CM-P00061616

Ref.TH.2123-CERN

Archives

INELASTIC DIFFRACTION IN THE QUARK-GLUE MODEL

L. Van Hove and K. Fiałkowski *)

CERN - Geneva

A B S T R A C T

Diffractional dissociation of hadrons is related to the quark-gluon picture of their structure. It is shown that the quark-gluon model can produce sufficiently large variations of opacities for diffractional eigenstates to account for the experimental value of diffractional cross-sections if unitarity is saturated in central gluon-gluon collisions. This requirement simplifies the collision picture and provides a natural explanation for the observed energy independence of proton opacity in central pp collisions.

*) On leave from and address after 1 February 1976 : Institute of Physics, Jagellonian University, 30-059 Kraków, Reymonta 4, Poland.

1. - INTRODUCTION

In the quark-gluon model ¹⁾⁻³⁾ the interaction properties of hadrons are to a large extent determined by their internal structure. Thus ^{2),3)} it is possible to relate non-diffractive hadron-hadron collisions and deep inelastic lepton-hadron collisions as being dependent on the same structure properties. In this note we extend the application of the model to diffractive processes.

As was shown recently ⁴⁾, the experimental estimates for the inelastic diffractive cross-sections are large and therefore difficult to explain. If elastic and inelastic diffraction is the shadow of non-diffractive production, and if all the relevant amplitudes are taken to be fully absorptive, the data indicate extremely strong variations in the opacity spectrum of eigenamplitudes contributing to the diffraction. This is incompatible with the standard quark picture, in which all the contributing eigenstates should have opacities of the same order, as they correspond to the same configuration of valence quarks. In the quark-gluon model, however, the opacity variations may be large since they are related to the corresponding variations of the gluon parameters in the incident hadrons. This is discussed in detail in this note. We obtain quantitative compatibility between the model and the ISR data on pp collisions. We are led to conclude that the unitarity limit is saturated in central gluon-gluon collisions, the lack of saturation in pp collisions being due to the transversal distribution of the gluon inside the protons. The parameter choice determined by the compatibility requirement produces, in turn, a simple explanation for the observed very slow energy dependence of the elastic pp amplitude at zero impact parameter.

The paper is organized as follows : in the next section we present the basic features of the quark-gluon picture and formulate our problem. The detailed assumptions used for our treatment of diffraction are summarized in the third section. The fourth section presents the results of our analysis. The discussion of possible modifications of our detailed assumptions and a brief conclusion are given in the last two sections.

2. - THE QUARK-GLUE MODEL : BASIC ASSUMPTIONS AND FORMULAE

In the quark-glue picture ¹⁾⁻³⁾ hadrons are built of valence quarks and glue. The former are responsible for the internal quantum numbers [meson-baryon distinction, SU(3) classification], the latter for the high energy non-diffractive hadron-hadron collisions in which valence quarks just fly through whereas the glue interacts and is emitted in the central region of rapidity. Obviously, these two components are not independent : e.g., to account for the successes of quark model relations for total cross-sections (as the 2/3 meson-baryon ratio) one should assume a dependence of the "glue density" on the number and properties of valence quarks. This is, however, unimportant for many applications including the present one. Also the presence of quark-antiquark pairs can be neglected.

Since the diffractive processes (elastic as well as inelastic) are usually interpreted as shadow effects of non-diffractive interactions, the model relates through unitarity the diffractive amplitudes to the glue interaction. In particular, the eigenstates of diffractive scattering may be easily defined as being the states of the two incident hadrons with given fraction of momentum carried by glue in each hadron and given impact parameter of the glue-glue collisions. The relevant quantities are illustrated in Fig. 1. In the model the momentum of each hadron is shared between the valence quarks and the glue

$$\begin{aligned} \vec{p} &= \sum \vec{p}_i + p_g = \sum x_i \vec{p} + x_g \vec{p} \quad (\sum x_i + x_g = 1) \\ \vec{p}' &= \sum \vec{p}'_i + \vec{p}'_g = \sum x'_i \vec{p} + x'_g \vec{p}' \quad (\sum x'_i + x'_g = 1) \end{aligned} \quad (1)$$

