CM-P00061610

Archives

Ref.TH.2104-CERN

MULTIPLICITY DISTRIBUTIONS IN INELASTIC REACTIONS ON NUCLEI

L. Caneschi and A. Schwimmer *)

CERN - Geneva

and

R. Jengo

Istituto di Fisica dell'Università, Trieste

ABSTRACT

We compute the multiplicity distribution of the number of knocked-out nucleons, and the correlation of the former with the multiplicity of the produced mesons, in inelastic particle nucleus scattering.

*) On leave of absence from the Weizmann Institute, Rehovot, Israel.

1. INTRODUCTION

The problem of consistently handling a Regge singularity (Pomeron) with intercept $\alpha(0) \simeq 1$ is one of the most challenging in the theory of high-energy hadronic reactions.

A promising approach is provided by the non-perturbative treatment of the Gribov-Reggeon calculus ¹⁾⁻⁵⁾. Unfortunately, the phenomenological value λ of the Pomeron interaction seems to be so small that for present energies a perturbative treatment (including up to second order term) is likely to provide the most sensible approach to the problem of particle-particle scattering ⁶⁾. A possibility of observing non-perturbative effects is, however, provided by the study of the interactions with heavy nuclei ^{4),7),8)}.

In the (admittedly crude) approximation of considering the nucleus as a collection of independent nucleons acting coherently, the effective coupling of the Pomeron to the nucleus is enhanced by a factor $A^{1/3}$ with respect to the Pomeron particle coupling g ⁴⁾. It is therefore possible to achieve a situation in which the product $gA^{1/3}\lambda$ is not small, and non-perturbative effects emerge. Since loop corrections are not enhanced by the $A^{1/3}$ factor, one is led to study the tree approximation.

In this paper we complete the analysis of Ref. 4) with a study of the distribution of the number of knocked-out nucleons (KON) that constitute an important phenomenological parameter to classify the inelastic events.

An important point of terminology is in order here. What we call KON are in fact the nucleons that have a high-energy multiparticle interaction with the incoming projectile. It is safe to guess that in the rest frame of the nucleus such nucleons will have an average energy of the order of 300 MeV, which is very large by the nuclear scale. Following the primary high-energy interaction, low-energy processes within the nucleus should operate a re-sharing of this energy among several nucleons, many of which can receive more than their binding energy. Therefore the number of actually observed evaporation nucleons is, we believe, proportional to our number of KON, but with a coefficient larger than unity. A possible way of measuring our number of KON would be to impose a minimum energy cut-off on the evaporated nucleons.

2. METHOD

We will consider here a Pomeron of intercept $\alpha(0) = 1 + \mu > 1$ in interaction with itself via a trilinear coupling. The Lagrangian that we will study is:

$$\mathcal{L}(\phi) = -\frac{i}{2} \phi^+ \overleftrightarrow{\frac{\partial}{\partial y}} \phi + \mu \phi^+ \phi - \frac{\lambda}{2} (\phi^+ + \phi) \phi^+ \phi \quad (1)$$

We have dropped the shrinkage term since the ratio of α' to the nuclear radius R is negligible in the range of reactions that we want to consider.

In the tree approximation, the partial wave amplitude T can be obtained from the field ϕ that solves the classical equation of motion ^{4),9)}:

$$\phi = \frac{\partial \phi}{\partial y} = \mu \phi - \frac{\lambda}{2} \phi^2 \quad (y = \ln s)$$

with the initial condition

$$\phi(0, \vec{b}) = g \frac{A^{1/3}}{r_0^2} \Theta(R - |\vec{b}|) \equiv G \quad (2)$$

where g is the Pomeron particle coupling, A the atomic number, and $R = r_0 A^{1/3}$ the nuclear radius.

From Eq. (2) we obtain

$$\phi(y, \vec{b}) = G e^{\mu y} \left(1 + G \frac{\lambda}{2\mu} (e^{\mu y} - 1) \right)^{-1} \quad (3)$$

The exact relation between ϕ and T is somehow model-dependent. In the limit of small g or small ϕ , T is simply given by $g\phi$, but higher powers are essential to enforce elastic unitarity when $g\phi$ is not small and a reasonable choice is ¹⁰⁾:

$$T = 1 - e^{-g\phi} \quad (4)$$

In the following we will use $T \simeq g\phi$, since none of our results are qualitatively affected by this simplification.

