions be weak at short distances. One motivation for this is provided by asymptotically free gauge theories ³⁾, which suggest that the neglect of quark-parton interactions may be a fair approximation over a range in Q^2 ($\equiv -q^2$) from 1 to 10 GeV^2 , say. This would correspond to $\mu \ll 1$, so that interaction effects would be small and only become clear over a larger range of Q^2 .

^{*}) or produced particle mass in e^+e^- annihilation.

The above strategy for distinguishing mass scales by computing exactly the effects of hadron and quark masses was initiated by Georgi and Politzer ^{4),5)}. They found subasymptotic mass corrections could be summarized in a generalized scaling variable ξ which approaches $x \equiv Q^2/2\nu$ in the scaling limit. Their analysis was made using short distance operator product expansions in the deep Euclidean region. Their original ξ variable differed from that derivable from the parton model ⁶⁾ or light-cone expansions under equivalent assumptions on the neglect of interactions and has now been corrected. We show that the variable

$$\xi = \frac{Q^2 + m_f^2 - m_i^2 + \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_H^2 Q^2}}{2(\nu + \sqrt{\nu^2 + m_H^2 Q^2})} \quad (1.1)$$

where m_i , m_f and m_H are the initial quark, final quark, and hadron target masses, respectively, can be derived from any of the parton, light-cone, or short distance expansion techniques ^{*}). The only difference between the results is that the parton and light-cone techniques impose a spectral condition $\xi \geq (m_i/m_H)^2$ which is absent in the operator product expansion approach. This reflects the fact that the parton and light-cone models work with the absorptive part of the current correlation function, which necessarily contains physical quarks if interactions are neglected, whereas the operator product expansion works with the real part of the amplitude in the deep Euclidean region. Another peculiarity of the parton and light-cone models is that they require the structure functions to have the symmetry $F(\xi) = F(m_i^2/m_H^2 \xi)$. We also show in Appendix B how similar ideas can be applied to the production of hadrons through heavy quarks in e^+e^- annihilation.

Unfortunately, the variable (1.1) is of limited application. One would expect, on general grounds, that the bound state nature of hadrons would be important near the kinematic boundaries $\xi \sim 0$, $\xi \sim 1$. In the low ξ region, Regge trajectories which are analytic continuations of crossed-channel bound states are expected to dominate, and it has been shown ⁹⁾ that paradoxes arise in electroproduction from free field assumptions if the Regge intercept $\alpha(0) > -1$. On the other hand, a threshold fixed in ξ near 1 is, in general, unphysical in the missing (mass)² W^2 , usually being Q^2 dependent, and we show that in a coloured quark-gluon theory the neglect of interactions is a non-uniform approximation which breaks down as $\xi \rightarrow 1$. Also, as remarked above, we would expect other mass parameters to be as important as the masses of light quarks $\leq 0(300 \text{ MeV})$.

^{*} As far as we know, this variable was first written down by Frampton ⁶⁾, see also Ref. 7). Some of our results were already presented in Ref. 8). For related analyses see Ref. 9).

More problems arise if one applies renormalization group arguments: renormalizing on mass shell introduces effects due to the large infra-red coupling, and is therefore of limited use. If one renormalizes at a Euclidean mass scale μ one generally finds $O(\mu^2/Q^2)$ and/or $O(m_1^2/Q^2)$ terms arising from unknown operator matrix elements, and which are in fact necessary for maintaining gauge invariance in the case of electroproduction. Hence other m^2/Q^2 effects can only be believed if $m^2 \gg \mu^2 : g^2(\mu^2)/4\pi \ll 1$. For example, only if the strong coupling constant is small at some mass scale $\mu \ll m_H$ will target hadron mass corrections in (1.1) be believable. However (1.1) should be useful for the neutrino production of new heavy quarks, such as charm ^{10),11)}.

In an intermediate range of x (or ξ) applying the variable (1.1) is not obviously unjustified, and we compare it with data on electroproduction -- including target mass effects, but neglecting light quark masses -- to see if it reproduces the observed scaling violations. We find that it fails to reproduce the trends of scaling violation at large Q^2 in W_1^P , νW_2^P and the ratio W_1^N/W_1^P , and predicts too small a value for the quantity R^P . We conclude that interaction effects are not negligible for $Q^2 \gtrsim 10 \text{ GeV}^2$, and believe ^{7),12)} scaling deviations associated with an interaction scale μ as expressed via the renormalization group ³⁾ are appearing. However, it will be a long time before data confirm asymptotically free gauge theory or any other renormalizable field theory.

Section 2 of this paper discusses mass corrections in the free parton and light-cone approaches; Section 3 uses free field theory short distance operator product expansions working in momentum space. Section 4 discusses what free field theory results can be motivated from asymptotic freedom, while Section 5 discusses generally limitations on the free field analyses. Section 6 contains a phenomenological analysis of deep inelastic electroproduction. There are appendices discussing the infinite momentum frame interpretation of the scaling variable, and e^+e^- annihilation.

2. DEEP INELASTIC SCATTERING FROM THE LIGHT CONE AND THE PARTON MODEL

The usual frameworks for discussing scaling in deep inelastic lepton-hadron scattering include:

- i) the light-cone (LC) analysis of the singularities of current commutators ¹³⁾;
- ii) the quark parton model (PM) ^{2),14),15)};
- iii) short distance operator product expansions (OPE) ¹⁶⁾.

Given the approximate validity of Bjorken scaling ¹⁾, it is worth while to consider within the above frameworks possible corrections to the naive picture, in particular at moderate Q^2 , where mass effects are still expected to be important. In this Section we analyze these corrections to scaling in the approximation of negligible interactions. This is motivated naively by the feeling that the observed precocious scaling requires interaction effects to be already small for $Q^2 \sim 1$. A more sophisticated discussion is referred to the renormalization group analysis of Section 4.

In the light-cone analysis the starting point is the connection between the standard structure functions and the matrix element of the current commutator

$$\begin{aligned}
 W_{\mu\nu}(q, p) &\equiv \frac{1}{2\pi} \int d^4y \ e^{iqy} \langle p | [J_\mu(y), J_\nu^+(0)] | p \rangle \equiv \\
 &\equiv -g_{\mu\nu} W_1(\nu, q^2) + \frac{p_\mu p_\nu}{m_H^2} W_2(\nu, q^2) + \frac{i}{2} \epsilon_{\mu\nu\sigma\lambda} \frac{q_\sigma p_\lambda}{m_H^2} W_3(\nu, q^2) \\
 &\quad + \dots
 \end{aligned} \tag{2.1}$$

where

$$J_\mu(y) = : \bar{\psi}_f(y) (a \gamma_\mu + b \gamma_\mu \gamma_5) \psi_i(y) : \equiv : \bar{\psi}_f(y) \Gamma_\mu \psi_i(y) : \tag{2.2}$$

is a general vector current composed of two fermion fields, p is the momentum of the hadron target of mass m_H , q is the spacelike momentum transfer to the hadrons ($-q^2 \equiv Q^2$), $\nu = p \cdot q$, spin averages have been taken, and we have suppressed in the right-hand side terms proportional to q_μ and q_ν which give terms proportional to the lepton mass when contracted with the leptonic currents.

In the standard analysis one assumes canonical commutation relations of the fermion fields on the light cone. To incorporate the effect of masses we are going to assume here that even for subasymptotic distances the interactions between the quarks are negligible, so that the fields ψ_i , ψ_f can be regarded as free fields.

