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SINGLE PION PRODUCTION AND THE STRUCTURE OF THE WEAK NEUTRAL CURRENT

G. Ecker

CERN -- Geneva

and

Institut für Theoretische Physik, University of Vienna

and

R. Fischer

Institut für Theoretische Physik, University of Vienna

ABSTRACT

We investigate the restrictions on the structure of the weak neutral current imposed by single pion production cross-sections on single nucleons. A general vector (V), axial-vector (A) neutral current with $|\Delta I| \leq 1$ is assumed, where the isovector V, A neutral currents are the neutral members of the isotriplets containing the charged weak currents. From neutrino cross-sections alone we derive bounds for the neutral current coupling constants. These bounds supplement the known constraints from inclusive scattering in a very useful way. More specific assumptions about the isoscalar neutral current are also considered. We discuss the resulting bounds using the existing neutrino data. Finally, it is shown that with the advent of antineutrino data for single pion production the neutral current coupling constants will be determined uniquely.

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1. INTRODUCTION

Single pion production by weak neutral currents has been investigated in detail ¹⁾ using low-energy theorems from current algebra. It is clear that such a method does not only depend on the structure of the neutral current, even though soft pion techniques have been rather successful in describing single pion production by charged currents ²⁾. Moreover, by their very nature, soft pion methods are expected to be applicable only near threshold. In a recent letter ³⁾, we proposed a more phenomenological approach which directly tests the underlying neutral current. The present paper is an extension of our previous work.

In order to be able to use the information from charged current interactions, we assume that the neutral current contains only vector (V) and axial-vector (A) currents and that the isovector ($I=1$) currents are in the same isotriplets as the corresponding charged weak currents. In general, we make no assumption about the isoscalar ($I=0$) neutral current, but we exclude currents with isospin larger than one. Although we shall not use any aspects of the quark model, our assumptions include the most general V,A neutral current that can be constructed from valence quarks. The general idea is to use all information from single pion production via electromagnetic, charged weak and neutral weak currents to determine the coupling constants g_V, g_A of the isovector neutral currents. It will turn out that neutrino cross-sections alone yield bounds on g_V and g_A which are especially useful together with the constraints from inclusive scattering ⁴⁾. Given both neutrino and antineutrino cross-sections, the coupling constants may be determined as the solutions of quadratic equations. The present analysis is relevant for pion production off single nucleons only if we refrain from performing nuclear corrections ⁵⁾ which seem difficult to control. A similar analysis which will apply to both exclusive and semi-inclusive pion production off complex nuclei is in preparation and will be presented elsewhere.

The paper is organized as follows. The first part contains a derivation of bounds for the neutral current coupling constants using neutrino cross-sections alone which are the only ones available at present. In a second part, we determine the coupling constants in terms of neutrino and antineutrino cross-sections together. In Section 2 we perform an isospin analysis for the charged current, neutral current and electromagnetic amplitudes for single pion production. Subsection 2.1 contains the bounds on g_V, g_A for a general isoscalar admixture. We discuss these bounds using data from bubble chamber experiments at Argonne ^{6),7)}, together with the

constraints from inclusive scattering ⁴⁾. In subsection 2.2 we consider a pure isovector neutral current. The resulting bounds allow for a resolution of the VA ambiguity ⁸⁾ in inclusive scattering to give a unique solution for g_V, g_A . Subsection 2.3 is concerned with models for the neutral current where the isoscalar weak current is proportional to the electromagnetic isoscalar current. Such models arise naturally in gauge theories based on $SU(2) \times U(1)$ when the isoscalar axial-vector is either absent or can be neglected. The connection between our coupling constants and the general $SU(2) \times U(1)$ current couplings in the quark model at low energies is exhibited. Our results may be interpreted to provide strong restrictions on right-handed couplings of the valence quarks. In this class the original Salam-Weinberg model ⁹⁾ is of particular interest. We find the model compatible with the existing data for $\sin^2 \theta_W \gtrsim 0.2$.

In Section 3 it is shown that with neutrino and antineutrino cross-sections, the structure of the isovector neutral current is completely determined, in the presence of an arbitrary isoscalar admixture (subsection 3.1), up to the usual ambiguities due to quadratic equations. For $\Delta I = 1$ (subsection 3.2) and Salam-Weinberg type models (subsection 3.3), we obtain consistency conditions among the various cross-sections which may serve as a test for these more specific assumptions. In Section 4 our results are summarized. The Appendices contain some definitions and technical details.

2. NEUTRINO CROSS-SECTIONS

We consider the following Lagrangians for neutrino scattering by charged and neutral weak currents

$$\mathcal{L}_{cc} = \frac{G}{\sqrt{2}} \bar{\ell} \gamma^\lambda (1 + \gamma_5) \nu (V_\lambda^{1+i2} + A_\lambda^{1+i2}) + h.c. \quad (2.1)$$

$$\mathcal{L}_{nc} = \frac{G}{\sqrt{2}} \bar{\nu} \gamma^\lambda (1 + \gamma_5) \nu (g_V V_\lambda^3 + g_A A_\lambda^3 + J_\lambda^{I=0}) \quad (2.2)$$

V_λ^3, A_λ^3 are the neutral components of the isotriplets containing the charged currents $V_\lambda^{1+i2}, A_\lambda^{1+i2}$, $J_\lambda^{I=0}$ is an arbitrary isoscalar neutral current and G is the Fermi coupling constant. In writing these Lagrangians, we have neglected the Cabibbo angle ¹⁰⁾. Let us emphasize that (2.2) encompasses the most general V, A neutral current with $\Delta S = \Delta C = 0$ that can be constructed with the usual u, d, s, c quarks.

In order to keep matters as simple as possible, we will also set the lepton mass $m_\mu = 0$. This is a less trivial restriction, especially near threshold. However, at least for the integrated cross-sections the corrections due to the finite muon mass are expected to be small, since the relevant scale is given by the target mass in that case. With this proviso, all our results will also be valid for the differential cross-sections, as long as the angle between the lepton and the hadron planes is integrated over. There are two reasons for doing this ¹¹⁾. Firstly, this integration eliminates the parity-violating terms in the hadronic VV and AA parts coming from the lepton tensor in the case of neutrino scattering. These terms are unwanted in our analysis because we intend to relate the VV part to the electromagnetic cross-sections. Secondly, only after integration over this angle, the neutrino and antineutrino cross-sections with the same hadronic matrix element differ only by the sign of the VA interference terms. This fact will be used in Section 3.

It is convenient to present the isospin analysis in terms of the following amplitudes for single pion production via charged currents (2.3), neutral currents (2.4) and electromagnetic currents (2.5):

$$M_1 = V_3 + A_3 \quad (2.3a)$$

$$M_2 = V_3 + A_3 - V_1 - A_1 \quad (2.3b)$$

$$M_3 = V_3 + A_3 + 2V_1 + 2A_1 \quad (2.3c)$$

$$M_4 = 2g_V V_3 + 2g_A A_3 + g_V V_1 + g_A A_1 - W_0 \quad (2.4a)$$

$$M_5 = g_V V_3 + g_A A_3 - g_V V_1 - g_A A_1 + W_0 \quad (2.4b)$$

$$M_6 = g_V V_3 + g_A A_3 - g_V V_1 - g_A A_1 - W_0 \quad (2.4c)$$

$$M_7 = 2g_V V_3 + 2g_A A_3 + g_V V_1 + g_A A_1 + W_0 \quad (2.4d)$$

$$M_8 = 2V_3 + V_1 - V_0 \quad (2.5a)$$

$$M_9 = V_3 - V_1 + V_0 \quad (2.5b)$$

$$M_{10} = V_3 - V_1 - V_0 \quad (2.5c)$$

$$M_{11} = 2V_3 + V_1 + V_0 \quad (2.5d)$$

V_3, A_3 (V_1, A_1) are the isovector amplitudes for the $I = 3/2$ ($I = 1/2$) final state, W_0 is the weak isoscalar amplitude and V_0 is the electromagnetic isoscalar amplitude.