where the index i runs over valence quarks and g refers to glue. Similarly, in the impact parameter plane the positions of quarks $\vec{\beta}_i$, centre of glue momentum $\vec{\beta}_g$ and centre of particle momentum $\vec{\beta}$ are related by

$$\begin{aligned} \sum x_i \vec{\beta}_i + x_g \vec{\beta}_g &= \vec{\beta} \\ \sum x'_i \vec{\beta}'_i + x'_g \vec{\beta}'_g &= \vec{\beta}' \end{aligned} \quad (2)$$

Thus the diffractive eigenstates in hadron-hadron collision are fully characterized by the glue momenta in both particles (x_g and x'_g), the glue-glue collision impact parameter

$$\vec{b}_g = \vec{\beta}_g - \vec{\beta}'_g \quad (3)$$

and the over-all energy s . In the following, we will not write down explicitly the energy dependence. Our phenomenological analysis will be restricted to the pp ISR data and the energy dependent effects relevant to this range will be discussed separately. The diffractive eigenamplitude is denoted by $t(x_g, x'_g, \vec{b}_g)$. It is assumed to be fully absorptive and our phase convention is to take it positive. We can determine from experiment the elastic amplitude $t_{el}(\vec{b})$ as function of the impact parameter of the hadron-hadron collision

$$\vec{b} = \vec{\beta} - \vec{\beta}' \quad (4)$$

This amplitude is given in the model by averaging the eigenamplitude $t(x_g, x'_g, \vec{b}_g)$ over the distribution function $P(x_g, x'_g, \vec{b} - \vec{b}_g)$, which by Eq. (2) is entirely determined by the internal hadron structure. Its value is

$$t_{el}(\vec{b}) = \bar{t} = \int t(x_g, x'_g, \vec{b}_g) P(x_g, x'_g, \vec{b} - \vec{b}_g) dx_g dx'_g d^2\vec{b}_g \quad (5)$$

As shown recently ⁴⁾, the inelastic diffractive cross-section at impact parameter $\vec{b}$ may be expressed similarly as the dispersion of the eigenamplitude distribution

$$2\sigma_D(\vec{b}) = D_t^2 = \int (t - \bar{t})^2 P dx_g dx'_g d^2\vec{b}_g = \int t^2 P dx_g dx'_g d^2\vec{b}_g - \bar{t}^2 \quad (6)$$

Thus we may ask if the model, with a reasonable form for the probability distribution P and with its free parameters restricted to fit the elastic data, is able to reproduce correctly the integrated inelastic diffractive cross-section

$$\sigma_D = \int \sigma_D(\vec{b}) d^2\vec{b} \quad (7)$$

as estimated in various experiments [see the compilation of σ_D/σ_{el} values in Ref. 4) and references therein] If this is possible, the parameter values determined by fitting the data and the resulting shape of the impact parameter profile (6) will give new information on the physical content of the model. Obviously, there is also the possibility of disproving the model in the present form, if the observed σ_D cannot be reproduced. In this respect our investigation presents a new independent test of the quark-gluon model of Ref. 1).

3. - DETAILED ASSUMPTIONS

To perform a quantitative analysis we have obviously to specify the dependence of the eigenamplitudes t on x_g, x'_g and $\vec{b}_g$, and to restrict the arbitrariness of the probability distribution P . We adopt the following assumptions.

- a) Since the diffractive scattering is assumed to be the shadow of inelastic production processes, eigenamplitudes may be parametrized by an eikonal form

$$t(x_g, x'_g, \vec{b}_g) = 1 - \exp[-\Omega(x_g, x'_g, \vec{b}_g)] \quad (8)$$

We assume that the $\vec{b}_g$ dependence factorizes from the x_g, x'_g dependence in the eikonal Ω as well as in the probability distribution P .