Free field commutation relations give the following expression for the current commutator

$$\begin{aligned}
 [J_\mu(y), J_\nu(0)] = & \\
 = -\partial_\alpha \Delta(y, m_\pi^2) : \bar{\psi}_f(y) \Gamma_\mu \gamma_\alpha \Gamma_\nu \psi_f(0) : & + \partial_\alpha \Delta(y, m_f^2) : \bar{\psi}_i(0) \Gamma_\nu \gamma_\alpha \Gamma_\mu \psi_i(y) : \\
 + i m_\pi \Delta(y, m_\pi^2) : \bar{\psi}_f(y) \Gamma_\mu \Gamma_\nu \psi_f(0) : & + i m_f \Delta(y, m_f^2) : \bar{\psi}_i(0) \Gamma_\nu \Gamma_\mu \psi_i(y) :
 \end{aligned}
 \tag{2.3}$$

where

$$\Delta(y, m^2) = \frac{-i}{(2\pi)^3} \int d^4k e^{-iky} \epsilon(k_0) \delta(k^2 - m^2)
 \tag{2.4}$$

The right-hand side of Eq. (2.3) corresponds to the handbag diagram of Fig. 1b. The purely disconnected piece of Fig. 1a does not contribute to deep inelastic scattering, while the cat's ears diagram of Fig. 1c does not contribute to the current commutator if quark-parton interactions are neglected.

By inserting Eq. (2.3) in the left-hand side of Eq. (2.1), after a straightforward reduction of the Dirac matrices, one easily expresses the matrix element of the current commutator, and therefore the structure functions themselves, in terms of the following matrix elements of bilocal operators

$$\begin{aligned}
 A_{i,f}^{\mu\nu}(p,y) &= \langle p | \{ : \bar{\psi}_{i,f}(y) \gamma^\mu \gamma^\nu \psi_{i,f}(0) : + : \bar{\psi}_{i,f}(0) \gamma^\nu \gamma^\mu \psi_{i,f}(y) : \} | p \rangle \\
 A_{i,f}^\mu(p,y) &= \langle p | \{ : \bar{\psi}_{i,f}(y) \gamma^\mu \psi_{i,f}(0) : - : \bar{\psi}_{i,f}(0) \gamma^\mu \psi_{i,f}(y) : \} | p \rangle \\
 A_{i,f}(p,y) &= \langle p | \{ : \bar{\psi}_{i,f}(y) \psi_{i,f}(0) : + : \bar{\psi}_{i,f}(0) \psi_{i,f}(y) : \} | p \rangle
 \end{aligned}
 \tag{2.5}$$

A full exploitation of the relationships implied for these matrix elements by the free field equation obeyed by the quark fields allows us to represent them in terms of only two scalar functions of one variable $f_{i,f}(x)$ -- one for each fermion field -- as follows

$$\begin{aligned}
 A_{i,f}(p,y) &= -m_{i,f} \int d^4k e^{-iky} f_{i,f}\left(\frac{k \cdot p}{m_{\#}^2}\right) \delta(k^2 - m_{i,f}^2) \\
 A_{i,f}^{\mu}(p,y) &= \int d^4k e^{-iky} k_{\mu} f_{i,f}\left(\frac{k \cdot p}{m_{\#}^2}\right) \delta(k^2 - m_{i,f}^2) \\
 A_{i,f}^{\mu\nu}(p,y) &= -m_{i,f} g^{\mu\nu} \int d^4k e^{-iky} f_{i,f}\left(\frac{k \cdot p}{m_{\#}^2}\right) \delta(k^2 - m_{i,f}^2)
 \end{aligned}
 \tag{2.6}$$

Notice the δ functions in these representations which keep on their mass shells the momenta carried by the quark field, and which are a direct consequence of the free field equations for ψ_f and ψ_i . Combining Eqs. (2.1), (2.3), (2.4), (2.5) and (2.6), we find

$$\begin{aligned}
 W_{\mu\nu}(p,q) &\propto \int d^4k f_i\left(\frac{k \cdot p}{m_{\#}^2}\right) \delta(k^2 - m_i^2) \delta[(k+q)^2 - m_f^2] \epsilon(k_0 + q_0) \cdot \\
 &\cdot \left\{ (a^2 + b^2) \left[q_{\mu} k_{\nu} + q_{\nu} k_{\mu} - 2k_{\mu} k_{\nu} + \frac{g_{\mu\nu}}{2} (Q^2 + m_f^2 + m_i^2) \right] \right. \\
 &\left. + 2iab \epsilon_{\mu\nu\lambda\rho} q_{\lambda} k_{\rho} - (a^2 - b^2) g_{\mu\nu} m_i m_f \right\} + (i \rightleftharpoons f)
 \end{aligned}
 \tag{2.7}$$

This equation (2.7) is just what we would have written down in a covariant formulation ¹⁵⁾ of the naive parton model ^{2),14)}, with both initial and final quark partons essentially free (and therefore on mass shell) -- as in a very simple-minded impulse approximation. Thus the light cone, under the additional assumptions stated above, and a naive parton formulation lead to the same effective result and are therefore equivalent. Notice the explicit gauge invariance of Eq. (2.7) for $b=0$ and $m_i = m_f$. In this free quark model, gauge invariance is ensured by treating on the same footing the initial and the struck quark ^{*)}.

Out of Eq. (2.7) the calculation of the various structure functions, in terms of the function $f_{i,f}(x)$, is straightforward. Disregarding for a moment the Lorentz structure, the typical expression for a corresponding scalar structure function is

*) See Section 4 for a discussion on this point.

$$W(\nu, Q^2) = \frac{1}{\sqrt{\nu^2 + m_H^2} Q^2} \left[F_i\left(\frac{E_+}{m_H}\right) + F_i\left(-\frac{E_-}{m_H}\right) + (i \neq f) \right] \quad (2.8)$$

where $F_i(z) \equiv \pi \int_{m_H z}^{\infty} dk^0 f_i(k^0/m_H)$, with k^0 the parton energy in the target rest frame. The limits $E_{\pm}$ are specified by the on-mass-shell conditions in Eq. (2.7); they are

$$E_{\pm} = \frac{1}{2Q^2 m_H} \left[-\nu(Q^2 + m_f^2 - m_i^2) \pm \sqrt{\nu^2 + Q^2 m_H^2} \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_i^2 Q^2} \right] \quad (2.9)$$

Figure 2 illustrates the ranges of $\pm E_{\pm}$ for different values of ν between $Q^2/2$ and ∞ . One sees that $-E_-$ varies strictly between the kinematic limits indicated, whereas E_+ can become as small as m_i when $\nu = m_H(Q^2 + m_f^2 - m_i^2)/2m_i$. The physical spectral condition $\nu \geq Q^2/2$ requires $F_i(E/m_H) = 0$ for $E \geq (m_H^2 + m_i^2)/2m_H$; this boundary is also indicated in the figure. This condition corresponds to the recoil momentum being timelike: $(p - k)^2 \geq 0$.

We see that for Q^2 greater than some limit value $Q_0^2 (= m_H^2 - m^2)^2 / 2(m_H^2 + m^2)$ for $m_i = m_f = m$ the $F_i(-E_-/m_H)$ term in Eq. (2.8) is zero. This equation can then be written as

$$W(\nu, Q^2) = \frac{1}{\sqrt{\nu^2 + Q^2 m_H^2}} F_i\left(\frac{1}{2}\left(\xi + \frac{m_i^2}{m_H^2 \xi}\right)\right) + (i \neq f) \equiv \frac{1}{\sqrt{\nu^2 + Q^2 m_H^2}} f_i(\xi) + (i \neq f) \quad (2.10)$$

where

$$\xi = \frac{Q^2 + m_f^2 - m_i^2 + \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_i^2 Q^2}}{2(\nu + \sqrt{\nu^2 + Q^2 m_H^2})} \quad (2.11)$$

We observe that $\xi \rightarrow x \equiv Q^2/2\nu$ as $Q^2 \rightarrow \infty$ ^{*)}. The physical spectral condition $E_+ \leq (m_H^2 + m_i^2)/2m_H$ requires $(m_i/m_H)^2 \leq \xi \leq 1$, whereas the condition $E_+ \geq m_i$ does not impose any constraint on ξ ^{**)} .

