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A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSONJohn Ellis, Mary K. Gaillard ^{*)} and D.V. Nanopoulos ⁺⁾

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A B S T R A C T

A discussion is given of the production, decay and observability of the scalar Higgs boson H expected in gauge theories of the weak and electromagnetic interactions such as the Weinberg-Salam model. After reviewing previous experimental limits on the mass of the Higgs boson, we give a speculative cosmological argument for a small mass. If its mass is similar to that of the pion, the Higgs boson may be visible in the reactions $\pi^- p \rightarrow H n$ or $\gamma p \rightarrow H p$ near threshold. If its mass is $\lesssim 300$ MeV, the Higgs boson may be present in the decays of kaons with a branching ratio $O(10^{-7})$, or in the decays of one of the new particles : $3.7 \rightarrow 3.1 + H$ with a branching ratio $O(10^{-4})$. If its mass is ≤ 4 GeV the Higgs boson may be visible in the reaction $pp \rightarrow H + X$, $H \rightarrow \mu^+ \mu^-$. If the Higgs boson has a mass $\leq 2m_u$, the decays $H \rightarrow e^+ e^-$ and $H \rightarrow \gamma \gamma$ dominate, and the lifetime is $O(2 \times 10^{-5} \text{ to } 2 \times 10^{-12})$ seconds. As thresholds for heavier particles (pions, strange particles, new particles) are crossed, decays into them become dominant, and the lifetime decreases rapidly to $O(10^{-20})$ sec for a Higgs boson of mass 10 GeV. Decay branching ratios in principle enable the quark masses to be determined.

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1. - INTRODUCTION

Many people now believe that weak and electromagnetic interactions may be described by a unified, renormalizable, spontaneously broken gauge theory ¹⁾. This view has not been discouraged by the advent of neutral currents, or the existence of the new narrow resonances ²⁾. These latter may well be a manifestation of some form of "charm", a new hadronic degree of freedom ³⁾ favoured by constructors of weak and electromagnetic interaction models. A comprehensive discussion of the phenomenology of conventional charm has been given by Gaillard, Lee and Rosner ⁴⁾. At the time of writing, the discovery of charm has not been confirmed, but gauge theorists are not yet discouraged.

Other particles have been suggested by gauge theorists, including heavy leptons ⁵⁾, Higgs bosons ⁶⁾ and intermediate vector bosons. Experimental searches for heavy leptons M^+ coupled to muon neutrinos have ruled out ⁷⁾ masses below 8 GeV. From e^+e^- collisions, the lower mass limit for charged heavy leptons is about $1\frac{1}{2}$ GeV ⁸⁾, but events are now ⁹⁾ being seen at storage rings which are consistent with the production and decays of heavy leptons with masses $O(2)$ GeV.

The situation with regard to Higgs bosons is unsatisfactory. First it should be stressed that they may well not exist. Higgs bosons are introduced to give intermediate vector bosons masses through spontaneous symmetry breaking. However, this symmetry breaking could be achieved dynamically ¹⁰⁾ without elementary Higgs bosons. Thus the confirmation or exclusion of their existence would be an important constraint on gauge theory model-building. Unfortunately, no way is known to calculate the mass of a Higgs boson, at least in the context of the popular Weinberg-Salam ¹¹⁾ model, and experimental lower limits ¹²⁾⁻¹⁴⁾ on its mass are around 15 MeV, piffling compared with the intermediate vector boson masses expected to be $O(50 \text{ to } 100)$ GeV. Furthermore, indirect evidence of the Higgs boson's existence ($g-2$ ¹⁵⁾, muonic atoms ^{15),16)}, etc.) is notoriously difficult to find. In this paper we do suggest one speculative argument why the Higgs mass may be small, based on considerations ^{17),18)} about the cosmological term in Einstein's equations for theories with spontaneous symmetry breaking.