- b) The eikonal is assumed to be proportional to the product $x_g x'_g$ and thus is written as

$$\Omega(x_g, x'_g, \vec{b}_g) = \lambda(\vec{b}_g) x_g x'_g \quad (9)$$

This may be justified by relating x_g to the density of gluons in the hadron. Our ansatz is then valid if, for given $\vec{b}_g$, the eikonal is proportional to the density convolution (as in the standard Chou-Yang model ⁵⁾ for hadrons). This is a very specific assumption and possible modifications will be briefly discussed later.

c) As shown in Ref. 2), the probability distribution F of x_g in each incident hadron is simply related in the model to the spectrum of leading particles in non-diffractive collisions, and thus known (at least for pp collisions) to be approximately flat. We here simply take

$$F(x_g) = F(x'_g) = 1 \quad (10)$$

Thus the distribution of the product $x_g x'_g$ is approximated by

$$F_g(x_g, x'_g) \equiv \int F(\bar{x}_g) F(\bar{x}'_g) \delta(\bar{x}_g \bar{x}'_g - x_g x'_g) d\bar{x}_g d\bar{x}'_g = -\ln(x_g x'_g) \quad (11)$$

From (8), (9) and (11) we obtain the following forms for our functions P and t

$$P(x_g, x'_g, \vec{b} - \vec{b}_g) = -\Gamma(\vec{b} - \vec{b}_g) \ln(x_g x'_g) \quad (12)$$

$$t(x_g, x'_g, \vec{b}_g) = 1 - \exp[-\lambda(\vec{b}_g) x_g x'_g]. \quad (13)$$

We are left with two unknown functions $\Gamma(\vec{b} - \vec{b}_g)$ and $\lambda(\vec{b}_g)$. Note that $\Gamma(\vec{b} - \vec{b}_g)$ is the convolution of glue distributions in the colliding hadrons:

$$\Gamma(\vec{b} - \vec{b}_g) = \int G(\vec{\beta} - \vec{\beta}_g) G'(\vec{\beta}' - \vec{\beta}'_g) \delta(\vec{b} - \vec{b}_g - \vec{\beta} + \vec{\beta}_g + \vec{\beta}' - \vec{\beta}'_g) d^2(\vec{\beta} - \vec{\beta}_g) d^2(\vec{\beta}' - \vec{\beta}'_g) \quad (14)$$

where G and G' describe the distribution of "glue CM" relative to "particle CM". Averaging t over $x_g x'_g$ by means of (12) gives

$$\int_0^1 t(x_g, x'_g, \vec{b}_g) P(x_g, x'_g, \vec{b}-\vec{b}_g) d(x_g x'_g) = \Gamma(\vec{b}-\vec{b}_g) [1 - \text{Ein}(\lambda(\vec{b}_g))] \equiv \Gamma(\vec{b}-\vec{b}_g) t_g(\vec{b}_g) \quad (15)$$

where

$$\text{Ein}(\lambda) = \lambda^{-1} [E_1(\lambda) + \gamma + \ln \lambda] \quad (16)$$

and $E_1(x)$ denotes, as usual, the exponential integral of the first order

$$E_1(x) = \int_x^\infty e^{-t} t^{-1} dt. \quad (17)$$

As seen from its definition in (15), the function $t_g(\vec{b}_g)$ is univocally related to $\lambda(\vec{b}_g)$ and is independent of Γ .