*) The variable ξ can in fact be interpreted as the fraction of longitudinal momentum carried by the parton in a target infinite momentum frame ^{7), 14)}, and $\sigma_i(\xi)$ is related to the probability of finding a quark with momentum fraction ξ as shown explicitly in Appendix A.

**) In a previous paper ⁸⁾ we had the constraint $\xi > m_i/m_H$. This is because we were sticking to the definition $\xi = E_+ + \sqrt{E_+^2 - m_i^2}$, which is actually not necessary.

nucleon states gives the forward Compton amplitude

$$T(\nu, Q^2) \equiv i \int d^4x e^{iqx} \langle p | T(J(x)J(0)) | p \rangle \quad (3.2)$$

Inserting (3.1) into (3.2) we find matrix elements of the traceless operators which may be written as

$$\langle p | O_{\mu_1 \dots \mu_n}^{(\alpha)} | p \rangle \equiv \langle p | O_n^{(\alpha)} | p \rangle \{ p_{\mu_1} \dots p_{\mu_n} - \text{trace terms} \} \quad (3.3)$$

The evaluation of (3.2), without neglecting any of the trace terms in (3.3), is complicated, but has been done by Nachtmann ⁷⁾. He finds

$$T(\nu, Q^2) = \sum_{n \text{ even}} C_n^1 \left(\frac{i\nu}{m_H Q} \right) \sin \frac{\pi}{2} (n+1) \left(\frac{Q}{m_H} \right)^{-n} \mu_n(Q^2) \quad (3.4)$$

where

$$\mu_n(Q^2) = \frac{i Q^{2n}}{\sin \frac{\pi}{2} (n+1)} \cdot \left(\frac{\partial}{\partial Q^2} \right)^n \int d^4x e^{iqx} \sum_{\alpha} f_n^{(\alpha)}(-x^2 + i\epsilon) \langle p | O_n^{(\alpha)} | p \rangle \quad (3.5)$$

and the $C_n^1(\eta)$ are Gegenbauer polynomials.

The deep inelastic structure function $W(\nu, Q^2)$ is given by

$$W(\nu, Q^2) = \frac{1}{2\pi} \text{Im} T(\nu, Q^2) \quad (3.6)$$

and the moments $\mu_n(Q^2)$ are related to the structure functions by the formulae

$$\mu_n(Q^2) = 8 \int_{\frac{Q^2}{2}}^{\infty} d\nu \frac{W(\nu, Q^2)}{Q^2} \left[\frac{Q^2}{\nu + \sqrt{\nu^2 + m_H^2 Q^2}} \right]^{n+1} \quad (3.7)$$

In the limit $m_H \rightarrow 0$, Eq. (3.7) reduces to the more familiar sum rules

$$\mu_n(Q^2) = 8 \int_{Q^2/2}^{\infty} d\nu \frac{W(\nu, Q^2)}{Q^2} x^{n+1} \quad (3.8)$$

where $x \equiv Q^2/2v$. Equations (3.4) and (3.7) give the target mass corrections necessary to project on to the moments corresponding to traceless operators when m_H^2/Q^2 is not negligible.

In our free field scalar boson example the traceless operators $O_{\mu_1 \dots \mu_n}$ will be constructed from the operators

$$\Theta_{\mu_1 \dots \mu_n} \equiv \phi_i(0) \overset{\leftrightarrow}{\partial}_{\mu_1} \dots \overset{\leftrightarrow}{\partial}_{\mu_n} \phi_i(0) \quad (3.9)$$

and their traces. The leading contributions to the $\mu_n(Q^2)$ will come from the $\Theta_{\mu_1 \dots \mu_n}$ themselves, and contributions $O(m_i^2/Q^2)$ will arise from the trace terms. They may then be calculable in free field theory using

$$\langle p | \text{Tr } \Theta^n | p \rangle = -m_i^2 \langle p | \Theta^{n-2} | p \rangle \quad (3.10)$$

in an obvious shorthand notation. However, the reader (and an author) may believe applying (3.10) to hadron targets involves making assumptions about relations between operator matrix elements which should not be abstracted from free field theory (see also Section 4). An exhausting discussion of the decomposition into traceless operators and the resulting inclusion of initial quark-mass effects for the spin $\frac{1}{2}$ case has been given by Georgi and Politzer^{4),5),*)}. The effects of the final quark mass arise from a factor of the scalar boson propagator $\Delta_F(x, m_F^2)$ in the c number singular functions $f_n^{(q)}(-x^2 + i\epsilon)$ in Eqs. (3.1) and (3.5). Fourier transforming the x space expressions via (3.5) yields the dependences of $\mu_n(Q^2)$ on m_i and m_F .

An equivalent, but more direct, method of extracting the $\mu_n(Q^2)$ comes from applying the Nachtmann⁷⁾ projection (3.7) to the free field theory structure function

$$W(\nu, Q^2) = \delta(2\nu - Q^2 - m_F^2 + m_i^2) \quad (3.11)$$

which yields

$$\mu_n^{FF}(Q^2) = \frac{8}{Q^2} \left[\frac{Q^2}{Q^2 + \sqrt{Q'^2 + 4m_i^2 Q^2}} \right]^{n+1} \quad (3.12)$$

*) See in Ref. 5) Eqs. (4.7), (4.8) and (6.5) to (6.10).

where

$$Q'^2 = Q^2 + m_f^2 - m_i^2 \quad (3.13)$$

Equation (3.12) is the scalar analogue of Eq. (6.9) in the analysis of Georgi and Politzer^{4),5)}. Note that the absorptive part (3.11) of the quark amplitude is only used to abstract the Q^2 dependence of the moments in the deep Euclidean region. To get to the moments $\mu_n(Q^2)$ for a hadron target neglecting interactions, we need only multiply (3.12) by the reduced matrix element of the traceless operator O_n between hadron states:

$$\mu_n(Q^2) = \mu_n^{FF}(Q^2) \langle p || O_n || p \rangle \quad (3.14)$$

Equations (3.12), (3.14) and (3.4) can then be used to recover the full amplitude $T(v, Q^2)$ and the structure function $W(v, Q^2)$. We represent the Gegenbauer polynomials in the form

$$C_n^1(\eta) = \frac{1}{2\pi i} \oint_C \frac{z^{-n-1} dz}{(1-2\eta z + z^2)} \quad (3.15)$$

where C is a contour enclosing the origin. Then from Eqs. (3.4) and (3.15) we have

$$T(v, Q^2) = \frac{1}{2\pi i} \oint_C dz \sum_{n \text{ even}} \frac{\mu_n(Q^2) \left(\frac{-iQz}{m_H} \right)^{-n}}{z \left(1 - 2 \left(\frac{iv}{Qm_H} \right) z + z^2 \right)} \quad (3.16)$$

By deforming the contour to enclose the poles arising from the factor $[1 - 2(iv/Qm_H)z + z^2]$ we find

$$T(v, Q^2) = \frac{Q^2}{\sqrt{v^2 + m_H^2} Q^2} \left\{ \sum_n \mu_n(Q^2) \left[(-z_0)^{-n-1} + (-z_1)^{-n-1} \right] \right\} \quad (3.17)$$

where

$$\begin{aligned} z_0 &= \frac{1}{m_H^2} \left[-v + \sqrt{v^2 + m_H^2} Q^2 \right] \\ z_1 &= \frac{1}{m_H^2} \left[v + \sqrt{v^2 + m_H^2} Q^2 \right] \end{aligned} \quad (3.18)$$

Using the form for $\mu_n(Q^2)$ given by (3.12) and (3.14) we see that

$$W(\nu, Q^2) = \frac{1}{\sqrt{\nu^2 + m_\pi^2 Q^2}} F(\xi) \quad (3.19)$$

where ξ is given by Eq. (1.1). The second term in (3.17) vanishes because of the physical spectral condition, as we discussed in Section 2.