Most of this paper is phenomenological, however, and we discuss ways of looking directly for the Higgs boson, pushing the experimental lower mass limit up to a few hundred MeV or a few GeV. We assume couplings of the Higgs boson to leptons and hadrons like those found in gauge theories including the $SU(2) \times U(1)$ Weinberg-Salam model ¹¹⁾. We are motivated to make this assumption by the consistency of the Weinberg-Salam model with present experimental data ¹⁹⁾. Then the Higgs couplings to fundamental fermions (leptons, quarks) are proportional to their masses, and amplitudes for processes involving the Higgs boson can be shown proportional to matrix elements of the trace Θ_{μ}^{μ} of the energy-momentum tensor. This enables phenomenological estimates using the ideas of broken scale invariance ²⁰⁾.

First we consider direct production of the Higgs boson. If its mass is $O(m_{\pi})$ then we find that in the reaction $\pi^- p \rightarrow H n$ close to threshold

$$\frac{d\sigma(\pi^- p \rightarrow H n)}{dt} = \frac{\alpha(10^{-7})}{s |p_{\pi}^{c.m.}|^2} \quad (1.1)$$

As a light Higgs boson often decays to e^+e^- it therefore might be visible in experiments ²¹⁾ designed to look for $\pi^- p \rightarrow e^+e^- n$ close to threshold. Another possibility is $\gamma p \rightarrow H p$ below or close to the pion production threshold. At higher energies, we find that under favourable circumstances a Higgs boson with mass $> 2m_{\pi}$ might be visible as a small peak above the continuum in the reaction $pp \rightarrow (\mu^+ \mu^-) + X$. On the other hand, direct production in electron-positron collisions is found very small

$$\frac{\sigma_{\text{peak}}(e^+e^- \rightarrow \text{Higgs boson})}{\sigma_{\text{peak}}(e^+e^- \rightarrow 3.1 \text{ resonance})} \approx \frac{10^{-7}}{m_H} : m_H \text{ in GeV} \quad (1.2)$$

Another possible way of producing Higgs bosons is by bremsstrahlung in processes involving massive particles such as an intermediate vector boson. Production by bremsstrahlung along with one of the new narrow resonances (e.g., $e^+e^- \rightarrow 3.1 + H$) does not seem to be large. However, one of the best ways of looking for the Higgs boson if it has a mass ≤ 500 MeV may be in decays of the 3.7 resonance : then we can estimate

$$\frac{\Gamma(3.7 \rightarrow 3.1 + H)}{\Gamma(3.7 \rightarrow \text{all})} = O(10^{-4}) \quad (1.3)$$

The branching ratio is relatively large because competing decay modes are suppressed by the Zweig²²⁾ mechanism. The Higgs boson may also show up in K decays if it has a mass ≤ 300 MeV :

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + H)}{\Gamma(K^+ \rightarrow \text{all})} = O(10^{-7}) \quad (1.4)$$

which is not far below limits from present K decay data²³⁾. On the other hand, the Higgs boson seems unlikely to turn up in the decays of other particles : for example we find

$$\frac{\Gamma(\gamma \rightarrow \pi^0 + H)}{\Gamma(\gamma \rightarrow \text{all})} = O(10^{-8}) \quad (1.5)$$

for $m_H < 400$ MeV, which is way below present experimental limits.

We also discuss the decays of Higgs bosons and their experimental signatures. If $2m_e < m_H < 2m_\mu$ then the dominant decays are $H \rightarrow e^+e^-$ and $\gamma\gamma$. The $\gamma\gamma$ branching ratio is quite large, thanks to contributions to the amplitude from hadrons and from a virtual intermediate vector boson loop. The e^+e^- and $\gamma\gamma$ rates are generally comparable in order of magnitude in this mass range (for details see Fig. 1). For $2m_\mu < m_H < 2m_\pi$, the $\mu^+\mu^-$ mode is dominant, while $\pi\pi$ takes over for $2m_\pi < m_H < 2m_K$. For $m_H > 2m_K \sim 1000$ MeV decays into strange particles are expected to dominate until the threshold of 4 GeV for new particles (hadrons, heavy leptons ?) is reached, whereafter they dominate the decays. This is because decay amplitudes are directly proportional to the masses of fundamental fermion fields. Thus branching ratios to $\mu^+\mu^-$ and different types of hadrons in principle enable the quark masses to be measured. For Higgs boson masses $\lesssim 4$ GeV the $\mu^+\mu^-$ decay mode is expected to be 0 (1 to 30) %, and may be the least invisible experimental signal. The main uncertainties in estimating decay rates and the total Higgs boson lifetime arise from our uncertainty as to the quark masses^{24),25)}. The calculated Higgs lifetime is plotted in Fig. 2, and we see that it varies between 2×10^{-5} and 2×10^{-12} sec for $2m_e < m_H < 2m_\mu$, so that the Higgs boson could leave a discernible track. For $m_H > 2m_\mu$ the lifetime decreases rapidly to $O(10^{-20})$ sec for $m_H \sim 10$ GeV.