A given tree diagram (e.g., the one of Fig. 1) corresponds to many s channel discontinuities that can be computed according to the generalization of the AGK cutting rules ¹¹⁾ obtained in the framework of a Dyson equation ¹²⁾, or in a three-Pomeron field Lagrangian formulation ^{13),14)}. Here we will follow the latter approach. Since for our purposes the distinction between the "positive" and "negative" field is not needed, we use the Lagrangian

$$\mathcal{L} = \mathcal{L}(\phi) + \mathcal{L}_0(\phi_c) - 2\lambda_A \phi_c^+ \phi_c (\phi + \phi^+) + \\ + \frac{\lambda_D}{2} (\phi_c^+ \phi^2 + \phi^{+2} \phi_c) + \lambda_I \phi_c^+ \phi_c (\phi_c^+ + \phi_c)$$

where

$$\mathcal{L}_0(\phi_c) = -\frac{1}{2} \phi_c^+ \frac{\overleftrightarrow{\partial}}{\partial y} \phi_c + (z + \mu) \phi_c^+ \phi_c \quad (5)$$

and $\lambda = \lambda_A = \lambda_D = \lambda_I > 0$ corresponds to the AGK cancellation rules. Our aim is to compute the generating function $Q(z, J)$ defined as follows

$$Q(z, J) = \frac{1}{\sigma_{Tst}} \sum_{n, N} \sigma_{nN} (1+z)^n J^N \quad (6)$$

where σ_{nN} is the cross-section for producing n mesons and knocking out N nucleons.

In order to have a KON in the final state, the cut-Reggeon field should interact with it. So we will identify the number N with the number of cut fields that interact with the nucleus, and therefore all the dependence on the parameter J can be injected in the normalization condition of the cut field ϕ_c :

$$\phi_c(0, \hat{b}) = J G \quad (7)$$

To obtain the multiplicity of produced mesons we follow the prescription of Ref. 12) and consider the mass of the cut-Pomeron field linearly dependent on z (for simplicity with coefficient 1):

$$\mu_c(z) = \mu + z \quad (8)$$

As usual the p^{th} moment of the meson multiplicity is obtained by taking the p^{th} derivative of the scattering amplitude at $z=0$ ¹²⁾. Using the Lagrangian of Refs. 13) and 14), the classical equation of motion (that describes the tree approximation) for ϕ_c is

$$\phi_c = (\mu + z) \phi_c - \frac{\lambda}{2} (4 \phi_c \phi - 2 \phi_c^2 - \phi^2) \quad (9)$$

corresponding to the well-known -4,2,1 relative weights of the three leading discontinuities of the two-Pomeron cut. Introducing now ^{*)} $\Delta = \phi_c - \phi$:

$$\Delta = (\mu + z) \Delta + z \phi + \lambda \Delta^2 \quad (10)$$

with

$$\Delta(0, \vec{b}) = G(J-1) \quad (11)$$

Here ϕ represents the uncut-Pomeron field and is given explicitly by Eq. (3).

3. CALCULATIONS: $z=0$

The general solution of (10) is not easy to obtain. Let us first concentrate on the distribution of the number of knocked-out nucleons N , summing over the multiplicity of the produced meson n . To obtain this we set $z=0$ in (10) and obtain the explicit solution $\Delta_0(y; \vec{b})$

$$\Delta_0(y, \vec{b}) = \frac{\mu}{\lambda} \left(\frac{1}{1 - \frac{G(J-1) e^{\mu y}}{\mu/\lambda + G(J-1)}} - 1 \right) \quad (12)$$

Therefore the generating function for the distribution of the KON is in the present approximation:

$$Q(z=0, J) = 1 + \frac{1}{2} \left(\frac{1}{1 - \frac{G(J-1) e^{-\mu y}}{\mu/\lambda + G(J-1)}} - 1 \right) \quad (13)$$

*) In the language of Ref. 13), Δ corresponds to the field ϕ_0 which operates the partial diagonalization of the three-Pomeron Lagrangian.