The analysis of spin $\frac{1}{2}$ partons proceeds as for the spin 0 case. But there are two series of operators which must now be considered

$$\begin{aligned} \theta_1^n &\equiv \bar{\Psi} \gamma_{\mu_1} \overset{\leftrightarrow}{\partial}_{\mu_2} \dots \overset{\leftrightarrow}{\partial}_{\mu_n} \Psi \\ \theta_2^n &\equiv \bar{\Psi} \overset{\leftrightarrow}{\partial}_{\mu_1} \dots \overset{\leftrightarrow}{\partial}_{\mu_n} \Psi \end{aligned} \quad (3.20)$$

Each moment $\mu_n(Q^2)$ will get contributions from both series of operators. In general their matrix elements are unrelated, each $\mu_n(Q^2)$ having an unknown Q^2 dependence, and no simple scaling variable will exist. But in free field theory

$$\langle p | \theta_2^n | p \rangle = m_\pi \langle p | \theta_1^n | p \rangle \quad (3.21)$$

and the operator matrix elements are related ^{*)}. The reader (and an author) may not believe this relation should be abstracted from free field theory. If we use (3.21), we then find for electroproduction with spin $\frac{1}{2}$ partons that Eq. (3.7) is replaced by the two moment equations ⁷⁾

$$\begin{aligned} \mu_{1,n}(Q^2) &= \int d\nu \frac{\nu^2}{Q^2} \left\{ 3W_1(\nu, Q^2) - \frac{(\nu^2 + m_\pi^2 Q^2)}{m_\pi^2 Q^2} W_2(\nu, Q^2) \right\} \\ &\cdot \left[\frac{Q^2}{\nu + \sqrt{\nu^2 + m_\pi^2 Q^2}} \right]^{n+1} \end{aligned} \quad (3.22)$$

and

*) Recall that the free field equations of motion applied to the bilocal operators yielded a similar uniqueness in the LC approach.

$$\mu_{2,n}(Q^2) = \int d\nu \frac{2}{Q^2} \left\{ -W_1(\nu, Q^2) + \frac{\nu^2 + m_H^2 Q^2}{m_H^2 Q^2} W_2(\nu, Q^2) \times \right. \\ \left. \times \left[1 + \frac{2(n+1)\nu(\sqrt{\nu^2 + m_H^2 Q^2} - \nu) - (n+2)Q^2 m_H^2}{(n+2)(n+3)(\nu^2 + m_H^2 Q^2)} \right] \right\} \left[\frac{Q^2}{\nu + \sqrt{\nu^2 + m_H^2 Q^2}} \right]^{n+1} \quad (3.23)$$

To extract the $\mu_{i,n}^{\text{FF}}(Q^2)$ corresponding to (3.12), we insert the free field structure functions

$$W_1 = \nu \delta(2\nu - Q^2) \\ W_2 = 2m_H^2 \delta(2\nu - Q^2) \quad (3.24)$$

into Eqs. (3.22) and (3.23). As in the spin zero case, the $\mu_{i,n}(Q^2)$ are, neglecting interactions,

$$\mu_{i,n}(Q^2) = \mu_{i,n}^{\text{FF}}(Q^2) \langle \phi | O_n | \phi \rangle \quad (3.25)$$

The resummation of the series (3.22) is identical to that of the scalar series (3.16), except that the left-hand side yields

$$3W_1(\nu, Q^2) - \frac{\nu^2 + m_H^2 Q^2}{m_H^2 Q^2} W_2(\nu, Q^2)$$

instead of $W(\nu, Q^2)$ in the analogue of Eq. (3.19). The resummation of the series (3.23) is more complicated because of the n dependent terms. We do not give the details here as they are equivalent to the analysis of Georgi and Politzer ^{4),5)}. The result of the resummation is the same as obtained in the LC and PM approaches Eqs. (2.12). But, as emphasized above, we do not get the parton model spectral condition $(m_1/m_\mu)^2 \leq \xi \leq 1$. This is because the moments $\mu_{i,n}(Q^2)$ have no memory of the parton model spectral condition, which is a restriction on the operator matrix elements $\langle p | |O_n| | p \rangle$ which need not be respected in the general OPE framework. The use of the partonian absorptive parts (3.24) was only a matter of calculational convenience for extracting the Q^2 dependence of the moments in the deep Euclidean region. Unlike the LC and PM analysis of Section 2, the

OPE does not have unphysical features of the quark-parton model's absorptive part -- i.e., physical quark partons, the relations (2.10) they imply between values of the structure functions at different values of ξ , and the spectral condition $\xi \geq (m_i/m_H)^2$.

4. THE RENORMALIZATION GROUP AND ASYMPTOTIC FREEDOM

In the previous Sections we used free field theory models for the structure of the operator product expansion of two currents, and the resulting form of the deep inelastic structure functions. The basic idea was that there may be a regime of Q^2 where the effective strong interaction coupling constant $\bar{g}(Q^2)$ is small, while $O(m^2/Q^2)$ effects are still substantial. We now discuss the justification for these analyses in the framework of an asymptotically free colour gauge theory³⁾.

We must first specify a renormalization prescription. One possibility is to renormalize Green's functions and operators on mass shell. This will yield Callan-Symanzik equations¹⁸⁾, where mass terms enter via inhomogeneous terms which should vanish asymptotically in the deep Euclidean region by Weinberg's theorem¹⁹⁾. Unfortunately the operator product expansion will then involve the strong coupling constant evaluated on mass shell where infra-red slavery presumably makes it non-negligible. We therefore renormalize at spacelike momentum: $p^2 = -\mu^2$. To define quark masses we follow Georgi and Politzer^{4),5)} and specify $m_q(\mu)$ by the identification

$$\bar{S}^{-1}(p) \Big|_{p^2 = -\mu^2} = \not{p} - m_q(\mu) \quad (4.1)$$

of the inverse propagator at the renormalization momentum. The resulting renormalization group equations for Green's functions in the Landau gauge are

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g, \frac{m}{\mu}) \frac{\partial}{\partial g} + \gamma_m(g, \frac{m}{\mu}) m \frac{\partial}{\partial m} + \sum_n \gamma(g, \frac{m}{\mu}) \right] \Gamma^n = 0 \quad (4.2)$$

The corresponding equations for the moments $\mu_n^{(\alpha)}(Q^2)$ corresponding to operators $O_n^{(\alpha)}$ of definite spin n are:

$$\left[\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \gamma_m m \frac{\partial}{\partial m} \right) \delta_{\alpha\alpha'} + \gamma_{\alpha\alpha'}^n \right] \mu_n^{(\alpha')} = 0 \quad (4.3)$$

where $-\gamma_{\alpha, \alpha'}^n$ is given by the transpose of the matrix of anomalous dimensions of the set of operators of spin n . These equations have the familiar solution

$$\mu_n^{(\alpha)}(Q^2, g(\mu), m(\mu)) = - \sum_{\alpha'} \mu_n^{(\alpha')} \left(\frac{Q^2}{\mu^2}, \bar{g}(Q), \bar{m}(Q) \right) \exp \left\{ T \int_{\mu}^Q \gamma^n(\bar{g}(\mu'), \frac{\bar{m}(\mu')}{\mu'}) \frac{d\mu'}{\mu'} \right\} \quad (4.4)$$

where T denotes that the matrices γ^n are ordered in μ' in the integral. In AFGT $\bar{g}(Q) \rightarrow 0$ as $Q \rightarrow \infty$ and we believe that for large enough Q^2 $\mu_n^{(\alpha)}[Q^2/\mu^2, \bar{g}(Q), \bar{m}(Q)]$ may be reliably estimated by perturbation series expansion in $\bar{g}(Q)$. Of course, as $Q \rightarrow \infty$ $\bar{m}(Q)$ contributes to $\mu_n^{(\alpha)}$ in $O[\bar{m}^2(Q)/Q^2]$ which also $\rightarrow 0$. The analyses of the previous Sections assumed there was some range of Q^2 where $\bar{m}^2(Q)/Q^2$ was $O(1)$ while $\bar{g}(Q)$ was small, so that

$$\mu_n^{(\alpha)} \left(\frac{Q^2}{\mu^2}, \bar{g}(Q), \bar{m}(Q) \right) \simeq \mu_n^{(\alpha)} \left(\frac{Q^2}{\mu^2}, 0, \bar{m}(Q) \right) \quad (4.5)$$

It was also assumed that the logarithmic scaling violations generated by the exponential in (4.4) are negligible over the range of Q^2 where $\bar{m}^2(Q)/Q^2$ effects are appreciable.