The organization of the paper is as follows. Section 2 discusses the rôle of the Higgs boson in gauge theories of weak and electromagnetic interactions, reviews present experimental limits on its mass, and discusses

the cosmological connection. Section 3 contains calculations of Higgs boson yields in various production and decay processes, while Section 4 considers its decay modes, rates and branching ratios. Section 5 discusses the conclusions and prospects for Higgs boson phenomenology.

2. - COUPLINGS AND MASS OF THE HIGGS BOSON

2.1. - The Weinberg-Salam ¹¹⁾ Model

Gauge theories of the weak and electromagnetic interactions generally start off with a collection of massless vector fields $W_{i\mu}$ and massless spinor fields q, e, μ , etc. for hadrons and leptons, arranged in a symmetric Lagrangian. The physical hadrons, leptons and presumably intermediate vector bosons have masses generated by breaking the gauge symmetry. This may be done either dynamically ¹⁰⁾ or by introducing a new scalar field and breaking the symmetry spontaneously - the Higgs ⁶⁾ mechanism. Consider for example the simple $SU(2) \times U(1)$ gauge model of Weinberg and Salam ¹¹⁾. This starts with a triplet (W^+, W^0, W^-) and a singlet Y^0 of zero-mass gauge fields, to which is added a doublet $\underline{H} = (H^+, H^0)$ of scalar Higgs fields. In the spontaneous symmetry breakdown $H^0 = (1/\sqrt{2})(v + H + i\tilde{H})$ where $v = \langle 0 | H^0 | 0 \rangle$, the vacuum expectation of H^0 , arises from the minimum of the potential

$$V(\underline{H}) = \mu^2 (\underline{H}^+ \cdot \underline{H}) + \lambda (\underline{H}^+ \cdot \underline{H})^2 \quad (\mu^2 < 0, \lambda > 0) \quad (2.1)$$

The field H^+ and its complex conjugate H^- are absorbed to give W^+ and W^- their physical masses, $\tilde{H}$ is absorbed to give one linear combination of W^0 and Y^0 a mass - the neutral intermediate vector boson Z^0 . The other linear combination of W^0 and Y^0 stays massless - the photon - and we are left with one neutral scalar field H representing a physical particle, the Higgs ⁶⁾ boson. The magnitude of v reflects the acquired mass of the $W^\pm$ bosons :

$$v^2 = \frac{1}{\sqrt{2} G_F} \quad (2.2)$$

with

$$m_w^2 = \frac{g^2 v^2}{4} \quad (2.3)$$

for the W boson mass, g being the non-Abelian weak SU(2) coupling constant, $G_F = 1.02 \times 10^{-5} / m_{\text{proton}}^2$, and the W^+W^-H coupling is specified :

$$g_{W^+W^-H} = \frac{2m_w^2}{v} \quad (2.4)$$

Correspondingly

$$m_z^2 = \frac{g^2}{\cos^2 \theta_w} \frac{v^2}{4} \quad (2.5)$$

where θ_w is the Weinberg angle, and

$$g_{Z^0 Z^0 H} = \frac{2m_z^2}{v} \quad (2.6)$$

for the neutral weak intermediate boson Z^0 . The parameters of the potential $V(H)$ (2.1) are related to v :