Finally, the elastic amplitude is the convolution of t_g and Γ

$$t_{el}(\vec{b}) = \int t_g(\vec{b}_g) \Gamma(\vec{b}-\vec{b}_g) d^2 \vec{b}_g. \quad (18)$$

Experimentally, $t_{el}(\vec{b})$ is known to be well approximated for moderate b by a Gaussian (see Fig. 2)

$$t_{el}(\vec{b}) = t_{el}(0) \exp(-b^2/R^2), \quad t_{el}(0) \simeq 0.75, \quad R^2 \simeq 0.9 \text{ fm}^2 \quad (19)$$

It is therefore a reasonable guess to parametrize t_g and Γ also by Gaussians

$$t_g(\vec{b}_g) = \tau \exp(-b_g^2/A^2) \quad (20)$$

$$\Gamma(\vec{b}-\vec{b}_g) = (a^2 \pi)^{-1} \exp[-|\vec{b}-\vec{b}_g|^2/a^2] \quad (21)$$

We have here taken into account the normalization of Γ . Of the three model parameters left (τ , A and a) only one remains free after fitting the measured parameters of the elastic amplitude:

$$R^2 = a^2 + A^2 \quad (22)$$

$$t_{el}(0) = \tau A^2 / R^2 \quad (23)$$

Moreover, since the eigenamplitudes t are not larger than 1, so is their average over x_g, x'_g for $b_g = 0$, which is just τ . In view of this restriction, it is meaningful to ask whether the model can reproduce the experimental inelastic diffractive cross-sections. This will be answered in the next section.

4. - PHENOMENOLOGICAL ANALYSIS

We shall discuss proton-proton data at ISR energies and shall use the elastic profile as presented at one energy ⁶⁾; some energy-dependent effects will be discussed later.

Let us begin, for convenience, with the simplified case of $a = 0$ in formula (21). The probability function $\Gamma(\vec{b}-\vec{b}_g)$ is then a two-dimensional Dirac δ function. This is equivalent to the statement that the "particle CM" and "glue CM" coincide for each of the colliding hadrons. The impact parameter for glue-glue collision $\vec{b}_g$ is then just the particle collision impact parameter $\vec{b}$, and both remaining parameters $A^2 = R^2$ and $\tau = t_{el}(0)$ are fixed directly from elastic data. In addition, t_g is now identical to t_{el} [see (18)], and we can determine $\lambda(\vec{b}_g)$ from (15)

$$\lambda(\vec{b}_g) = E \ln^{-1} [1 - t_{el}(\vec{b}_g)] \quad (24)$$

For the inelastic diffractive profile one finds according to formula (6)

$$2\sigma_D(\vec{b}) = \int_0^1 t^2(x_g x'_g, \vec{b}) F_g(x_g x'_g) d(x_g x'_g) - t_{el}^2(\vec{b}) = E \ln [2\lambda(\vec{b})] - \{E \ln [\lambda(\vec{b})]\}^2 \quad (25)$$

The result is shown in Fig. 2 as the dotted line marked σ_D^1 . The diffractive profile is peripheral, but for all $\vec{b}$ the inelastic diffractive cross-section is smaller than the properly normalized elastic cross-section $\sigma_{el}(\vec{b}) = \frac{1}{2}[t_{el}(\vec{b})]^2$ shown also as the broken line. The integrated value is

$$\sigma_D = \int \sigma_D(\vec{b}) d^2\vec{b} \simeq 0.35 \sigma_{el} \simeq 2.7 \text{ mb} \quad (26)$$

i.e., much too low to explain the experimental estimates ⁴⁾. This shows that the spread in the eigenstates' opacities introduced by the x_g, x'_g distributions is not sufficient to produce large enough a dispersion D_t^2 . Note that we have not yet used the Gaussian approximation of $t_{el}(\vec{b})$ - the values could be just read out of the data.

We now consider non-zero values of a , thus introducing additional variations of the eigenamplitudes t . They are related to the displacements of "glue CM" relative to "particle CM". This will increase D_t^2 and, consequently, the integrated diffractive cross-section. Instead of formulae (24), (25), we get

$$\lambda(\vec{b}_g) = Ein^{-1} [1 - t_g(\vec{b}_g)] \quad (27)$$

$$2\sigma_D(\vec{b}) = \int D_g^2(\vec{b}_g) \Gamma(\vec{b}-\vec{b}_g) d^2\vec{b}_g + \int [t_{el}(\vec{b}) - t_g(\vec{b}_g)]^2 \Gamma(\vec{b}-\vec{b}_g) d^2\vec{b}_g \quad (28)$$

where

$$D_g^2(\vec{b}_g) = Ein[2\lambda(\vec{b}_g)] - \{Ein[\lambda(\vec{b}_g)]\}^2 \quad (29)$$