Mainly in order to simplify our notation, we introduce a Hilbert space H_Ω of amplitudes with a scalar product defined by

$$(M_i, M_j) = \int d\Omega M_i^* M_j \quad (2.6)$$

where the integration extends over an arbitrary region of phase space, including however always the full integration over the angle between lepton and hadron planes, as explained before. The integral may also contain an integration over the neutrino spectrum and a summation over spins is understood. The norms in H_Ω are then directly proportional to the cross-sections, and we refer to Appendix A for the details. The bounds to be derived will essentially result from frequent use of the Cauchy inequality in H_Ω .

Using the charged current and electromagnetic amplitudes, we may write the neutral current amplitudes as

$$M_4 = \frac{1}{2}(g_V - g_A)(M_8 + M_{11}) + g_A M_0 - W_0 \quad (2.7a)$$

$$M_5 = \frac{1}{2}(g_V - g_A)(M_9 + M_{10}) + g_A M_2 + W_0 \quad (2.7b)$$

$$M_6 = \frac{1}{2}(g_v - g_A)(M_9 + M_{10}) + g_A M_2 - W_0 \quad (2.7c)$$

$$M_7 = \frac{1}{2}(g_v - g_A)(M_8 + M_{11}) + g_A M_0 + W_0 \quad (2.7d)$$

where

$$M_0 \equiv 3M_1 - M_2 \quad (2.8)$$

The following relations are a consequence of the assumed isospin structure of the currents only. They imply the well-known triangle and quadrangle inequalities ¹²⁾ for cross-sections

$$3M_1 = 2M_2 + M_3 \quad (2.9)$$

$$M_4 + M_5 = M_6 + M_7 \quad (2.10)$$

$$M_8 + M_9 = M_{10} + M_{11} \quad (2.11)$$

Because of (2.9), M_0^2 is determined by the three charged current cross-sections ¹³⁾.

2.1 General $\Delta I = 0, 1$ neutral current

In this subsection no assumption is made concerning the weak isoscalar amplitude W_0 . Defining, for convenience, two amplitudes A, B by

$$A \equiv \frac{1}{2}(g_v - g_A)(M_8 + M_{11}) + g_A M_0 \quad (2.12a)$$

$$B \equiv \frac{1}{2}(g_v - g_A)(M_9 + M_{10}) + g_A M_2 \quad (2.12b)$$

we find for the scalar products involving W_0

$$(A, W_0) + (W_0, A) = \frac{1}{2}(M_7^2 - M_4^2) \quad (2.13)$$

$$(B, W_0) + (W_0, B) = \frac{1}{2} (M_5^2 - M_6^2) \quad (2.14)$$

$$W_0^2 = \frac{1}{2} (M_4^2 + M_7^2) - H^2 \quad (2.15)$$

and in addition

$$2B^2 = 2H^2 + M_5^2 + M_6^2 - M_4^2 - M_7^2 \quad (2.16)$$

Given the norms of the amplitudes $M_1, M_2, \dots, M_{11}$, in other words the cross-sections, what are the restrictions on the coupling constants g_V, g_A ?

From (2.11) and (2.12a) one gets, using the Cauchy inequality

$$a_- \leq H^2 \leq a_+ \quad (2.17)$$

where

$$a_+ = \left[|g_H M_0| + \frac{1}{2} |g_V - g_H| (M_8^2 + M_{11}^2 + w_1)^{1/2} \right]^2 \quad (2.18a)$$

$$a_- = \begin{cases} \left[\frac{1}{2} |g_V - g_H| (M_8^2 + M_{11}^2 + w_2)^{1/2} - |g_H M_0| \right]^2 & \text{for } 2|g_H M_0| \leq |g_V - g_H| (M_8^2 + M_{11}^2 + w_2)^{1/2} \\ 0 & \text{for } |g_V - g_H| (M_8^2 + M_{11}^2 + w_2)^{1/2} \leq 2|g_H M_0| \leq |g_V - g_H| (M_8^2 + M_{11}^2 + w_1)^{1/2} \\ \left[|g_H M_0| - \frac{1}{2} |g_V - g_H| (M_8^2 + M_{11}^2 + w_1)^{1/2} \right]^2 & \text{for } 2|g_H M_0| \geq |g_V - g_H| (M_8^2 + M_{11}^2 + w_1)^{1/2} \end{cases} \quad (2.18b)$$

where

$$w_1 = \min (2|M_8||M_{11}|, M_8^2 + M_{11}^2 - (|M_9| - |M_{10}|)^2) \quad (2.19a)$$

$$w_2 = \max(-2|M_8||M_{11}|, M_8^2 + M_{11}^2 - (|M_9| + |M_{10}|)^2) \quad (2.19b)$$

With the obvious replacements, a similar inequality holds for B^2

$$b_- \leq B^2 \leq b_+ \quad (2.20)$$

On the other hand, the Cauchy inequality $[(A, W_0) + (W_0, A)]^2 \leq 4A^2 W_0^2$ which may be written in either of two ways

$$(M_7^2 - M_4^2)^2 \leq 8F^2 (M_4^2 + M_7^2 - 2F^2) \quad (2.21)$$

$$(M_7^2 - M_4^2)^2 \leq 4(2B^2 + M_4^2 + M_7^2 - M_5^2 - M_6^2)(M_5^2 + M_6^2 - 2B^2) \quad (2.22)$$

leads to the constraints

$$(|M_4| - |M_7|)^2 \leq 4F^2 \leq (|M_4| + |M_7|)^2 \quad (2.23)$$

and

$$2M_5^2 + 2M_6^2 - (|M_4| + |M_7|)^2 \leq 4B^2 \leq 2M_5^2 + 2M_6^2 - (|M_4| - |M_7|)^2 \quad (2.24)$$

In exactly the same way the Cauchy inequality for $(B, W_0) + \text{c.c.}$ implies

$$2M_4^2 + 2M_7^2 - (|M_5| + |M_6|)^2 \leq 4F^2 \leq 2M_4^2 + 2M_7^2 - (|M_5| - |M_6|)^2 \quad (2.25)$$

and

$$(|M_5| - |M_6|)^2 \leq 4B^2 \leq (|M_5| + |M_6|)^2 \quad (2.26)$$

There is one further constraint due to Eq. (2.16). Using the definitions for A , B and the isospin relation (2.11), one obtains

$$2(B^2 - F^2) = (g_v - g_\pi)^2 (M_\rho^2 + M_{10}^2 - M_8^2 - M_{11}^2) + \\ + 2g_\pi^2 (M_2^2 - M_0^2) + g_\pi(g_v - g_\pi) [(M_2, M_\rho + M_{10}) - (M_0, M_8 + M_{11}) + \text{c.c.}] \quad (2.27)$$

The only undetermined quantity in this expression is

$$d \equiv (M_2, M_\rho + M_{10}) - (M_0, M_8 + M_{11}) + \text{c.c.} \quad (2.28)$$

which is bounded by

$$|d| \leq 2|M_2|(M_\rho^2 + M_{10}^2 + t_1)^{1/2} + 2|M_0|(2M_8^2 + 2M_{11}^2 - M_\rho^2 - M_{10}^2 + t_1)^{1/2} \quad (2.29)$$

where

$$t_1 = \min(2|M_\rho||M_{10}|, M_\rho^2 + M_{10}^2 - (|M_8| - |M_{11}|)^2) \quad (2.30)$$

However, the bound (2.29) is not optimal in general because of the isospin relation (2.11). We defer a discussion of this point to Appendix B and denote $d_{\max}$ to be the actual maximum of d .