$$\lambda v^2 = -\mu \quad (2.7)$$

The potential acquires a non-zero vacuum expectation value after spontaneous breakdown :

$$\langle 0 | V | 0 \rangle = \frac{\mu^2 v^2}{4} \quad (2.8)$$

and the Higgs boson mass is given by

$$m_H^2 = -2\mu^2 \quad (2.9)$$

After spontaneous breakdown the couplings of the Higgs field to fundamental fermions f are

$$g_{\bar{f}fH} \bar{f}f (v+H) \quad (2.10)$$

which gives Higgs-fermion-antifermion couplings $g_{\bar{f}fH}$ proportional to the fermion masses m_f :

$$g_{\bar{f}fH} = \frac{m_f}{v} \quad (2.11)$$

In all the above equations (2.1) to (2.11) there is one unknown parameter : μ or equivalently the Higgs boson mass m_H .

2.2. - Ambiguities

The model described above is the simplest version of the Weinberg-Salam model. As soon as we consider more complicated versions of this model, or other models of weak and electromagnetic interactions, then considerable ambiguities arise in the Higgs boson couplings. For example :

- i) - Even in the context of the Weinberg-Salam ¹¹⁾ model we can choose to have several Higgs fields H_i belonging to several multiplets i with weak isospins I_i . Then if the uncharged member H_i^0 of each multiplet has as its third component of isospin I_{3i} and acquires a vacuum expectation value $\langle 0|H_i^0|0 \rangle = v_i$ we find

$$m_w^2 = \frac{g^2}{2} \sum_i v_i^2 (I_i^2 + I_i - I_{3i}^2) \quad (2.12)$$

and

$$m_z^2 = \frac{g^2}{\cos^2 \theta_w} \sum_i v_i^2 I_{3i}^2 \quad (2.13)$$

After the spontaneous breakdown of the theory in general there remain physical charged Higgs bosons as well as neutral ones H_i . The couplings (2.4), (2.6) and (2.11) are then shared out between the physical neutral Higgs particles :

$$\sum_i g_{w^+w^-H_i} v_i = 2m_w^2$$

$$\sum_i g_{Z^0Z^0H_i} v_i = 2m_Z^2$$

$$\sum_i g_{\bar{f}fH_i} v_i = m_f$$

(2.14)

There is no reason except optimism to believe that the couplings (2.4), (2.6) and (2.11) would be even qualitatively correct for any of the physical neutral Higgs bosons in such a theory. In this paper we are optimistic, and assume either that Nature chooses the simple spontaneous breaking via a Higgs doublet as done by Weinberg and Salam ¹¹⁾ and discussed in Section 2.1, or, if not, that the couplings in Nature are qualitatively similar to this simplest possibility.

- ii) - If weak and electromagnetic interactions are not unified via the Weinberg-Salam model or some extension thereof, then the Higgs couplings may be very different from those assumed above. A statistical sampling ²⁶⁾ of such models shows for example that in theories with heavy leptons the Higgs coupling to electrons may be governed by a "heavy electron" mass. In this case most of our remarks about the Higgs boson in situations involving electrons would be invalid. Some recent experimental data ⁹⁾ may be connected with a heavy lepton, but this is as yet unclear, and the Weinberg-Salam model can also incorporate sequential heavy leptons without affecting our conclusions. We remain buoyed by the general consistency of the Weinberg-Salam model with experimental data ¹⁹⁾.
- iii) - It may be that the spontaneous breakdown responsible for the intermediate vector boson masses does not arise from the introduction of a fundamental Higgs field, but from a dynamical mechanism ¹⁰⁾. Indeed there are theorems ²⁷⁾ that if, for example, the Weinberg-Salam model is embedded in a model with a higher simple group

symmetry broken by the Higgs mechanism down to $SU(2) \times U(1)$, then this residual symmetry cannot also be broken by the Higgs mechanism. This and other aesthetic reasons including economy lead some people to prefer dynamical symmetry breaking. Unfortunately no calculable model exists, and the couplings of any physical composite Higgs fields in such a situation are unknown. If experiments do not find a Higgs boson of the type discussed here, dynamical symmetry breaking may be more attractive.