and corresponds to the following asymptotic behaviour of σ_N (N fixed, $y \rightarrow \infty$), $\sigma_0 \rightarrow \text{const} = \frac{1}{2}\sigma_{\text{tot}}$

$$\sigma_N = \frac{g G \left(\frac{\mu}{\lambda}\right)^2 e^{-\mu y} [G(e^{-\mu y} - 1)]^{n-1}}{[\frac{\mu}{\lambda} + G(e^{-\mu y} - 1)]^{n+1}} \rightarrow \frac{\sigma_{\text{tot}}}{2} \left(\frac{\mu}{\lambda}\right) e^{-\mu y} \left(g \frac{A^{1/3}}{z_0^2}\right)^{-1} \quad (14)$$

Taking the derivatives of (13) at $J=1$, we obtain the moments:

$$\begin{aligned} \langle N(N-1) \dots (N-p) \rangle &= \\ &= \frac{1}{2} \left(\frac{\lambda}{\mu}\right)^p p! e^{p\mu y} \left(\frac{g A^{1/3}}{z_0^2}\right)^p \end{aligned} \quad (15)$$

Some remarkable features of the results thus obtained are:

- a) Only σ_0 approaches a constant for large y , and in fact it approaches $\frac{1}{2}\sigma_{\text{tot}}$. This feature does not correspond to a saturation of elastic unitarity; remember that on the one hand the partial waves in our model approach a limit that is smaller than 1 (grey disk), and on the other hand σ_0 contains the elastic, as well as many different inelastic, channels (e.g., the ones of Figs. 2a and 2f).
- b) All the other cross-sections vanish asymptotically as s^{-u} , i.e., with a power that is higher, the higher the Pomeron intercept is above 1. This feature has a simple explanation, in terms of a quasi-Goldstone-like phenomenon, which will be discussed in Section 5.
- c) Asymptotically all σ_N are equal and satisfy a KNO-like scaling¹⁵⁾ relation of the form

$$\sigma_N = \frac{\sigma_{\text{tot}}}{2 \langle N \rangle} e^{-N} \quad (16)$$

- d) The dependence of $\sigma_N/\sigma_{\text{tot}}$ on $A^{1/3}$ is very simple: all $\sigma_N/\sigma_{\text{tot}}$ decrease with increasing A , like $A^{-1/3}$. This fact corresponds to the intuitive expectation that $\langle N \rangle$ increases linearly with $A^{1/3}$ [and the higher moments p increase like $A^{p/3}$, see Eq. (15)] with

a small coefficient that turns out to be of order σ_{DD}/σ_T . (We would remind the reader, however, of our definition of KON given in the Introduction.)

4. CALCULATIONS: $z \neq 0$

We now want to study the properties of the meson production and its correlation with N . To do this, we have to solve (10) with $z \neq 0$. This is easily done if we choose for G the specific value $2\mu/\lambda$, such that ϕ does not depend any more on y [see Eq. (3)]. With this choice we can no longer study the dependence on A . However, in this model the dependence on A is confined to a finite region of rapidity in the nucleus fragmentation and affects only the distribution studied in the previous section and, at most, a constant term in the meson multiplicity. With this choice, Eq. (10) can again be solved:

$$\Delta(z, J) = \frac{2a}{1 - \frac{(b-a) + G(J-1)}{a+b + G(J-1)} e^{2a\lambda y}} - (a+b) \quad (17)$$

with

$$a = \frac{1}{2\lambda} \sqrt{z^2 - 6\mu z + \mu^2}$$

$$b = \frac{z + \mu}{2\lambda} \quad (18)$$

The average multiplicity is given by

$$\langle n \rangle = \frac{1}{J_{T,t}} \frac{d}{dz} \Delta \Big|_{\substack{z=0 \\ J=1}} = \frac{e^{\mu y} - 1}{\mu} \quad (19)$$

and shows the features that were expected because of the AGK cancellations¹¹⁾ (power behaviour s^μ that turns into a $\ln s$ behaviour when $\mu \rightarrow 0$).