Expressing everything in terms of cross-sections (see Appendix A), the results of this Section may be written in terms of the following inequalities

$$H_+ \geq \max\left\{(\sigma_4^{1/2} - \sigma_7^{1/2})^2, 2\sigma_4 + 2\sigma_7 - \frac{1}{2}(\sigma_5^{1/2} + \sigma_6^{1/2})^2\right\} \quad (2.31)$$

$$H_- \leq \min\left\{(\sigma_4^{1/2} + \sigma_7^{1/2})^2, 2\sigma_4 + 2\sigma_7 - \frac{1}{2}(\sigma_5^{1/2} - \sigma_6^{1/2})^2\right\} \quad (2.32)$$

$$B_+ \geq \max\left\{\frac{1}{2}(\sigma_5^{1/2} - \sigma_6^{1/2})^2, \sigma_5 + \sigma_6 - (\sigma_4^{1/2} + \sigma_7^{1/2})^2\right\} \quad (2.33)$$

$$B_- \leq \min\left\{\frac{1}{2}(\sigma_5^{1/2} + \sigma_6^{1/2})^2, \sigma_5 + \sigma_6 - (\sigma_4^{1/2} - \sigma_7^{1/2})^2\right\} \quad (2.34)$$

$$\begin{aligned}
 -|g_H(g_V - g_H)|D_{\max} &\leq 2\sigma_4 + 2\sigma_7 - \sigma_5 - \sigma_6 + g_H^2(2\sigma_2 - \sigma_1 - \sigma_3) + \\
 + (g_V - g_H)^2(\sigma_9 + \sigma_{10} - 2\sigma_8 - 2\sigma_{11}) &\leq |g_H(g_V - g_H)|D_{\max}
 \end{aligned} \tag{2.35}$$

where

$$H_+ = [|g_H|(\sigma_1 - \sigma_2 + \sigma_3)^{1/2} + |g_V - g_H|(\sigma_8 + \sigma_{11} + W_1)^{1/2}]^2 \tag{2.36a}$$

$$H_- = \begin{cases} [|g_V - g_H|(\sigma_8 + \sigma_{11} + W_2)^{1/2} - |g_H|(\sigma_1 - \sigma_2 + \sigma_3)^{1/2}]^2 \\ \quad \text{for } |g_H|(\sigma_1 - \sigma_2 + \sigma_3)^{1/2} \leq |g_V - g_H|(\sigma_8 + \sigma_{11} + W_2)^{1/2} \\ 0 \quad \text{for } |g_V - g_H|(\sigma_8 + \sigma_{11} + W_2)^{1/2} \leq |g_H|(\sigma_1 - \sigma_2 + \sigma_3)^{1/2} \leq |g_V - g_H|(\sigma_8 + \sigma_{11} + W_1)^{1/2} \\ [|g_H|(\sigma_1 - \sigma_2 + \sigma_3)^{1/2} - |g_V - g_H|(\sigma_8 + \sigma_{11} + W_1)^{1/2}]^2 \\ \quad \text{for } |g_H|(\sigma_1 - \sigma_2 + \sigma_3)^{1/2} \geq |g_V - g_H|(\sigma_8 + \sigma_{11} + W_1)^{1/2} \end{cases} \tag{2.36b}$$

$$B_+ = [|g_H|\sigma_2^{1/2} + |g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_1)^{1/2}]^2 \tag{2.37a}$$

$$B_- = \begin{cases} [|g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_2)^{1/2} - |g_H|\sigma_2^{1/2}]^2 \\ \quad \text{for } |g_H|\sigma_2^{1/2} \leq |g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_2)^{1/2} \\ 0 \quad \text{for } |g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_2)^{1/2} \leq |g_H|\sigma_2^{1/2} \leq |g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_1)^{1/2} \\ [|g_H|\sigma_2^{1/2} - |g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_1)^{1/2}]^2 \\ \quad \text{for } |g_H|\sigma_2^{1/2} \geq |g_V - g_H|(\sigma_9/2 + \sigma_{10}/2 + T_1)^{1/2} \end{cases} \tag{2.37b}$$

$$W_1 = \min \left\{ 2(\sigma_8 \sigma_{11})^{1/2}, \sigma_8 + \sigma_{11} - \frac{1}{2}(\sigma_9^{1/2} - \sigma_{10}^{1/2})^2 \right\} \quad (2.38a)$$

$$W_2 = \max \left\{ -2(\sigma_8 \sigma_{11})^{1/2}, \sigma_8 + \sigma_{11} - \frac{1}{2}(\sigma_9^{1/2} + \sigma_{10}^{1/2})^2 \right\} \quad (2.38b)$$

$$T_1 = \min \left\{ (\sigma_9 \sigma_{10})^{1/2}, \sigma_9/2 + \sigma_{10}/2 - (\sigma_8^{1/2} - \sigma_{11}^{1/2})^2 \right\} \quad (2.38c)$$

$$T_2 = \max \left\{ -(\sigma_9 \sigma_{10})^{1/2}, \sigma_9/2 + \sigma_{10}/2 - (\sigma_8^{1/2} + \sigma_{11}^{1/2})^2 \right\} \quad (2.38d)$$

and

$$D_{\max} = 2d_{\max}/9 \quad (2.39)$$

Before confronting these bounds on g_V, g_A with the available data, let us consider the two special cases where we can actually determine the coupling constants. For $g_V = g_A$

$$g_V^2 (\sigma_1 - 2\sigma_2 + \sigma_3) = 2\sigma_4 + 2\sigma_7 - \sigma_5 - \sigma_6 \quad (2.40)$$

while for $g_A = 0$

$$g_V^2 (2\sigma_8 + 2\sigma_{11} - \sigma_9 - \sigma_{10}) = 2\sigma_4 + 2\sigma_7 - \sigma_5 - \sigma_6 \quad (2.41)$$

However, these values for g_V, g_A must still be checked if they fulfil the inequalities (2.31)-(2.34).

Let us now turn to a numerical discussion of the bounds. As already mentioned, we want to avoid the uncertainties connected with nuclear corrections so that we can only use the ANL bubble chamber experiments^{6),7)} on hydrogen and deuterium for data on the charged and neutral current cross-sections. Moreover, in the Argonne experiment, $\sigma_7 = \sigma(\nu n \rightarrow \nu n \pi^0)$ was not measured. We therefore use the quadrangle inequalities resulting from the isospin relation (2.10) to bound σ_7 :

$$(2\sigma_7)^{1/2} \geq \max \left\{ (2\sigma_4)^{1/2} - \sigma_5^{1/2} - \sigma_6^{1/2}, \sigma_5^{1/2} - (2\sigma_4)^{1/2} - \sigma_6^{1/2}, \sigma_6^{1/2} - (2\sigma_4)^{1/2} - \sigma_5^{1/2} \right\} \quad (2.42a)$$

$$(2\sigma_7)^{1/2} \leq (2\sigma_4)^{1/2} + \sigma_5^{1/2} + \sigma_6^{1/2} \quad (2.42b)$$

In contrast to our previous work ³⁾ where only upper bounds for $\sigma_8 + \sigma_{11}$ and $\sigma_9 + \sigma_{10}$ were considered, we now make full use of the electromagnetic cross-sections. Applying the narrow width approximation to data from DESY ¹⁴⁾ and CEA ¹⁵⁾ and including some background, we have calculated the ratios σ_8/σ_1 , σ_9/σ_1 , σ_{10}/σ_1 , σ_{11}/σ_1 with an estimated error of less than 25%. The values used in the figures are shown in the Table together with the data for neutrino scattering. It turns out that the relevant bounds are rather insensitive to the exact values of the electromagnetic cross-sections. In view of the rather large errors in the neutral current cross-sections, we do not have to worry about the errors in the electromagnetic cross-sections at this time. The situation is similar for the charged current cross-section ratios σ_2/σ_1 and σ_3/σ_1 .

In Fig. 1 we display the bounds on g_V , g_A in the general case of a $\Delta I = 0, 1$ neutral current. Forgetting about errors altogether for the moment, we would conclude that $g_V = g_A$, i.e., a V-A isovector part of the neutral current, is not compatible with the data, while $g_A = 0$ is allowed. Note that the gap between forbidden domains on the g_V axis is due to our ignorance of σ_7 . In order to get some feeling for the uncertainties involved in Fig. 1 we remark that the size of the central region is determined essentially by the quantity

$$S_1 = \left[(2\sigma_4 - \sigma_5 - \sigma_6 + 2(\sigma_4^{1/2} - (\sigma_5/2)^{1/2} - (\sigma_6/2)^{1/2})^2) / \sigma_1 \right]^{1/2} \quad (2.43)$$

while the domain excluding $g_V = g_A$ is governed by

$$S_2 = (\sigma_5/2\sigma_1)^{1/2} + (\sigma_6/2\sigma_1)^{1/2} \quad (2.44)$$

Using the experimental errors, we calculate

$$S_1 = .64 \pm .38 \quad (2.45a)$$

$$S_2 = .52 \pm .08 \quad (2.45b)$$

where the large error in S_1 is mainly due to the error for σ_4/σ_1 . Note that S_1 is a relevant observable to check for a purely isoscalar neutral current ($g_V = g_A = 0$) because $S_1 = 0$ in that case. The value given in (2.45a) may be considered as evidence against such a neutral current in agreement with the results from Gargamelle¹⁶⁾. In assessing the relevance of Fig. 1 one should keep in mind that the central domain will actually be larger for any reasonable value of σ_7 above its isospin limit.