2.3. - Restrictions on the Higgs Boson Mass

If we accept the simplest model discussed in Section 2.1, what theoretical and phenomenological arguments restrict the Higgs mass m_H ? Jackiw and Weinberg¹⁵⁾ considered the effect of the Higgs on calculations of the muon magnetic moment. They found that for $m_H \lesssim O(m_\mu)$

$$(\Delta g_\mu)_H = O\left(\frac{G_F m_\mu^2}{\pi^2}\right) = O(10^{-8}) \quad (2.15)$$

comparable with the effects of virtual $W^\pm$ and Z^0 exchanges, and impossible to disentangle from hadronic contributions in standard QED. If $m_H \gg m_\mu$, then $(\Delta g_\mu)_H$ is still smaller, so no limit on m_H from the muon magnetic moment. Several authors^{12), 15), 16)} have wondered whether the Higgs boson could be responsible for apparent discrepancies in X-rays from muonic atoms. With the couplings of Section 2.1 this could be arranged with $m_H \lesssim O(20)$ MeV. However, the discrepancies now seem to be disappearing²⁸⁾, and such a Higgs could have difficulties with data on the fine structure of muonic Helium²⁹⁾.

Other restrictions on the existence of a low-mass Higgs boson include the validity of the usual gravitational coupling which excludes¹²⁾ a long-range scalar force and entails $m_H > 10^{-4}$ eV. More stringently, data on neutron-electron scattering require¹⁶⁾ $m_H > 0.7$ MeV. Following a suggestion by Sundaresan and Watson¹²⁾, Kohler, Watson and Becker¹³⁾ searched unsuccessfully in $0^+ \rightarrow 0^+$ nuclear transitions for Higgs production and subsequent decay into e^+e^- . The absence of a signal excluded $2m_e < m_H < 18$ MeV. Another argument from nuclear physics is given by Barbieri and Ericson¹⁴⁾, who argue that a low-mass Higgs is inconsistent with angular distribution measurements in low energy neutron-nucleus scattering. They conclude that almost certainly $m_H > 5$ MeV and very probably > 13 MeV. Present experimental constraints on m_H are tabulated in Fig. 3.

Thus phenomenological arguments from macroscopic, atomic and nuclear physics require $m_H > 0(15) \text{ MeV}$. The main purpose of this paper will be to study how this limit could be improved using the mass scales of high energy physics. But first we would like to mention one theoretical reason that may motivate a low value of m_H .

2.4. - Cosmology and the Higgs Boson Mass

A connection between the Higgs boson and general relativity and cosmology has been made independently by Linde, Dreitlein and Veltman¹⁷⁾. They point out that a constant term C in the world's Lorentz invariant Lagrangian $\mathcal{L}$ gives a term $C\sqrt{g}$ in the generally covariant Lagrangian, where $g = \det(g_{\mu\nu})$ and $g_{\mu\nu}$ is the metric tensor. Such a term in turn introduces a cosmological term into the general relativistic equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{1}{2} \kappa^2 C g_{\mu\nu} = \kappa^2 \Theta_{\mu\nu} \quad (2.16)$$

where $\Theta_{\mu\nu}$ is the energy momentum tensor, $R_{\mu\nu}$ is the Riemann tensor, and $\kappa \simeq 5.8 \times 10^{-22} \text{ MeV}^{-1}$ is the gravitational coupling. Cosmological considerations limit $\kappa^2 C < 10^{-57} \text{ cm}^2$, which implies

$$C < 0.23 \times 10^{-35} \text{ MeV}^4 \quad (2.17)$$

In a spontaneously broken gauge theory, Eqs. (2.8) and (2.9) give

$$C_H \equiv \langle 0|V|0 \rangle = - \frac{m_H^2 v^2}{8} \quad (2.18)$$

and using Eq. (2.2) and the value of G_F , the cosmological constraint (2.17) implies $m_H < 2.35 \times 10^{-27} \text{ MeV}$ if $C = C_H$. There are at least three possible attitudes to this result.