However, if we compute the average multiplicity in events with a given number N of KON, we obtain

$$\langle n \rangle_{N=0} = \frac{\lambda^2}{\mu} ; \quad \langle n \rangle_{N \neq 0} = 3 \ln s \quad (20)$$

independently of N . Also for any given N the generating function of the n distribution is of the typical short-range correlation type:

$$\frac{1}{\sigma_N} \frac{1}{N!} \frac{d^N}{dJ^N} \Phi(z, J) \Big|_{J=0} = s^{\mu + \alpha(z)} \quad (21)$$

with $-\alpha(z) = 2\lambda a$ of Eq. (18), independently of N .

5. A DIFFERENT METHOD

The results obtained so far can look somewhat mysterious, but have, in fact, a quite natural justification which we will discuss here. Let us first consider the cut-Reggeon propagator of mass $\mu + z$ in interaction with an arbitrary number of uncut fields, and sum the graphs of Fig. 3 to obtain a "partially dressed" cut field propagator (Fig. 4). The mass of this field is easily computed to be:

$$\mu_D = \mu + z - 2\lambda \frac{2\mu}{\lambda} = -3\mu + z$$

In order to compute, e.g., the $N=1$ cross-section, we have to consider the propagator μ_D (modified, however, by its interactions of the form of Fig. 5) that we will symbolically represent as in Fig. 6. This corresponds therefore again to a geometrical series and to a simple mass shift

$$\mu_D \rightarrow \mu_F = \mu_D + 2\lambda C \quad (22)$$

[In Eq. (22) the coupling constant is enhanced by a factor 2 with respect to the usual polyperipheral coupling, because at each vertex either one of the two cut fields can propagate as such.]

The value of C is the solution of the equation ¹⁶⁾:

$$\frac{\partial \mathcal{L}}{\partial \phi^+} = -J$$

that interpolates to 0 when $J \rightarrow 0$, taken at the value

$$J = \left(\frac{2\mu}{\lambda} \right)^2 \frac{\lambda}{2} = \frac{2\mu^2}{\lambda}$$

i.e.

$$C = \frac{1}{2\lambda} \left(3\mu - z - \sqrt{\mu^2 - 6\mu z + z^2} \right)$$

and inserting in (22), we obtain

$$\mu_F = - \sqrt{\mu^2 - 6\mu z + z^2} \equiv \alpha(z)$$

of Section 4.

In this way all our results become very transparent. For $M=0$ the dominant term is depicted in Fig. 7a. Since the propagator ϕ_F corresponds to a negative power $\alpha(z)$, only σ_0 goes to a constant (and $\langle n \rangle$ does the same with a coefficient $\sim 1/\mu$). All the other cross-sections behave like $s^{+\alpha(0)}$ and the multiplicity is logarithmic and equal for all M , since for $M \neq 1$ the large rapidity gap will be filled by only one complete propagator ϕ_F . Obviously, also the coefficient of $\ln s$ is recovered taking the derivative of $\alpha(z)$ at $z=0$.

6. A MORE GENERAL APPROACH

For the moment we have worked with the very specific Lagrangian (1). One may wonder how dependent are our results on this specific choice, i.e., on the trilinear nature of the interaction and on the relative weights of the vertices [and, in particular, on the validity of the cancellation rules ¹¹⁾].

Let us now substitute Eq. (10) with a general form

$$\Delta = \beta(z, \Delta) \quad (23)$$

and show that all our qualitative results follow simply from the fact that for small z , β has a zero at $\Delta = 0$ and at least one more for $\Delta = \bar{\Delta} \neq 0$ ($\bar{\Delta} = -\mu/\lambda$ in our example), and that the former zero is repulsive

$$\left. \frac{\partial \beta}{\partial \Delta} \right|_{\Delta=0} > 0$$

and the latter is attractive

$$\left. \frac{\partial \beta}{\partial \Delta} \right|_{\Delta=\bar{\Delta}} \equiv \beta' < 0 \quad ; \quad \beta' = -\mu$$

in our case). In fact, under these conditions, for $J \neq 1$

$$\lim_{y \rightarrow \infty} (\Delta - \bar{\Delta}) \simeq e^{\beta' y}$$

Integrating (23), we obtain

$$t = \int_{G(J-1)}^{\Delta(t)} \frac{dx}{\beta(x)} \quad (24)$$

where $\beta(x)$ depends on z but not on J .