The restrictions on g_V, g_A become much tighter if we include the constraints⁴⁾ imposed by the inclusive cross-sections. In terms of the neutral to charged current ratios $R, \bar{R}$ for neutrino and antineutrino scattering, these constraints may be expressed in their simplest form¹⁷⁾ as two inequalities

$$8R \geq (g_V + g_A)^2 + r(g_V - g_A)^2 \quad (2.46a)$$

$$8r\bar{R} \geq r(g_V + g_A)^2 + (g_V - g_A)^2 \quad (2.46b)$$

where r is the ratio of antineutrino to neutrino charged current cross-sections. In Fig. 2 the two ellipses in the g_V, g_A plane are shown using the latest results of the Gargamelle experiment¹⁸⁾: $r = 0.37(\pm 0.03)$, $R = 0.25(\pm 0.04)$, $\bar{R} = 0.39(\pm 0.06)$. It should be emphasized that the world averages of three experiments¹⁸⁾ for $R, \bar{R}$ are now practically identical to the Gargamelle values.

Putting the bounds from single pion production and inclusive scattering together, we arrive at Fig. 3. The main observation to be made is the complementarity between the two sets of constraints: while inclusive scattering provides upper¹⁹⁾ bounds for the isovector coupling constants, single pion production gives essentially lower bounds in the g_V, g_A plane. Even with the large errors involved in single pion production cross-sections, we may tentatively draw two conclusions about the isovector neutral current, irrespective of the structure of the isoscalar current:

- i) the isovector current is not pure vector ($g_A \neq 0$);
- ii) a dominant axial-vector isovector current is compatible with the data.

2.2 Pure isovector neutral current

The Argonne data ⁷⁾ are perfectly compatible with the hypothesis of a purely isovector neutral current, i.e., $W_0 = 0$. In this case

$$M_4 = M_7 = \frac{1}{2} (g_v - g_H) (M_8 + M_{11}) + g_H M_0 \quad (2.47)$$

$$M_5 = M_6 = \frac{1}{2} (g_v - g_H) (M_9 + M_{10}) + g_H M_2 \quad (2.48)$$

so that

$$\sigma_4 = \sigma_7 \quad (2.49)$$

$$\sigma_5 = \sigma_6 \quad (2.50)$$

We can also immediately write down the following bounds using the constraints on A^2 , B^2 from the last Section

$$A_- \leq 4\sigma_4 \leq A_+ \quad (2.51)$$

$$B_- \leq 2\sigma_5 \leq B_+ \quad (2.52)$$

with $A_{\pm}$, $B_{\pm}$ defined in Eqs. (2.36) and (2.37). These bounds are equivalent to the four inequalities

$$4 \left[\sigma_4^{1/2} + \frac{1}{2} |g_H| (\sigma_1 - \sigma_2 + \sigma_3)^{1/2} \right]^2 \geq (g_v - g_H)^2 (\sigma_8 + \sigma_{11} + W_2) \quad (2.53)$$

$$4 \left[\sigma_4^{1/2} - \frac{1}{2} |g_H| (\sigma_1 - \sigma_2 + \sigma_3)^{1/2} \right]^2 \leq (g_v - g_H)^2 (\sigma_8 + \sigma_{11} + W_4) \quad (2.54)$$

$$4 \left[\sigma_5^{1/2} + \frac{1}{2} |g_H| \sigma_2^{1/2} \right]^2 \geq (g_v - g_H)^2 (\sigma_9 + \sigma_{10} + 2T_2) \quad (2.55)$$

$$4 \left[\sigma_5^{1/2} - \frac{1}{2} |g_\pi| \sigma_2^{1/2} \right]^2 \leq (g_v - g_\pi)^2 (\sigma_9 + \sigma_{10} + 2T_1) \quad (2.56)$$

where $W_{1,2}$ and $T_{1,2}$ are defined in Eqs. (2.38). In addition, we have the equivalent of (2.35)

$$\begin{aligned} -|g_\pi(g_v - g_\pi)| D_{\max} &\leq 4\sigma_4 - 2\sigma_5 + g_\pi^2 (2\sigma_2 - \sigma_1 - \sigma_3) + \\ &+ (g_v - g_\pi)^2 (\sigma_9 + \sigma_{10} - 2\sigma_8 - 2\sigma_{11}) \leq |g_\pi(g_v - g_\pi)| D_{\max} \end{aligned} \quad (2.57)$$

The bounds for a $\Delta I = 1$ neutral current are shown in Fig. 4. However, in this case the inequalities (2.46) from inclusive scattering turn into equalities, so that the possible values for g_V, g_A are given by the four intersection points of the two ellipses in Fig. 2. If we now use the information from single pion production as given in Fig. 4, we conclude that up to a trivial sign a unique solution for g_V, g_A emerges, i.e., the one with a dominant axial vector current. In other words, the existing data on single pion production resolve the VA ambiguity⁸⁾ in inclusive scattering for an isovector neutral current. The resulting unique solution for g_V, g_A is practically equivalent to the Salam-Weinberg model⁹⁾, even though the neutral current in this model is not purely isovector. We now turn to a discussion of models of this type.

2.3 Weak isoscalar proportional to electromagnetic isoscalar

This case is of particular importance nowadays because all gauge models based on $SU(2) \times U(1)$ are of this type, as long as the isoscalar axial vector is either zero or can be neglected and as long as we stick to the valence quark model. In $SU(2) \times U(1)$ gauge theories, the neutral current is determined by the weak isospin assignments for the valence quarks u and d . In view of the large interest in gauge models of the $SU(2) \times U(1)$ variety, we consider it worth while to write down explicitly the connection between the coupling constants used in this paper and the most general form of the $SU(2) \times U(1)$ neutral current.

If we denote $I_{3,L}^q$ ($I_{3,R}^q$) to stand for the third component of weak isospin of the left-handed (right-handed) quark field q and if we assume the simplest Higgs structure²⁰⁾ as in the original Salam-Weinberg model, we

obtain

$$g_V = I_{3,L}^u - I_{3,L}^d + I_{3,R}^u - I_{3,R}^d - 2 \sin^2 \theta_W \quad (2.58a)$$

$$g_H = I_{3,L}^u - I_{3,L}^d - I_{3,R}^u + I_{3,R}^d \quad (2.58b)$$

$$Z = 3(I_{3,L}^u + I_{3,L}^d + I_{3,R}^u + I_{3,R}^d) - 2 \sin^2 \theta_W \quad (2.58c)$$

$$S = 3(I_{3,L}^u + I_{3,L}^d - I_{3,R}^u - I_{3,R}^d) \quad (2.58d)$$

where the neutral current is defined as

$$J_\mu^{NC} = g_V V_\mu^3 + g_H H_\mu^3 + Z V_\mu^0 + S H_\mu^0 \quad (2.59)$$

θ_W is the Weinberg angle and the quark currents in (2.59) are

$$V_\mu^3 = \frac{1}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) \quad (2.60a)$$

$$H_\mu^3 = \frac{1}{2} (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d) \quad (2.60b)$$

$$V_\mu^0 = \frac{1}{6} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d) \quad (2.60c)$$

$$H_\mu^0 = \frac{1}{6} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d) \quad (2.60d)$$

In this Section we are concerned with models where the only isoscalar current is the electromagnetic isoscalar, i.e., $W_0 = zV_0$. For definiteness, we want to emphasize that our coupling constants g_V, g_A, z are defined completely independent of any underlying quark structure.