There are other places where the interactions affect the discussion of the OPE as given in Section 3. First, we observe that the operators $\Theta_{1,2}^n$ must be made gauge invariant by replacing

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ig T^a A_\mu^a \quad (4.6)$$

where T^a is a colour group representation matrix and A_μ^a : $a = 1, \dots, 8$ are the gluons. Then since the operators are normalized at $p^2 = -\mu^2$, the mass terms on the right-hand side of the spin $\frac{1}{2}$ analogues of the trace Eqs. (3.10) get altered: $m_i^2 \rightarrow -\mu^2$. Also the interactions generate new terms in the trace equations: for example

$$\langle p | \text{Tr } \Theta_2^n | p \rangle = \mu^2 \langle p | \Theta_2^{n-2} | p \rangle + (n-2) g(\mu) \langle p | \Phi^{n-2} | p \rangle \quad (4.7)$$

where

$$\begin{aligned} \theta_2^n &= S \bar{\psi} \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n} \psi \\ \text{and} \quad \phi_2^n &= S \bar{\psi} T^b \overleftrightarrow{D}_\nu \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_{n-1}} F_{\mu_n}^{\nu b} \psi \end{aligned} \quad (4.8)$$

is a non-leading twist operator, and S denotes symmetrization in the indices $\mu_1 \dots \mu_n$ and division by $n!$. Equation (4.7) means that the $O(m_i^2/Q^2)$ effects in (2.12) generated by the construction of traceless operators described in Section 3 are replaced by $O(\mu^2/Q^2)$ effects. On the other hand, the m_i effects from the free field Eqs. (3.21), and the m_F effects from the propagator in Fig. 1b, remain because of the renormalization prescription (4.1). Explicit gauge invariance will then be violated to the extent that $-\mu^2 \neq m_i^2$ ($m_i^2 = m_F^2$ for electroproduction), but this is not unreasonable as the hadronic matrix elements are unknown to the same extent, even if the hadronic wave function is constructed from effectively free quarks. The analysis of the previous Sections also requires the Φ^n (and other operator towers) contributions, as seen in (4.7), to be small. We must therefore choose μ so that $g(\mu)$ is not too big and the matrix elements $\langle p | \Phi^n | p \rangle$ are not badly normalized (do not blow up). Since the Φ^n are normalized at $p^2 = -\mu^2$, the $\langle p | \Phi^n | p \rangle$ will probably be well behaved if the average quark momenta in the proton are $O(\mu)$. Thus $\mu = O(\text{typical hadronic size}) = O(\langle p_T \rangle) = O(300 \text{ MeV})$ might be preferred as a renormalization point. But it is then not obvious that $g(\mu)$ is small: probably it is not.

Therefore, because of the terms on the right-hand side of the trace Eq. (4.7), the results (2.12) will only be true to $O(\mu^2/Q^2)$ and the m_i^2/Q^2 pieces are generally unreliable. Also, we will want to choose μ as small as possible so that $g(\mu)$ is still negligible. Even then we are reduced to piously hoping that the matrix elements $\langle p | \Phi^{n-2} | p \rangle$, etc., are not too large. If $g(\mu)$ is small, then $\bar{m}(Q)$ can be replaced by $m(\mu)$ in Eq. (4.5) for $\mu_n^{(\alpha)}(Q^2)$. Only the effects of masses $m \gg \mu$: $g(\mu)$ is small can be believed in Eqs. (2.12). Thus the m_F^2/Q^2 effects may be believable for heavy quarks¹¹⁾, as in charm production in neutrino interactions¹⁰⁾. Whether the $O(m_H^2/Q^2)$ effects are believable, e.g., for a nucleon target, depends crucially on the value of $g(\mu \sim 1 \text{ GeV})$. But independently of the approximation of neglecting interactions or the formulae (2.12), the moments (3.22) and (3.23) are the theoretically correct way of extracting the Q^2 dependence of specific spins, and would be ideal quantities to compute from the experimental data.

5. LIMITATIONS OF THE ANALYSIS

In the previous Sections, while deriving the wholly scaling variable formulae (2.12), we have emphasized some of the limitations on their applicability. In this Section we will try to discuss them all systematically: for another discussion, see Ref. 20).

5.1 $\xi \rightarrow 1$

This is the threshold region where you might naively expect direct channel bound state ²¹⁾ effects to be important (see Fig. 3a). They are indeed reflected in our analysis in several ways. First we noticed a negative parton energy contribution $F_i(-E_-)$ to the structure function in (2.8). If one seeks to apply (2.12) out to $\xi = 1$, then ν must be restricted to $\gtrsim m_H^2/2$. More importantly, the formulae (2.12) only admits a threshold fixed in ξ , which means a threshold dependent on Q^2 in general and usually unphysical. The physical condition $Q^2 \leq 2\nu$ corresponds to

$$\xi \leq \frac{Q^2 + m_F^2 - m_i^2 + \sqrt{(Q^2 + m_F^2 - m_i^2)^2 + 4m_i^2 Q^2}}{Q^2 + \sqrt{Q^4 + 4m_H^2 Q^2}}$$

and in the case $m_i = m_F = 0$ the physical asymptotic spectral condition $\xi \leq 1$ corresponds to

$$W^2 \equiv (p+q)^2 = m_H^2 + 2\nu - Q^2 \geq 0$$

for all Q^2 ^{*)}. Furthermore, the Adler sum rule

$$A = \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \nu W_2 \quad (5.1)$$

should be a constant independent of Q^2 since it is derivable from a current algebra equal time commutator. In quark-parton language it just counts the number of quarks in the target. We have verified that A is a constant only if the threshold ν_0 in (5.1) is taken to be fixed in ξ (and so varying with Q^2), and that then it indeed counts the number of quarks given by the density $f_i(k \cdot p/m_H^2)$ in Eq. (2.7) (see Appendix A).

We expect these threshold anomalies to be cured by interactions, and asymptotically free gauge theory ³⁾ indicates at least two ways in which the neglect of interactions becomes a worse approximation as $x(\xi) \rightarrow 1$. First it is known ^{3), 17), 22)} that the structure function at a given x is

*) These problems are removed by Derman's ¹⁰⁾ choice of variable which has a less direct field theoretical significance than does ξ .

dominated by moments $\mu_n(Q^2)$ of order $n \sim (4G \ln \alpha_s / \ln x)$, where $\alpha_s = g^2/4\pi$ and G is determined by the colour group and quark representations. It can be shown that interaction corrections go like $[\alpha_s (\ln n)^3]^i$ $i=1,2,3 \dots$, and so are important unless

$$\left(\alpha \ln \left(\frac{4G \ln x}{\ln x} \right) \right)^3 \ll 1 \quad (5.2)$$

which excludes the region $x(\xi) \rightarrow 1$. There are also the $g(\mu) \xi^{n-2}$ interaction corrections in the trace relation (4.7) $\alpha(n-2)$, which indicate that the perturbation series in $\bar{g}(Q)$ is only reasonable for $\alpha_s(n-2)^2 \ll 1$. We, in fact, find that for a structure function $\alpha(1-\xi)^a$ the corrections from this source are $(1-\xi)^a \times O[\alpha_s/(1-\xi)]$ as $\xi \rightarrow 1$. These are obviously not negligible as $\xi \rightarrow 1$, and not obviously negligible anywhere. The problem is more acute for higher twist operators contributing in higher order in $(1/Q^2)$.