Even in the Gaussian approximation t_g and Γ are not fully determined ; they are related to data only by formula (18). One can, however, find rather easily how the results depend on the value of a . The second term of

formula (28) can be estimated analytically as the difference of two Gaussians and its integrated contribution to σ_D appears to be a monotonically increasing function of a in the relevant region. The first term may be estimated using a linear combination of Gaussians to approximate $D_g^2(\vec{b}_g)$ and performing then analytically the integration for different values of a . Its contribution to σ_D is found to be very weakly dependent on a . Thus the maximal value of σ_D will be obtained for maximal a . As seen from (22), (23), this is obtained for maximal τ , $\tau = 1$, and the maximal value of a is

$$a^2 = R^2 [1 - t_{el}(0)]. \quad (30)$$

The physical meaning of this limit is obvious : it saturates unitarity in the glue-glue collision at zero impact parameter. In other words, glue is completely "black" for central collisions, but the protons are not, because for $a \neq 0$ a central proton-proton collision corresponds to a non-zero range of glue impact parameter.

Taking the relevant parameters in agreement with data [see, e.g. (19)] we get

$$a \simeq \frac{1}{2} R, \quad A \simeq \frac{\sqrt{3}}{2} R \quad (31)$$

The resulting $\sigma_D(\vec{b})$ is peripheral and significantly larger than in the case $a = 0$. It is shown in Fig. 2 as the solid line marked σ_D^2 . The integrated value of σ_D is found to be

$$\sigma_D \simeq 0.67 \sigma_{el} \simeq 5.1 \text{ mb} \quad (32)$$

in which both terms of formula (28) contribute about equally.

We have thus found the maximal value of σ_D in our model. It is compatible with the lowest estimates from experimental data (NAL estimates of σ_D vary between 65 and 150 % of the elastic cross-section⁴⁾). We really need τ very close to 1 to be in marginal agreement with the data : as mentioned above, for $\tau = t_{el}(0) = 0.75$ our σ_D is already twice too small.

Our result allows us to answer simply the old question : why do pp collisions in the ISR range obey approximate geometrical scaling with $t_{el}(0)$ significantly smaller than one and virtually energy independent ? In fact the central opacity value $t_{el}(0)$ does not increase significantly in the entire ISR energy range (whereas the average radius squared R^2 does) ⁷⁾. The reason is that the unitarity limit is already saturated for the glue-glue interaction in our model. Further increase of $t_{el}(0)$ may occur only according to

$$t_{el}(0) = A^2/R^2 = 1 - (a^2/R^2) \quad (33)$$

with fixed a and increasing R . However, even the fastest increase of R compatible with the Froissart bound (which is $R \propto \ln s$), when combined with energy independent a , produces only a mild energy dependence

$$1 - t_{el}(0) \propto (\ln s)^{-2} \quad (34)$$

This should be contrasted with the typical Cheng-Wu type variation

$$1 - t_{el}(0) \propto \exp(-c s^\alpha) \quad (35)$$

Note that the energy dependence from formula (34) is very weak numerically : increasing R^2 by 10 % (which corresponds roughly to the full ISR range) one increases $t_{el}(0)$ only by 2 % in very good agreement with the data ⁷⁾.

Summarizing, we have shown that the model can explain the experimental value of the inelastic diffractive cross-section if one assumes saturation of the unitarity limit for central glue-glue collisions. This, in turn, provides a simple explanation for the approximate geometrical scaling observed before saturation of the unitarity limit for central proton-proton collisions. Thus the description of elastic and inelastic diffraction in the quark-glue model is self-consistent and provides a simple geometrical picture of the interaction.