We can now write the neutral current amplitudes in terms of charged current and electromagnetic amplitudes

$$M_4 = \frac{1}{2}(g_V - g_A + z)M_8 + \frac{1}{2}(g_V - g_A - z)M_{11} + g_A M_0 \quad (2.61a)$$

$$M_5 = \frac{1}{2}(g_V - g_A + z)M_9 + \frac{1}{2}(g_V - g_A - z)M_{10} + g_A M_2 \quad (2.61b)$$

$$M_6 = \frac{1}{2}(g_V - g_A - z)M_9 + \frac{1}{2}(g_V - g_A + z)M_{10} + g_A M_2 \quad (2.61c)$$

$$M_7 = \frac{1}{2}(g_V - g_A - z)M_8 + \frac{1}{2}(g_V - g_A + z)M_{11} + g_A M_0 \quad (2.61d)$$

In squaring these amplitudes, five independent scalar products appear, i.e.,

$$\begin{aligned} u &= (M_8, M_0) + \text{c.c.} & x &= (M_9, M_2) + \text{c.c.} \\ v &= (M_{11}, M_0) + \text{c.c.} & y &= (M_{10}, M_2) + \text{c.c.} \\ w &= (M_8, M_{11}) + \text{c.c.} \end{aligned} \quad (2.62)$$

because $t \equiv (M_9, M_{10}) + \text{c.c.}$ is linearly dependent on w :

$$t = w + M_9^2 + M_{10}^2 - M_8^2 - M_{11}^2 \quad (2.63)$$

It is not difficult to convince oneself that the simple Cauchy inequalities for these scalar products will not lead to optimal constraints in general, because the system is highly constrained. It turns out that the problem is equivalent to finding an optimal inequality for the three scalar products of three different vectors. Referring to Appendix C for a proof, we obtain the two following inequalities

$$4M_0^2 M_8^2 M_{11}^2 - u^2 M_{11}^2 - v^2 M_8^2 - w^2 M_0^2 + uvw \geq 0 \quad (2.64)$$

$$4M_2^2 M_9^2 M_{10}^2 - x^2 M_{10}^2 - y^2 M_9^2 - t^2 M_2^2 + xyt \geq 0 \quad (2.65)$$

There is one more inequality of this type which takes account of the fact that in view of the isospin relation (2.9) we also know the scalar product $m \equiv (M_0, M_2) + c.c.$ This leads to a third inequality

$$4M_0^2 M_2^2 (M_8^2 + M_{11}^2 - w) - M_2^2 (u-v)^2 - M_0^2 (y-x)^2 - m^2 (M_8^2 + M_{11}^2 - w) + (u-v)(y-x)m \geq 0 \quad (2.66)$$

Using Eqs. (2.61), the three inequalities given above imply constraints for the coupling constants g_V, g_A, z . We have not succeeded in solving the problem analytically. Instead, we proceed in the following way: for $g_V \neq g_A, g_A \neq 0, z \neq 0$ we can express each of the four scalar products u, v, x, y in terms of w and cross-sections as functions of g_V, g_A, z . The strategy is now to scan the (g_V, g_A, z) space to find the regions compatible with the three inequalities given above. More precisely, the criterium for an allowed point (g_V, g_A, z) is: for at least one possible value of w , i.e., for $w_2 \leq w \leq w_1$ [Eq. (2.19)], the four scalar products u, v, x, y as functions of g_V, g_A, z must obey the inequalities (2.64), (2.65) and (2.66).

Before discussing our results, we briefly consider the three special cases where the above program fails:

a) $z = 0$

In this case the neutral current is purely isovector (subsection 2.2).

b) $g_V = g_A$

We have already seen in subsection 2.1 that with the present data (without errors) this possibility is excluded. With our assumption about the isoscalar current we find two additional consistency constraints for the cross-sections

$$(\sigma_4 - \sigma_7)^2 (\sigma_1 - 2\sigma_2 + \sigma_3)^2 \leq (\sigma_1 - \sigma_2 + \sigma_3) (2\sigma_4 + 2\sigma_7 - \sigma_5 - \sigma_6) [(\sigma_1 - \sigma_2 + \sigma_3)(\sigma_5 + \sigma_6) - 2\sigma_2(\sigma_4 + \sigma_7)] \quad (2.67)$$

$$(\sigma_5 - \sigma_6)^2 (\sigma_1 - 2\sigma_2 + \sigma_3)^2 \leq 4\sigma_2 (2\sigma_4 + 2\sigma_7 - \sigma_5 - \sigma_6) \left[(\sigma_1 - \sigma_2 + \sigma_3)(\sigma_5 + \sigma_6) - 2\sigma_2(\sigma_4 + \sigma_7) \right] \quad (2.68)$$

neither of which is fulfilled.

c) $g_A = 0$

In addition to Eq. (2.41) for g_V^2 we find in this case

$$g_V z (\sigma_9 - \sigma_{10}) = \sigma_5 - \sigma_6 \quad (2.69)$$

and a consistency condition

$$(\sigma_4 - \sigma_7)(\sigma_9 - \sigma_{10}) = (\sigma_5 - \sigma_6)(\sigma_8 - \sigma_{11}) \quad (2.70)$$

The present data for single pion production are compatible with $g_A = 0$. However, as already discussed in subsection 2.1, this possibility is excluded considering also the inclusive cross-sections.

For the numerical solution we are again faced with the problem of not knowing σ_7 . As we do not have an analytic solution we have varied σ_7 . It turns out that the forbidden central domain in (g_V, g_A, z) space is minimal for $\sigma_7 \simeq \sigma_4$. This observation does depend, however, on the exact values of the electromagnetic cross-sections σ_8 and σ_{11} . A more precise statement is the following: for $\sigma_8 = \sigma_{11}$ (e.g., for $V_0 = 0$) the constraints for the weak coupling constants are least restrictive for $\sigma_7 = \sigma_4$. Since experimentally the isoscalar amplitude V_0 is known to be small around the Δ resonance and therefore $\sigma_8 \simeq \sigma_{11}$ (see the Table), we have chosen $\sigma_7 = \sigma_4$ for the figures.

In Fig. 5 we display the bounds for g_V, g_A in the plane $z = -1$. As can be seen by comparing Figs. 4 and 5, the isoscalar admixture does not change the picture appreciably. In other words, the central region in the g_V, g_A plane depends only very little on z . This is again due to the small isoscalar amplitude as expressed in the electromagnetic cross-sections.

An interesting observation ²¹⁾ from Fig. 5 is the following: the restrictions are minimal for $g_A \simeq 1$. We therefore show in Fig. 6 the bounds in the plane $g_A = 1$. Also shown in Fig. 6 is a straight line which defines the Salam-Weinberg model ⁹⁾: $g_A = 1$, $g_V = 1 + z = 1 - 2 \sin^2 \theta_W$. Neglecting errors, we conclude that single pion production data are compatible with the Salam-Weinberg model for $\sin^2 \theta_W \gtrsim 0.2$, in agreement with practically every other evidence ²²⁾ for neutral currents.

Let us come back once more to gauge models. Since the weak isospins of the left-handed quark fields are determined by low-energy phenomenology ($I_{3,L}^u = -I_{3,L}^d = \frac{1}{2}$), we may write

$$g_V = 1 - 2 \sin^2 \theta_W + I_{3,R}^u - I_{3,R}^d \quad (2.71a)$$

$$g_A = 1 - I_{3,R}^u + I_{3,R}^d \quad (2.71b)$$

$$z = 3 (I_{3,R}^u + I_{3,R}^d) - 2 \sin^2 \theta_W \quad (2.71c)$$

Our results can therefore be regarded to imply strong restrictions on possible right-handed non-singlets containing valence quarks which are of interest for models with more than 4 quarks ²³⁾. As an example, we consider one version of a model proposed by Gürsey and Sikivie ²⁴⁾ with $I_{3,R}^u = \frac{1}{2}$, $I_{3,R}^d = 0$, neglecting however the isoscalar axial vector matrix element. In terms of our coupling constants, the model is defined by $g_A = \frac{1}{2}$, $g_V = z = 3/2 - 2 \sin^2 \theta_W$. The results are shown in Fig. 7. The straight line defining the model for different $\sin^2 \theta_W$ is seen to lie entirely in the forbidden region. Even without drawing any definite conclusions on the basis of present data, it is evident that single pion production yields strong constraints for model building.