5.2 $\xi \rightarrow 0$

In this region, usually expected to be related to Regge behaviour, you might naively expect crossed channel bound state effects to be important (see Fig. 3b). We saw this directly in the LC and PM approaches where the formulae (2.12) were restricted to $\xi \geq (m_i/m_H)^2$ because the quark-parton being struck was real. The OPE approach allowed $\xi < (m_i/m_H)^2$, but in a picture based on physical partons this possibility requires interaction effects. In Appendix A of his paper, Broadhurst⁹⁾ has used equivalent free field assumptions to ours to show that if

$$W_L(\nu, Q^2) \equiv \left(1 + \frac{\nu^2}{Q^2 m_H^2} \right) W_2(\nu, Q^2) - W_1(\nu, Q^2) \quad (5.3)$$

and if $\lim_{\nu \rightarrow \infty} W_2(\nu, Q^2) \propto \nu^{\alpha-2}$ in electroproduction, then

$$R(\nu, Q^2) \equiv \frac{W_L(\nu, Q^2)}{W_1(\nu, Q^2)} = \frac{\sigma_L(\nu, Q^2)}{\sigma_T(\nu, Q^2)}$$

has the limiting behaviour

$$\lim_{Q^2 \rightarrow 0} \lim_{\nu \rightarrow \infty} R(\nu, Q^2) = -(\alpha+1) \quad (5.4)$$

which is absurd if $\alpha > -1$ since R must be ≥ 0 . Thus using free field assumptions near $v \rightarrow \infty$ ($\xi \rightarrow 0$) is inconsistent with the structure functions having Regge behaviour with any intercept $\alpha > -1$ *).

In AFGT it is well known ²³⁾ that the perturbation expansion of $\mu_n(Q^2)$ in α_s breaks down in the Regge limit. For SU(flavour) singlet structure functions the $O(\alpha_s)$ contributions to the anomalous dimensions of operators of spin n have a pole at $n = -1$ and it is found that interaction terms of order α_s and above also have singularities at $n = -1$ proportional to

$$\left(\frac{\alpha_s}{n+1} \right)^i, \quad i = 1, 2, \dots$$

For small x the effective value of n is of order

$$n+1 = \sqrt{\frac{\ln \alpha_s}{|\ln x|}} \quad (5.5)$$

and so interaction corrections will be important unless

$$\left(\frac{|\ln x|}{9 \ln \left(\frac{1}{\alpha_s} \right)} \right)^{3/2} \ll 1 \quad (5.6)$$

The condition (5.6) excludes the Regge region $x(\xi) \rightarrow 0$: similar considerations apply to the non-singlet structure functions.

5.3 m_i effects

We saw in Section 2 that $\xi \geq (m_i/m_H)^2$ in the PM and LC approaches, implying that these analyses were totally inapplicable when $m_i > m_H$. Also we saw in Section 4 that in an asymptotically free gauge theory $O(m_i^2/Q^2)$ pieces in (2.12) are suspect, many being replaced by $O(\mu^2/Q^2)$ effects where presumably $\mu \gtrsim m_i$.

5.4 Small m_i, m_f

In all our analyses we have neglected other strong interaction mass scales, some probably non-perturbative like f_π, α' , and possibly $\langle p_T \rangle$. Effects due to these scales and the mass scale μ probably dominate mass corrections due to very light quarks such as up, down, and probably also strange quarks. For this reason details of the formulae (2.12) should not be taken very seriously in this case.

*) The paper of Dash ⁹⁾ implicitly contains a variable similar to ξ in the Regge limit. He did not encounter the problems found by Broadhurst because he did not use free field theory for the spin kinematics.

6. PHENOMENOLOGICAL APPLICATIONS

Having calculated hadronic and quark-parton mass corrections to scaling as best we can, we will now make some comparisons with the observed²⁴⁾⁻²⁷⁾ patterns of deviations from scaling in deep inelastic scattering. We will ignore other corrections to scaling, wanting to know whether the trivial mass factors of (2.12) can explain all or some of the scale breaking.

We discuss mainly electroproduction and restrict ourselves to moderate values of $x(\xi)$ because of the arguments of Section 5. We will mainly be concerned with data taken at SLAC, hence only need the formulae (2.12) when $m_i \simeq m_f \simeq 0$. We then have

$$\begin{aligned} W_1(\nu, Q^2) &= \frac{Q^2}{2\sqrt{\nu^2 + m_H^2} Q^2} [F(\xi) + G(\xi)] \\ W_2(\nu, Q^2) &= \frac{Q^4 m_H^2}{2(\nu^2 + m_H^2 Q^2)^{3/2}} [F(\xi) + 3G(\xi)] \end{aligned} \quad (6.1)$$

where

$$G(\xi) = \frac{m_H^2}{\sqrt{\nu^2 + m_H^2} Q^2} \int_{\xi}^1 d\xi' F(\xi') + \frac{m_H^4}{\nu^2 + m_H^2 Q^2} \int_{\xi}^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'') \quad (6.2)$$

and

$$\xi = \frac{Q^2}{\nu + \sqrt{\nu^2 + m_H^2} Q^2}$$

in this case. In the range $1/3 \leq x \leq 2/3$ where we may try to apply these formulae, the structure functions are generally expressed as polynomials in $(1-x)^n$. If we take $F(\xi) = (1-\xi)^n$ we get

$$\begin{aligned} W_1 &= \frac{Q^2 (1-\xi)^n}{2\sqrt{\nu^2 + m_H^2} Q^2} \left[1 + \frac{m_H^2}{\sqrt{\nu^2 + m_H^2} Q^2} \frac{1-\xi}{n+1} + \frac{m_H^4}{\nu^2 + m_H^2 Q^2} \frac{(1-\xi)^2}{(n+1)(n+2)} \right] \\ W_2 &= \frac{Q^4 m_H^2 (1-\xi)^n}{2(\nu^2 + m_H^2 Q^2)^{3/2}} \left[1 + \frac{3m_H^2}{\sqrt{\nu^2 + m_H^2} Q^2} \frac{1-\xi}{n+1} + \frac{3m_H^4}{\nu^2 + m_H^2 Q^2} \frac{(1-\xi)^2}{(n+1)(n+2)} \right] \end{aligned} \quad (6.3)$$

The most precise data in the suitable range of x comes from MIT-SLAC ²⁴⁾ and SLAC ²⁵⁾ groups. They report that their data are best fitted by $n \simeq 4$ in polynomial expansions with various different scaling variables. We have checked that $n=4$ in Eqs. (6.3) also fits best the x dependence of the data at $Q^2 \sim 6 \text{ GeV}^2$ for $1/2 \leq x \leq 2/3$. Figure 4a compares the formulae (6.3) with MIT-SLAC data ²⁴⁾ on W_1^P and $\sqrt{W_2^P}$ for $x=1/2, 2/3$ normalized to the values at $Q^2 = 6 \text{ GeV}^2$. We observe that the experimental trend of scale breaking is not well reproduced by the mass corrections to scaling in (6.3), particularly in the case of $\sqrt{W_2^P}$. In Fig. 4b we have compared SLAC ²⁵⁾ data on W_1^P for $0.5 < x < 0.7$ for $Q^2 \leq 24$ to the Q^2 dependence contained in (6.3) with an arbitrary normalization, and again we find that (6.3) does not fit the data well. In Fig. 4c we have compared the FNAL data of Anderson et al. ²⁶⁾ with (6.3) in the kinematic range $1/3 < x < 1$. On the scale the data is presented, the variation in the theoretical formula is negligible, whereas the experimental value of $\sqrt{W_2^D}$ falls by a factor ~ 2 .