5. - DISCUSSION

One can ask whether the conclusions obtained above depend strongly on the detailed assumptions of Section 3. In particular, one would like to know of other possibilities to increase in the model the inelastic diffractive cross-section, which we have seen to be at the lower end of the range of experimental estimates.

The factorization assumption a), although difficult to justify from first principles, seems reasonable at least for the purpose of approximate calculations. E.g., the unavoidable non-factorization of P for $x_g, x'_g \sim 1$ which results from the angular momentum rule (2) is estimated to result in rather small corrections. Thus we shall discuss separately possible modifications of the x_g and b dependences.

For the sake of simplicity we have chosen the eikonal proportional to the product of glue momenta [Eq. (9)]. One can check that any dependence weaker than linear would result in a reduced σ_D . For example, a square root dependence on the product of glue momenta

$$\Omega(x_g, x'_g, \vec{b}_g) = \lambda(\vec{b}_g) (x_g x'_g)^{1/2} \quad (36)$$

gives σ_D not larger than 4 mb, which seems to be already incompatible with the data. A stronger dependence on $x_g x'_g$ (e.g., power-like with exponent above one or a rapid decrease of Ω for increasing $x_g x'_g$) may increase σ_D , but there is no obvious justification for such choices.

The constant probability distribution assumed for the x_g variables is obviously very simplified. One can check, however, that modifying the spectrum (10) to take into account the dip at x_g close to zero expected from the non-diffractive proton spectrum leads only to a slight reduction of σ_D .

Finally, non-Gaussian shapes for t_g or Γ are not expected to introduce major changes as long as both functions are monotonical and similar in shape to $t_{el}(b)$.

6. - CONCLUSION

We have shown that the quark-gluon model provides sufficiently large variations of opacities for diffractive eigenstates to account for the experimental value of the inelastic diffractive cross-section σ_D . To obtain an acceptable value of σ_D we have to assume unitarity saturation ("total blackness") in central gluon-gluon collisions. This fixes the free parameters of the model in the Gaussian approximation, provides a natural explanation for the observed energy independence of the proton opacity in central collisions, and simplifies significantly the collision picture in the model. Modifications of the detailed assumptions used are unlikely to change this picture.

REFERENCES

- 1) S. Pokorski and L. Van Hove - Acta Phys.Polonica B5, 229 (1974).
- 2) L. Van Hove and S. Pokorski - Nuclear Phys. B86, 243 (1975).
- 3) L. Van Hove - Acta Phys.Polonica, to be published.
- 4) K. Fiałkowski and H.I. Miettinen - Nuclear Phys., to be published.
- 5) T.T. Chou and C.N. Yang - Phys.Rev. 170, 1591 (1968).
- 6) P. Pirilä, P.V. Ruuskanen and H.I. Miettinen - as quoted in Ref. 4).
- 7) R. Henzi - Phys.Letters, to be published.

FIGURE CAPTIONS

- Figure 1 Diagram defining the variables. All momentum vectors are parallel to the incident momenta $\vec{p}$ and $\vec{p}' = -\vec{p}$ (transverse momenta are neglected) ; all the remaining vectors are two-dimensional, being defined in the impact parameter plane perpendicular to the incident momenta.
- Figure 2 Impact parameter profiles for pp collisions at ISR energies. Black dots and dash-dotted line represent the experimental total cross-section, equal to the purely absorptive elastic amplitude $t_{el}(b)$, and the Gaussian fit of formula (19) ; open circles and dashed line are for the elastic profile defined as $\sigma_{el} = \frac{1}{2}(t_{el})^2$ and corresponding Gaussian. Dotted line σ_D^1 and solid line σ_D^2 show the results of the model for the diffractive inelastic profile with $a = 0$, Eq. (25), and maximal a , Eq. (28), respectively. The black disc limit and the upper unitarity limit for the diffractive inelastic profile σ_D^{max} are also shown.

FIG. 1

FIG. 2