Finally, we recall that for models of the type we are considering in this Section one can use the relation

$$g_V g_A (1 - \lambda) = 2 (R - \lambda \bar{R}) \quad (2.72)$$

to determine the Weinberg angle from inclusive cross-sections. From the

Gargamelle data ¹⁸⁾ one obtains $\sin^2 \theta_W = 0.33$ ($g_V = 0.34$) for the Salam-Weinberg model and $\sin^2 \theta_W = 0.41$ ($g_V = 0.67$) for the Gürsey-Sikivie model, neglecting errors. These values are relevant for Figs. 6 and 7, respectively.

3. NEUTRINO AND ANTINEUTRINO CROSS-SECTIONS

The results of Section 2 are valid for neutrino or antineutrino scattering. At present, only neutrino data exist for single pion production off single nucleons. We want to show in this Section that with antineutrino data becoming available, the neutral current coupling constants will be uniquely determined except for the usual ambiguities connected with quadratic equations. The inequalities derived in the last Section may help to resolve such ambiguities. In principle, it may turn out that neither of the solutions is consistent with all constraints; in this case, the neutral current Lagrangian (2.2) used in this paper would have to be reconsidered.

We define seven new amplitudes which differ from the corresponding amplitudes for neutrino scattering in the sign of the axial amplitudes:

$$\overline{M}_1 = V_3 - A_3 \quad (3.1a)$$

$$\overline{M}_2 = V_3 - A_3 - V_1 + A_1 \quad (3.1b)$$

$$\overline{M}_3 = V_3 - A_3 + 2V_1 - 2A_1 \quad (3.1c)$$

$$\overline{M}_4 = 2g_V V_3 - 2g_A A_3 + g_V V_1 - g_A A_1 - \overline{W}_0 \quad (3.2a)$$

$$\overline{M}_5 = g_V V_3 - g_A A_3 - g_V V_1 + g_A A_1 + \overline{W}_0 \quad (3.2b)$$

$$\overline{M}_6 = g_V V_3 - g_A A_3 - g_V V_1 + g_A A_1 - \overline{W}_0 \quad (3.2c)$$

$$\overline{M}_7 = 2g_V V_3 - 2g_A A_3 + g_V V_1 - g_A A_1 + \overline{W}_0 \quad (3.2d)$$

In order to prevent any misunderstandings, we emphasize that the amplitudes $\bar{M}_1, \bar{M}_2, \dots, \bar{M}_7$ are not necessarily the amplitudes for single pion production by antineutrinos, since in general the antineutrino amplitude is not simply obtained by changing the signs of the axial matrix elements in the neutrino amplitude. However, all we need to know is that the norms of $\bar{M}_1, \bar{M}_2, \dots, \bar{M}_7$ in the previously defined Hilbert space H_Ω are proportional to the relevant antineutrino cross-sections (Appendix A). This is true, provided we integrate over the angle between lepton and hadron planes, because then the corresponding neutrino and antineutrino cross-sections only differ in the sign of the VA interference terms ¹¹⁾. The isoscalar amplitude $\bar{W}_0$ is likewise defined to differ from W_0 only in the sign of the axial amplitude.

If we define an amplitude $\bar{M}_0$ as before by

$$\bar{M}_0 \equiv 3\bar{M}_1 - \bar{M}_2 \quad (3.3)$$

we may write the neutral current amplitudes in the following way

$$M_4 = \frac{1}{2}(g_\nu + g_\pi)M_0 + \frac{1}{2}(g_\nu - g_\pi)\bar{M}_0 - W_0 \quad (3.4a)$$

$$M_5 = \frac{1}{2}(g_\nu + g_\pi)M_2 + \frac{1}{2}(g_\nu - g_\pi)\bar{M}_2 + W_0 \quad (3.4b)$$

$$M_6 = \frac{1}{2}(g_\nu + g_\pi)M_2 + \frac{1}{2}(g_\nu - g_\pi)\bar{M}_2 - W_0 \quad (3.4c)$$

$$M_7 = \frac{1}{2}(g_\nu + g_\pi)M_0 + \frac{1}{2}(g_\nu - g_\pi)\bar{M}_0 + W_0 \quad (3.4d)$$

$$\bar{M}_4 = \frac{1}{2}(g_\nu - g_\pi)M_0 + \frac{1}{2}(g_\nu + g_\pi)\bar{M}_0 - \bar{W}_0 \quad (3.5a)$$

$$\bar{M}_5 = \frac{1}{2}(g_\nu - g_\pi)M_2 + \frac{1}{2}(g_\nu + g_\pi)\bar{M}_2 + \bar{W}_0 \quad (3.5b)$$

$$\bar{M}_6 = \frac{1}{2}(g_v - g_A)M_2 + \frac{1}{2}(g_v + g_A)\bar{M}_2 - \bar{W}_0 \quad (3.5c)$$

$$\bar{M}_7 = \frac{1}{2}(g_v - g_A)M_0 + \frac{1}{2}(g_v + g_A)\bar{M}_0 + \bar{W}_0 \quad (3.5d)$$

Besides the scalar products containing the isoscalar amplitudes W_0 , $\bar{W}_0$, only two scalar products appear in the norms of these amplitudes, i.e., $(M_0, \bar{M}_0) + c.c.$ and $(M_2, \bar{M}_2) + c.c.$ Moreover, these two scalar products are not independent. Using the isospin relation (2.11) and expressing $M_0 + \bar{M}_0$, $M_2 + \bar{M}_2$ in terms of electromagnetic amplitudes, it is easy to show that

$$\begin{aligned} & (M_0, \bar{M}_0) - (M_2, \bar{M}_2) + c.c. = \\ & = 2(M_8^2 + M_{11}^2 - M_9^2 - M_{10}^2) - M_0^2 - \bar{M}_0^2 + M_2^2 + \bar{M}_2^2 \end{aligned} \quad (3.6)$$

We are now in a position to derive the solutions for the different cases considered in the last Section.

3.1 General $\Delta I = 0, 1$ neutral current

In terms of cross-sections (Appendix A) we obtain two quadratic equations for g_V, g_A

$$\begin{aligned} & 4(2\sigma_4 + 2\sigma_7 - \sigma_5 - \sigma_6) = (g_v + g_A)^2(\sigma_1 - 2\sigma_2 + \sigma_3) + \\ & + (g_v - g_A)^2(\bar{\sigma}_1 - 2\bar{\sigma}_2 + \bar{\sigma}_3) + (g_v^2 - g_A^2)(8\sigma_8 + 8\sigma_{11} - \\ & - 4\sigma_9 - 4\sigma_{10} + 2\sigma_2 - \sigma_1 - \sigma_3 + 2\bar{\sigma}_2 - \bar{\sigma}_1 - \bar{\sigma}_3) \end{aligned} \quad (3.7a)$$

$$\begin{aligned} & 4(2\bar{\sigma}_4 + 2\bar{\sigma}_7 - \bar{\sigma}_5 - \bar{\sigma}_6) = (g_v - g_A)^2(\sigma_1 - 2\sigma_2 + \sigma_3) + \\ & + (g_v + g_A)^2(\bar{\sigma}_1 - 2\bar{\sigma}_2 + \bar{\sigma}_3) + (g_v^2 - g_A^2)(8\sigma_8 + 8\sigma_{11} - \\ & - 4\sigma_9 - 4\sigma_{10} + 2\sigma_2 - \sigma_1 - \sigma_3 + 2\bar{\sigma}_2 - \bar{\sigma}_1 - \bar{\sigma}_3) \end{aligned} \quad (3.7b)$$

We note another interesting relation of the form

$$\begin{aligned} & 2(\sigma_4 + \sigma_7 - \bar{\sigma}_4 - \bar{\sigma}_7)(\sigma_2 - \bar{\sigma}_2) - (\sigma_5 + \sigma_6 - \bar{\sigma}_5 - \bar{\sigma}_6)(\sigma_1 - \sigma_2 + \sigma_3 - \bar{\sigma}_1 + \bar{\sigma}_2 - \bar{\sigma}_3) = \\ & = \frac{4}{9} (W_0^2 - \bar{W}_0^2) (2\sigma_2 - \sigma_1 - \sigma_3 - 2\bar{\sigma}_2 + \bar{\sigma}_1 + \bar{\sigma}_3) \end{aligned} \quad (3.8)$$

In principle, this equation furnishes information about the vector/axial vector structure of the isoscalar current. If the left-hand side of (3.8) is different from zero, the isoscalar neutral current cannot be pure vector or pure axial vector. Without any antineutrino data available, we shall not bother to discuss the feasibility of such a test.