Another application of the formula (6.3) is to measurements of

$$R \equiv \frac{W_2 \left(\frac{\nu^2}{Q^2 m_H^2} + 1 \right) - W_1}{W_1} \quad (6.4)$$

In Fig. 5 we have compared SLAC data on R^P with the formulae (6.3). In the relevant range $1/3 \leq x < 2/3$ the experimental ^{24), 25)} average value of R^P is $\sim (10 \text{ to } 15)\%$. The mass corrections to scaling just give $R^P \sim (1-3)\%$.

A final application of these results is to the ratio of neutron and proton structure functions. Since $m_p \simeq m_N$, the pattern of scale breaking for neutron and proton would be identical if $W^N(x)/W^P(x)$ were constant at any Q^2 . In fact, the N/P ratio varies between $1/2$ and $1/4$ in the x range measured, so that the neutron and proton structure functions have different functional forms, and a variation in the N/P ratio with Q^2 is, in principle, possible. The data ²⁴⁾ on W_1^N are reasonably described by a combination of $(1-x)^4$ and $(1-x)^5$. Even if the $(1-x)^5$ term were dominant, which it is not, the variation in the N/P ratio would be

$$\frac{W_1^N(Q^2=6)}{W_1^P(Q^2=6)} \bigg/ \frac{W_1^N(Q^2=20)}{W_1^P(Q^2=20)} = 1.04 \quad (6.5)$$

The experimental data ²⁴⁾ show a variation by $\sim 30\%$ in this ratio between $Q^2 = 6$ and 20 GeV^2 .

The conclusion of these phenomenological comparisons is that simple target and parton mass effects do not adequately describe scaling breakdowns in the electroproduction data. Since $\langle p_T \rangle^2 / Q^2 \simeq 1/100$ when $Q^2 \simeq 10$, we do not believe hadron "size" effects are likely to make up the discrepancies at larger Q^2 . We conclude that renormalization group effects are probably significant in the observed scaling deviations, and possibly dominant in the quantities R^P and W_1^N/W_1^P . These conclusions are not novel: Nachtmann ⁷⁾ has been saying it for two years, and Baluni and Eichten and Glück and Reya ¹²⁾ have recently reached similar conclusions.

In the case of neutrino-production of new heavy quarks from light initial quarks in a target of negligible mass, our formulae reduce to those of Barnett ¹⁰⁾:

$$\begin{aligned} W_1 &= 2\xi F(\xi) \\ \nu W_2 &= 4\xi^2 F(\xi) \\ \nu W_3 &= \pm 4\xi F(\xi) \end{aligned} \qquad \xi = \frac{Q^2 + m_\ell^2}{2\nu} \qquad (6.6)$$

As observed by Barnett, the Callan-Gross relation becomes $\nu W_2 = 2\xi W_1$ sub-asymptotically. We have not attempted detailed comparisons with neutrino-production data because a large fraction of the new particle production comes from small values of x where the neglect of parton interactions is dubious, and renormalization group (asymptotic freedom) effects are likely to be dominant.

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APPENDIX A

In order to establish contact with the more familiar infinite momentum frame parton model ^{2),14)}, consider an invariant distribution of quark momenta k in a proton of momentum p :

$$d^4k \, g\left(\frac{p \cdot k}{m_H^2}\right) \delta(k^2 - m^2) = \frac{1}{2} d\varphi d\cos\theta \, k dk_0 \, g\left(\frac{k_0}{m_H}\right) \quad (A.1)$$

where the right-hand side is evaluated in the target rest frame, $p = (m_H, 0)$. The longitudinal fraction of parton momentum in an infinite momentum frame $p \rightarrow (\gamma m_H, \gamma \beta m_H, 0, 0)$, $\beta \rightarrow 1$, is given in terms of the rest frame variables by

$$x = (k_0 + k \cos\theta) / m_H$$

Then the x distribution corresponding to the invariant distribution (A.1) is

$$\begin{aligned} F(x) &= \pi \int k dk_0 d\cos\theta \, \delta\left(x - \frac{k_0 + k \cos\theta}{m_H}\right) g\left(\frac{k_0}{m_H}\right) \\ &= \frac{\pi}{2} m_H^2 \int_x^{y_{\max}} dy \left(1 - \frac{m^2}{m_H^2 y^2}\right) g\left(\frac{1}{2}\left(y + \frac{m^2}{m_H^2 y}\right)\right); \quad k_0 = \frac{m_H}{2} \left(y + \frac{m^2}{m_H^2 y}\right) \end{aligned} \quad (A.2)$$

Up to a mass factor, (A.2) is the defining integral for the function appearing in Eq. (2.10)

$$J(\xi) = \frac{1}{m_H} F(\xi) \quad (A.3)$$

if the distribution $g(p \cdot q / m_H^2)$ in (A.1) is identified with that in (2.7). The normalization is determined by the definition

$$\langle p | [J_\mu(x), J_\nu^\dagger(0)] | p \rangle = \int d^4k \, g\left(\frac{k \cdot p}{m_H^2}\right) \delta(k^2 - m^2) \langle k | [J_\mu(x), J_\nu^\dagger(0)] | k \rangle$$

and the covariant state normalization $\langle p | p' \rangle = 2E \delta^3(p - p')$ such that (for unit charges)

$$\int d^4k \, g\left(\frac{k \cdot p}{m_H^2}\right) \frac{k \cdot p}{m_H^2} \delta(k^2 - m^2) = 1 \quad (\text{A.4})$$

Using the spectral condition

$$\frac{m^2}{m_H^2} < \xi < 1 \quad \left(k_0 < \frac{m_H^2 + m^2}{2m_H} \right),$$

we may use the definition (A.2) to express (A.4) in the form:

$$1 = -\frac{1}{2} \int_{\left(\frac{m}{m_H}\right)^2}^1 d\xi \left(\xi^2 - \frac{m^4}{m_H^4 \xi^2} \right) F'(\xi) = \int_{\left(\frac{m}{m_H}\right)^2}^1 d\xi \left(\xi + \frac{2m^4}{m_H^4 \xi^3} \right) F(\xi) \quad (\text{A.5})$$

Therefore the function

$$\phi(\xi) = m_H \left(\xi + \frac{2m^4}{m_H^4 \xi^3} \right) F(\xi)$$

may be interpreted as the probability of finding a parton with momentum fraction ξ in the infinite momentum frame. We find no restriction on the p_T of the parton necessary for the validity of this interpretation of ξ . One may verify that in the limits m^2/Q^2 , m_H^2/Q^2 , $m/m_H \rightarrow 0$, the expressions (2.12) reduce to the usual ones, e.g.,

$$\xi \rightarrow x = \frac{Q^2}{2\nu} \quad , \quad \nu W_2 \longrightarrow x^2 F(x) = x \phi(x)$$

and that the right-hand side of the Adler sum rule is proportional to (A.5) as required by current algebra.