Differentiating (24) with respect to J at $J=0$:

$$\beta(-G) \Delta_1(y) = G \beta(\Delta(y)) \quad (25)$$

where

$$\Delta_N(y) = \left. \frac{d^N}{dJ^N} \Delta(y) \right|_{J=0} = N! \tau_N(y) \quad (26)$$

Since $\beta[\Delta(y)] \simeq s^{\beta'}$, and $\beta(-G)$ is (barring accidents) non-vanishing, $\Delta_1(y)$ also behaves like $s^{\beta'}$. Taking further derivatives of (24) with respect to J at $J=0$, we reach the conclusion that all Δ_N behave like $s^{\beta'}$ and that the following recursion relation holds (up to terms of order $s^{2\beta'}$)

$$\sum_{k=0}^{N-1} \binom{N-1}{k} \Delta_{N-k} \beta_k = G \beta' \Delta_{N-1} \quad (27)$$

with

$$\beta_k = \frac{d^k}{dJ^k} \beta(\Delta(u)) \Big|_{J=0}$$

In the case of Eq. (10): $\beta_0 = -G\mu + \lambda G^2$, $\beta_1 = -G\mu - 2\lambda G^2$, $\beta_2 = 2\lambda G^2$, $\beta_i = 0$, $i \geq 3$, hence (27) yields

$$\begin{aligned} N \sigma_N (-G\mu + \lambda G^2) + (N-1) \sigma_{N-1} (G\mu - 2\lambda G^2) + \\ + (N-2) \lambda G^2 \sigma_{N-2} = -G\mu \sigma_{N-1} \end{aligned} \quad (28)$$

whence we see again that all σ are equal asymptotically [but this is obviously a special feature of (10)].

Let us now consider the multiplicity of the produced particles, and define

$$\Omega^n = \frac{d^n}{dz^n} \Delta \Big|_{z=0} \Big|_{J=1} \quad (29)$$

and

$$\Omega_N^n = \frac{1}{N!} \frac{d^N}{dJ^N} \frac{d^n}{dz^n} \Delta \Big|_{z=0} \Big|_{J=0} \quad (30)$$

and assume for $\beta(z, \Delta)$ a general dependence of the type

$$\beta = (u+z) \Delta + c z + P(\Delta) \quad (31)$$

where $P(\Delta)$ is a polynomial that starts with a quadratic term.

Taking the derivative of (23), with respect to z , we obtain

$$\dot{\Omega}^{(1)} = \Delta + c + \Omega^{(1)} \beta'(\Delta) \quad (32)$$

The behaviour of the total multiplicity is determined by the point $\Delta = 0$ (since $J=0$), and $\langle n \rangle \simeq s^{\beta'(0)}$ since $\beta'(0)$ is, by assumption, positive. The behaviour of the higher moments $\Omega^{(n)}$ can also be determined taking derivatives of (32), and turns out to depend on whether the coefficients

$$\beta^{(n)}(0) = \left. \frac{d^n}{d\Delta^n} \beta(\Delta) \right|_{\Delta=0}$$

are not vanishing.

For the multiplicity associated with zero KON, we have again Eq. (32), but this time asymptotically evaluated at $\Delta = \bar{\Delta}$ where $\beta'(0) = \beta' < 0$. Hence

$$\dot{\Omega}^{(1)} \rightarrow \frac{\bar{\Delta} + c}{-\beta'} = \text{const}$$

The multiplicity of the mesons produced together with N nucleons can be obtained by taking derivatives of (32), always at $\Delta = \bar{\Delta}$. Hence in all cases the homogeneous part of the equation reads

$$\dot{\Omega}_N^{(1)} = \beta' \Omega_N^{(1)}$$

which would suggest $\Omega_N \simeq e^{\beta' y}$. However, since the non-homogeneous part behaves also in the same way, the solution is generally of the form $\Omega_N = C_N y e^{\beta' y}$, and since σ_N also behaves like $e^{\beta' y}$, $\langle n \rangle_N \simeq \ln s$. For instance, the first equation is

$$\dot{\Omega}_1^{(1)} = \Delta_1 + \beta' \Omega_1 + \beta'' \Omega_2 \Delta_1 \quad (33)$$

whence $\langle n \rangle_1 = a \ln s$, with $a = (1 + \beta'' \Omega_2^{(1)})$.