3.2 Pure isovector neutral current

With $W_0 = \bar{W}_0 = 0$ we get immediately

$$\bar{\sigma}_4 = \bar{\sigma}_7 \quad (3.9)$$

$$\bar{\sigma}_5 = \bar{\sigma}_6 \quad (3.10)$$

in addition to Eqs. (2.49) and (2.50). We are then left with four potentially different cross-sections depending on two scalar products with one relation (3.6) among them. Therefore, there are three additional equations, one of which is a further consistency relation, i.e.,

$$2(\sigma_4 - \bar{\sigma}_4)(\sigma_2 - \bar{\sigma}_2) = (\sigma_5 - \bar{\sigma}_5)(\sigma_1 - \sigma_2 + \sigma_3 - \bar{\sigma}_1 + \bar{\sigma}_2 - \bar{\sigma}_3) \quad (3.11)$$

which is the analogue of (3.8) for a pure isovector current. The remaining two equations determine g_V, g_A

$$\begin{aligned} & 8(2\sigma_4 - \sigma_5) = (g_V + g_A)^2(\sigma_1 - 2\sigma_2 + \sigma_3) + (g_V - g_A)^2(\bar{\sigma}_1 - 2\bar{\sigma}_2 + \bar{\sigma}_3) + \\ & + (g_V^2 - g_A^2)(8\sigma_8 + 8\sigma_{11} - 4\sigma_9 - 4\sigma_{10} + 2\sigma_2 - \sigma_1 - \sigma_3 + 2\bar{\sigma}_2 - \bar{\sigma}_1 - \bar{\sigma}_3) \end{aligned} \quad (3.12a)$$

$$2(\sigma_5 - \bar{\sigma}_5) = g_V g_A (\sigma_2 - \bar{\sigma}_2) \quad (3.12b)$$

3.3 Weak isoscalar proportional to electromagnetic isoscalar

In this case we find three equations for g_V, g_A, z , one of which may be taken as Eq. (3.7a). The two other equations are

$$\sigma_5 + \sigma_6 - \bar{\sigma}_5 - \bar{\sigma}_6 = g_V g_A (\sigma_2 - \bar{\sigma}_2) \quad (3.13a)$$

$$\sigma_4 - \sigma_7 + \bar{\sigma}_4 - \bar{\sigma}_7 = 2g_V z (\sigma_8 - \sigma_{11}) \quad (3.13b)$$

In addition, two consistency relations for cross-sections may be derived, i.e.,

$$2(\sigma_4 + \sigma_7 - \bar{\sigma}_4 - \bar{\sigma}_7)(\sigma_2 - \bar{\sigma}_2) = (\sigma_5 + \sigma_6 - \bar{\sigma}_5 - \bar{\sigma}_6)(\sigma_1 - \sigma_2 + \sigma_3 - \bar{\sigma}_1 + \bar{\sigma}_2 - \bar{\sigma}_3) \quad (3.14)$$

$$(\sigma_4 - \sigma_7 + \bar{\sigma}_4 - \bar{\sigma}_7)(\sigma_9 - \sigma_{10}) = (\sigma_5 - \sigma_6 + \bar{\sigma}_5 - \bar{\sigma}_6)(\sigma_8 - \sigma_{11}) \quad (3.15)$$

As already shown, relation (3.14) follows from the absence of an isoscalar VA interference term. Equation (3.15) is a specific test for the assumption that the isoscalar weak current is proportional to the isoscalar electromagnetic current.

4. CONCLUSION

Our aim in this paper was to make the general expectation somewhat more quantitative that single pion production provides interesting information on the structure of the neutral current. The over-all conclusion from our work is that the neutral current coupling constants defined in the Lagrangian (2.2) are indeed strongly restricted, especially if both pion production and inclusive cross-sections are taken into account.

The main assumptions incorporated in the neutral current interaction (2.2) are:

- i) V, A currents;
- ii) $|\Delta I| \leq 1$;
- iii) the isovector neutral currents are in the same isotriplets as the corresponding charged currents.

These assumptions allow for the most general neutral current that can be constructed out of valence quarks.

In the main part of this paper, bounds for the neutral current couplings were derived using the presently existing neutrino data on single pion production off single nucleons only. From inclusive and pion production cross-sections together, we may draw the following conclusions which are still tentative in view of the large errors for the exclusive cross-sections:

- i) irrespective of the structure of the isoscalar neutral current, the isovector neutral current cannot be pure vector, whereas a dominant axial isovector current is preferred;
- ii) for a pure isovector current, a unique solution for g_V, g_A is obtained resolving the VA ambiguity of inclusive scattering;
- iii) considering models where the only isoscalar current is the electromagnetic one, we find the Salam-Weinberg model compatible with the data on single pion production for $\sin^2 \theta_W \gtrsim 0.2$. Models where the isovector axial coupling differs significantly from the Salam-Weinberg value are more likely to be in difficulties. In terms of gauge theory language, our bounds may be interpreted to restrict the weak isospin assignments for the right-handed valence quark fields.

In the second part of this paper we anticipated the existence of both ν and $\bar{\nu}$ data on single pion production. Our main result in this case is that g_V and g_A are uniquely determined by cross-sections, except for the usual quadratic ambiguities. Tests are presented to check on additional assumptions such as a pure isovector neutral current, or absence of any isoscalar except the electromagnetic isoscalar current.

Most experiments on single pion production have been performed on complex nuclei. The present analysis does not directly apply to those experiments. However, a similar investigation which is not troubled by the uncertainties of nuclear physics is possible for scattering on complex nuclei and will be presented elsewhere.

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APPENDIX A

The cross-sections for single pion production by weak and electromagnetic currents and their relations to the amplitudes defined in Sections 2 and 3 are

$$\sigma_1 \equiv \int d\Omega |\mathcal{M}_1|^2 = 2 \int d\Omega |\mathcal{M}_1|^2 \quad (\text{A.1a})$$

$$\sigma_2 \equiv \int d\Omega |\mathcal{M}_2|^2 = \frac{4}{9} \int d\Omega |\mathcal{M}_2|^2 \quad (\text{A.1b})$$

$$\sigma_3 \equiv \int d\Omega |\mathcal{M}_3|^2 = \frac{2}{9} \int d\Omega |\mathcal{M}_3|^2 \quad (\text{A.1c})$$

$$\bar{\sigma}_1 \equiv \int d\Omega |\bar{\mathcal{M}}_1|^2 = 2 \int d\Omega |\bar{\mathcal{M}}_1|^2 \quad (\text{A.2a})$$

$$\bar{\sigma}_2 \equiv \int d\Omega |\bar{\mathcal{M}}_2|^2 = \frac{4}{9} \int d\Omega |\bar{\mathcal{M}}_2|^2 \quad (\text{A.2b})$$

$$\bar{\sigma}_3 \equiv \int d\Omega |\bar{\mathcal{M}}_3|^2 = \frac{2}{9} \int d\Omega |\bar{\mathcal{M}}_3|^2 \quad (\text{A.2c})$$

$$\sigma_4 \equiv \int d\Omega |\mathcal{M}_4|^2 = \frac{1}{9} \int d\Omega |\mathcal{M}_4|^2 \quad (\text{A.3a})$$