In the case of e^+e^- annihilation, the distribution (A.2) applies for the momentum fraction of the decay hadron of mass m from a quark of infinite momentum and mass m_H with $y \equiv (p_0 + p)/m_H$ defined in terms of the decay hadron energy in the quark rest frame. If k_0 , k are the quark energy and momentum in the hadron rest frame, we have:

$$\frac{k_0}{p_0} = \frac{k}{p} = \frac{m_H}{m}.$$

Defining

$$x = \frac{k_0 - k}{m} = \frac{m_H}{p_0 + p} = \frac{1}{y} ,$$

we may rewrite (B.2) as

$$F(x) \equiv G(\omega) = \frac{\pi}{2} m^2 \int_{\omega_{\min}}^{\omega} dz \left(\frac{m_H^2}{m^2 z^2} - 1 \right) g \left(\frac{1}{2} \left(z + \frac{m_H^2}{m^2 z} \right) \right) , \quad (A.6)$$

$\omega = 1/x$

Again up to a mass factor, $G(\xi)$ is the function $g(\xi)$ appearing in Eq. (B.2) with the roles of m_H and m exchanged.

e^+e^- annihilation

In the context of the free covariant quark-parton model, the discussion of one-particle inclusive e^+e^- annihilation is very similar to that given in Section 2 for deep inelastic scattering. The counterpart of Eq. (2.7) is here

$$\begin{aligned}\bar{W}_{\mu\nu}(p, q) &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \bar{W}_1(\nu, q^2) + \left(p_\mu - \frac{\nu q_\mu}{q^2}\right) \left(p_\nu - \frac{\nu q_\nu}{q^2}\right) \bar{W}_2(\nu, q^2) \\ &= \int d^4k \, g\left(\frac{k \cdot p}{m_H^2}\right) \delta(k^2 - m^2) \delta((q-k)^2 - m^2) (q_\mu k_\nu + q_\nu k_\mu - 2k_\mu k_\nu - \frac{q^2}{2} g_{\mu\nu})\end{aligned}\tag{B.1}$$

The function $g(k \cdot p/m_H^2)$ in Eq. (B.1) can be interpreted as the spin-averaged probability for the fragmentation of an on-shell quark of mass m and momentum k into a hadron of momentum p (again $p^2 = m_H^2$) + anything. Clearly the picture of the process, as expressed in Eq. (B.1), imposes the constraints $q^2 \geq 4m^2 \geq 4m_H^2$. We also have the general kinematic limits $q^2 \geq 2\nu$, $\nu \geq m_H \sqrt{q^2}$. There is also the condition $(k-p)^2 \geq 0$ which ensures that the "anything" produced by the quark decay has timelike momentum.

The k integral in Eq. (B.1) is most conveniently performed in the rest frame of the timelike photon. Discarding again for a moment the Lorentz structure, one finds for the corresponding scalar structure function

$$\bar{W}(\nu, q^2) = \frac{1}{\sqrt{\nu^2 - q^2 m_H^2}} \left[G\left(\frac{\bar{E}_-}{m_H}\right) - G\left(\frac{\bar{E}_+}{m_H}\right) \right]\tag{B.2}$$

where

$$G\left(\frac{\bar{E}}{m_H}\right) = \pi \int_{\bar{E}/m_H}^{\infty} dx \, g(x)\tag{B.3}$$

and

$$\bar{E}_\pm = \frac{1}{2m_H q^2} \left[\nu q^2 \pm \sqrt{q^2(q^2 - 4m^2)} \sqrt{\nu^2 - q^2 m_H^2} \right]\tag{B.4}$$

The spectral boundary condition $v = \sqrt{q^2} m_H$ corresponds to $E_{\pm} = \frac{1}{2}\sqrt{q^2}$, while $v = q^2/2$ corresponds to

$$E_{\pm} \approx \begin{cases} m^2/m_H & \text{when } q^2 = 4m^2 \text{ (threshold)} \\ \frac{q^2/2m_H}{\frac{m_H^2 + m^2}{2m_H}} & \text{when } q^2 \rightarrow \infty \end{cases} \quad (\text{B.5})$$

For intermediate values of v , E_{+} varies strictly between these kinematic limits, while E_{-} can become as small as m when $v = m_H q^2/2m$. The timelike recoil condition $(k-p)^2 \geq 0$ ensures $G(E/m_H) = 0$ for

$$E \geq \frac{m_H^2 + m^2}{2m_H} \quad (\text{B.6})$$

In Fig. 6 we have plotted the ranges of $E_{\pm}$ corresponding to the physical spectral conditions $m_H \sqrt{q^2} \leq v \leq q^2/2$, and indicated the on-shell parton model spectral conditions $m \leq E \leq (m_H^2 + m^2)/2m_H$.

We see that for $q^2 \gg (m_H^2 + m^2)^2/m_H^2$ the $G(E_+/m_H)$ term in (B.2) is zero. This reflects the physical fact that the decay products of a free massive particle can only populate the hemisphere into which the parent is moving, if it is travelling fast enough. The formula (B.2) can then be written as

$$\begin{aligned} \overline{W}(v, q^2) &= \frac{\pi}{\sqrt{v^2 - m_H^2} q^2} G\left(\frac{1}{2}\left(\bar{\xi} + \frac{m^2}{m_H^2 \bar{\xi}}\right)\right) = \\ &= \frac{\pi}{\sqrt{v^2 - m_H^2} q^2} g(\bar{\xi}) \end{aligned} \quad (\text{B.7})$$

where

$$\bar{\xi} = \frac{q^2 + \sqrt{q^2(q^2 - 4m^2)}}{2(v + \sqrt{v^2 - m_H^2} q^2)} \quad (\text{B.8})$$

is the e^+e^- analogue of the deep inelastic ξ variable (2.11). We observe that $\bar{\xi} \rightarrow \bar{x} \equiv q^2/2v$ as $q^2 \rightarrow \infty$, and that Eqs. (B.7) and (B.8) are restricted to $1 \leq \bar{\xi} \leq (m/m_H)^2$. As in deep inelastic scattering the structure function has the invariance property $G(\bar{\xi}) = G(m^2/m_H^2 \bar{\xi})$. Just like the corresponding electroproduction prediction, this result should not be taken too seriously.

The appropriate extension of (B.7) to the spin $\frac{1}{2}$ case (B.1) is found to be

$$\overline{W}_1 = \frac{q^2}{2\sqrt{v^2 - m_H^2} q^2} \left[g(\overline{\xi}) + \mathcal{L} \right] \quad (\text{B.9a})$$

$$\overline{W}_2 = \frac{q^4 m_H^2}{2(v^2 - m_H^2 q^2)^{3/2}} \left\{ \left(\frac{\overline{R}}{q^2} \right)^2 g(\overline{\xi}) + 3\mathcal{L} \right\} \quad (\text{B.9b})$$

with

$$\begin{aligned} \mathcal{L} = & \frac{\overline{R} m_H^2}{q^2 \sqrt{v^2 - m_H^2} q^2} \int_1^{\overline{\xi}} d\overline{\xi}' \left(1 - \frac{m^2}{m_H^2 \overline{\xi}'^2} \right) g(\overline{\xi}') + \\ & + \frac{m_H^4}{v^2 - m_H^2 q^2} \int_1^{\overline{\xi}} d\overline{\xi}' \left(1 - \frac{m^2}{m_H^2 \overline{\xi}'^2} \right) \int_1^{\overline{\xi}'} d\overline{\xi}'' \left(1 - \frac{m^2}{m_H^2 \overline{\xi}''^2} \right) g(\overline{\xi}'') \end{aligned} \quad (\text{B.9c})$$

and

$$\overline{R} = \sqrt{q^2(q^2 - 4m^2)} \quad (\text{B.9d})$$

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FIGURE CAPTIONS

- Fig. 1 : The (a) disconnected (b) handbag, and (c) cat's ears terms in the operator product expansion of two currents.
- Fig. 2 : The ranges of the variables $\pm E_{\pm}$ in Eq. (2.9) with the physical ($\nu \geq Q^2/2$) and free parton [$E \leq (m_H^2 + m_i^2)/2m_H$] spectral condition indicated. The curves corresponding to $\nu = Q^2/2$ are appropriate to the case $m_f = m_i$.
- Fig. 3 : Possible interaction corrections to the free field approximation.
- Fig. 4 : Comparisons of the formulae (6.3) for $n=4$ with
 (a) MIT-SLAC data on W_1^P and $\sqrt{W_2^P}$ ²³⁾;
 (b) SLAC data on W_1^P ²⁴⁾;
 (c) the data of Anderson et al. ²⁶⁾ (solid dots) on $\sqrt{W_2^D}$.
- Fig. 5 : Comparison of R^P from Eq. (6.4) with SLAC data ²⁴⁾.
- Fig. 6 : The ranges of the variables $E_{\pm}$ in Eq. (B.2), with the physical (B.5) and free parton (B.6) spectral conditions indicated.

FIG.1

FIG.2

FIG.3

FIG. 4

FIG.5

FIG.6