Taking further derivatives of (33) with respect to J , and neglecting terms that behave like $s^{2\beta'}$ $\ln s$, we obtain

$$\mathcal{R}_N^{(1)} = \mu \Delta_N + \beta' \mathcal{R}_N + \beta'' \mathcal{R}_0^{(1)} \Delta_N$$

from which we recover generally the feature that $\langle n \rangle / \ln s$ is N independent asymptotically.

7. CONCLUSIONS

Let us summarize our results as follows.

- 1) The cross-section for having a given number of KON in the final state decreases with s and A like $\sigma_N / \sigma_{\text{tot}} \approx s^{-\mu} A^{-1/3}$, with $\mu = \alpha(0) - 1$.
- 2) The multiplicity moments of the number of KON behave like powers of s and of $A^{1/3}$

$$\langle N^P \rangle \approx A^{P/3} s^{\mu P}$$

- 3) The multiplicity of the produced particles associated with a given number N of KON increases only logarithmically with s , whereas the total multiplicity is power-behaved in our example. The coefficient of the $\ln s$ term is different from the coefficient of $\ln s$ in the multiplicity in particle-particle scattering, and is independent of the number of KON N and of the atomic number A . Of course, an important N and A dependence is expected in $\langle n \rangle$, but this should be limited to terms that do not increase with energy as soon as s is larger than some $\bar{s}$ (this can in turn depend on A).
- 4) All these features do not depend on the details of the Lagrangian that we chose, and in particular, on the validity of the cancellation rules, but only on the existence of two fixed points (one repulsive at $\Delta = 0$, and one attractive at some $\Delta \neq 0$).

REFERENCES

- 1) V.N. Gribov and A.A. Migdal, Soviet Physics, JETP 28, 784 (1969).
- 2) A.A. Migdal, A.M. Poliakoff and K.A. Ter Martirosyan, Phys. Letters 48B, 239 (1974).
- 3) H. Abarbanel and J.B. Bronzan, Phys. Letters 48B, 354 (1974).
- 4) A. Schwimmer, Nuclear Phys. B94, 445 (1975).
- 5) D. Amati, L. Caneschi and R. Jengo, Nuclear Phys. (in press).
- 6) D. Amati and R. Jengo, Phys. Letters 56B, 465 (1975);
W. Frazer and M. Moshe, Phys. Letters 57B, 475 (1975).
- 7) O.V. Kancheli, Soviet Physics, JETP 18, 273 (1973).
- 8) E.S. Lehman and G.A. Winbow, Phys. Rev. D10, 1962 (1974).
- 9) Y. Nambu, Phys. Letters 26B, 626 (1968).
- 10) L. Caneschi and A. Schwimmer, Nuclear Phys. (in press).
- 11) V.A. Abramovskii, V.N. Gribov and O.V. Kancheli, Proceedings XVI
International Conference on High-Energy Physics, Batavia,
Vol. 1, p. 389 (1972).
- 12) L. Caneschi and R. Jengo, Nuclear Phys. B89, 19 (1975).
- 13) M. Ciafaloni, G. Marchesini and G. Veneziano, Nuclear Phys. B98,
493 (1975).
- 14) P. Suranyi, Phys. Rev. D (in press).
- 15) Z. Koba, H. Nielsen and P. Olesen, Nuclear Phys. B40, 317 (1972).
- 16) S. Coleman, "Secret Symmetry", Lectures given at the XI School on
Subnuclear Physics, Erice (1973), A. Zichichi, Editor (in press).

* * *

FIGURE CAPTIONS

- Fig. 1 : A tree diagram.
- Fig. 2 : The s channel discontinuities of the diagram of Fig. 1.
- Fig. 3 : A contribution to σ_1 .
- Fig. 4 : Definition of φ_D .
- Fig. 5 : Changing φ_D into the final φ_F .
- Fig. 6 : A two-step summation of the diagrams of Fig. 5.
- Fig. 7 : The leading contributions to (a) $N = 0$, (b) $N = 1$.

FIG.1

FIG. 2

FIG.3

FIG.4

FIG.5

FIG.6

a)

b)

FIG. 7