$$\sigma_5 \equiv \int d\Omega |\mathcal{M}_5|^2 = \frac{2}{9} \int d\Omega |\mathcal{M}_5|^2 \quad (\text{A.3b})$$

$$\sigma_6 \equiv \int d\Omega |\mathcal{M}_6|^2 = \frac{2}{9} \int d\Omega |\mathcal{M}_6|^2 \quad (\text{A.3c})$$

$$\sigma_7 \equiv \int d\Omega |\mathcal{M}_7|^2 = \frac{1}{9} \int d\Omega |\mathcal{M}_7|^2 \quad (\text{A.3d})$$

$$\overline{\sigma}_4 \equiv \int d\Omega (\bar{\nu} p \rightarrow \bar{\nu} p \pi^0) = \frac{1}{9} \int d\Omega |\bar{M}_4|^2 \quad (\text{A.4a})$$

$$\overline{\sigma}_5 \equiv \int d\Omega (\bar{\nu} p \rightarrow \bar{\nu} n \pi^+) = \frac{2}{9} \int d\Omega |\bar{M}_5|^2 \quad (\text{A.4b})$$

$$\overline{\sigma}_6 \equiv \int d\Omega (\bar{\nu} n \rightarrow \bar{\nu} p \pi^-) = \frac{2}{9} \int d\Omega |\bar{M}_6|^2 \quad (\text{A.4c})$$

$$\overline{\sigma}_7 \equiv \int d\Omega (\bar{\nu} n \rightarrow \bar{\nu} n \pi^0) = \frac{1}{9} \int d\Omega |\bar{M}_7|^2 \quad (\text{A.4d})$$

$$\sigma_8 \equiv \frac{G^2}{8\pi^2\alpha^2} \int Q^4 d\sigma (\bar{e} p \rightarrow \bar{e} p \pi^0) = \frac{1}{9} \int d\Omega |M_8|^2 \quad (\text{A.5a})$$

$$\sigma_9 \equiv \frac{G^2}{8\pi^2\alpha^2} \int Q^4 d\sigma (\bar{e} p \rightarrow \bar{e} n \pi^+) = \frac{2}{9} \int d\Omega |M_9|^2 \quad (\text{A.5b})$$

$$\sigma_{10} \equiv \frac{G^2}{8\pi^2\alpha^2} \int Q^4 d\sigma (\bar{e} n \rightarrow \bar{e} p \pi^-) = \frac{2}{9} \int d\Omega |M_{10}|^2 \quad (\text{A.5c})$$

$$\sigma_{11} \equiv \frac{G^2}{8\pi^2\alpha^2} \int Q^4 d\sigma (\bar{e} n \rightarrow \bar{e} n \pi^0) = \frac{1}{9} \int d\Omega |M_{11}|^2 \quad (\text{A.5d})$$

q is the momentum carried by the currents and $Q^4 = (-q^2)^2$.

Throughout the paper an abbreviated notation is used where M^2 stands for the square of the norm in H_Ω , i.e., $M^2 \equiv ||M||_\Omega^2 = \int d\Omega |M|^2$.

APPENDIX B

We briefly discuss the problem of the optimal bound for the scalar product d defined in (2.28). We may employ geometrical notions because all scalar products involved are real by definition.

The bound (2.29) is optimal only if $M_9 + M_{10}$ is parallel to M_2 and $M_8 + M_{11}$ is antiparallel to M_0 at the same time, or vice versa. However, because of the isospin relation (2.11) and the given angle ²⁵⁾ between M_0 and M_2 , this will not be possible in general. Consider the extreme case, where, e.g., $|M_8| = |M_9| + |M_{10}| + |M_{11}|$ so that all electromagnetic amplitudes are collinear; unless M_0 and M_2 are also collinear, the bound (2.29) cannot be attained in this case.

On the other hand, if $|M_8| = |M_{11}|$, $|M_9| = |M_{10}|$, one can always draw a parallelogram with the electromagnetic amplitudes so that $M_8 + M_{11}$ is antiparallel to M_0 and $M_9 + M_{10}$ is parallel to M_2 , or vice versa, no matter what the angle between M_0 and M_2 is. Since this is nearly the case for the electromagnetic cross-sections, it is clear that the optimal bound for d will not differ much from the bound given in (2.29). A numerical calculation using the cross-sections of the Table shows that $d_{\max}$ is only about 2% smaller than the trivial bound.

In subsection 2.3 we use an inequality involving the scalar products of three vectors M_0, M_8, M_{11} .

Proof:

Define two vectors X, Y by

$$M_8 = \frac{u}{2M_0^2} M_0 + X \quad (C.1)$$

$$M_{11} = \frac{v}{2M_0^2} M_0 + Y \quad (C.2)$$

Using the definitions (2.62) for u, v we find

$$X^2 = M_8^2 - \frac{u^2}{4M_0^2} \quad (C.3)$$

$$Y^2 = M_{11}^2 - \frac{v^2}{4M_0^2} \quad (C.4)$$

Moreover,

$$w = \frac{uv}{2M_0^2} + (X, Y) + (Y, X) \quad (C.5)$$

Applying the Cauchy inequality to $(X, Y) + c.c.$, one obtains

$$\left| w - \frac{uv}{2M_0^2} \right| \leq 2 \left(M_8^2 - \frac{u^2}{4M_0^2} \right)^{1/2} \left(M_{11}^2 - \frac{v^2}{4M_0^2} \right)^{1/2} \quad (C.6)$$

which is equivalent to the inequality (2.64). The proof shows that the inequality turns into an equality if the three vectors M_0, M_8, M_{11} lie in a plane.

It is easily shown that (2.63) implies that $4M_0^2 M_8^2 - u^2$, $4M_0^2 M_{11}^2 - v^2$ and $4M_8^2 M_{11}^2 - w^2$ have the same sign. Since we only consider values for w such that $w^2 \leq 4M_8^2 M_{11}^2$ the inequality (2.63) ensures the Cauchy inequalities for u and v .

Table

Data 6),7) for charged and neutral current pion production
and estimated values for electromagnetic cross - sections
as defined in Appendix A

σ_2/σ_1	σ_3/σ_1	σ_4/σ_1	σ_5/σ_1	σ_6/σ_3
0.37 ± 0.10	0.37 ± 0.09	0.40 ± 0.22	0.13 ± 0.06	0.38 ± 0.11

σ_8/σ_1	σ_4/σ_1	σ_{10}/σ_1	σ_{11}/σ_1
0.09	0.07	0.06	0.10

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FIGURE CAPTIONS

- Fig. 1 : Constraints from single pion production for g_V, g_A with an arbitrary isoscalar admixture in the neutral current. Numerical input is taken from the Table (errors not shown). The hatched regions are forbidden.
- Fig. 2 : Constraints ⁴⁾ from inclusive scattering for g_V, g_A with an arbitrary isoscalar admixture using ¹⁸⁾ $R = 0.25, \bar{R} = 0.39, r = 0.37$. Allowed points lie inside the common interior of the two ellipses.
- Fig. 3 : Restrictions on g_V, g_A from both inclusive scattering and single pion production. Allowed regions are cross-hatched.
- Fig. 4 : Constraints for g_V, g_A from single pion production for a purely isovector neutral current. Allowed points lie outside the hatched domains.
- Fig. 5 : Constraints from single pion production in the case where the weak isoscalar is proportional to the electromagnetic isoscalar current for $z = -1$. Allowed points lie outside the hatched regions.
- Fig. 6 : As in Fig. 5, but for $g_A = 1$. The straight line in the centre defines the Salam-Weinberg model ⁹⁾: $g_V = 1 + z = 1 - 2 \sin^2 \theta_W$.
- Fig. 7 : As in Fig. 6, but for $g_A = 0.5$. The straight line in the centre defines one version of the Gürsey-Sikivie model ²⁴⁾:
 $g_V = z = \frac{3}{2} - 2 \sin^2 \theta_W$.

FIG.1

FIG. 2

FIG. 3

FIG.4

FIG. 5

FIG. 6

FIG. 7