- (1) Accept it, and take it as an argument against the simple Higgs mechanism of spontaneous symmetry breaking. One might then seek either a more complicated Higgs structure for the theory¹⁸⁾, or perhaps a dynamical symmetry breaking mechanism¹⁰⁾ instead.

- (2) Circumvent the result by supposing that the world's Lagrangian $\mathcal{L}$ had a constant term $\simeq -\langle 0|V|0 \rangle$ before spontaneous symmetry breakdown, which was then essentially cancelled by the Higgs term $C_H = \langle 0|V|0 \rangle$ to produce a net $C \simeq 0$. Perhaps the gauge theory's spontaneous breakdown was cosmologically necessary?
- (3) Observe that other dynamical terms in $\mathcal{L}$ can have non-zero vacuum expectation values, and suggest that an interplay between them is necessary to ensure $C = \langle 0|\mathcal{L}|0 \rangle \simeq 0$. In this case the vanishing of the cosmological term would not be the responsibility of weak and electromagnetic interactions alone, or the result of a fiat, but a consistency requirement to be imposed on the ensemble of the world's interactions.

Such extra terms in $\langle 0|\mathcal{L}|0 \rangle$ will arise from any other spontaneous symmetry breaking in $\mathcal{L}$, an example being chiral symmetry breaking in the strong interactions. If the strong interaction part of $\mathcal{L}$ is split into chiral $SU(N) \times SU(N)$ symmetric and breaking pieces:

$$\mathcal{L}_{\text{strong}} = \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{break}} \quad (2.19)$$

then $\langle 0|\mathcal{L}_{\text{break}}|0 \rangle \neq 0$ in general ³⁰⁾. For instance, in an $SU(3) \times SU(3)$ theory, assuming pole dominance for the axial current divergences

$$\partial_\mu A_\pi^\mu \equiv 1 + i2 \quad \text{and} \quad \partial_\mu A_K^\mu \equiv 4 + i5, 6 + i7$$

with π and K quantum numbers gives ^{20), 30)}

$$\langle 0|\mathcal{L}_{\text{break}}|0 \rangle \simeq -\frac{1}{4}(2m_K^2 + m_\pi^2)f_\pi^2 \quad (2.20)$$

where

$$\langle 0|\partial_\mu A_{\pi,K}^\mu|\pi,K \rangle = f_\pi m_\pi^2, f_K m_K^2$$

There is also no reason why $\langle 0|\mathcal{L}_{\text{sym}}|0 \rangle \neq 0$, and this happens in spontaneously broken scale invariance ²⁰⁾ models.

This observation suggests another speculative argument on m_H .
Let us suppose that in order to ensure $C \simeq 0$ we must impose ^{*)}

$$\langle 0 | \mathcal{L}_{\text{sym}} | 0 \rangle + \langle 0 | \mathcal{L}_{\text{break}} | 0 \rangle + \langle 0 | V | 0 \rangle \simeq 0 \quad (2.21)$$

We do not know how to calculate $\langle 0 | \mathcal{L}_{\text{sym}} | 0 \rangle$, so we assume it is similar in order of magnitude to the other strong interaction piece $\langle | \mathcal{L}_{\text{break}} | 0 \rangle$:

$$\langle 0 | \mathcal{L}_{\text{sym}} | 0 \rangle = O(\langle 0 | \mathcal{L}_{\text{break}} | 0 \rangle) \quad (2.22)$$

To estimate $\langle 0 | \mathcal{L}_{\text{break}} | 0 \rangle$ we naïvely extend Eq. (2.20) to $SU(4)^{3),4)}$, assuming pole dominance for the axial divergences $\partial_\mu A_D^\mu$ and $\partial_\mu A_F^\mu$ coupled to D and F mesons by

$$\langle 0 | \partial_\mu A_{D,F}^\mu | D, F \rangle = f_D m_D^2, f_F m_F^2$$

and taking $f_D, f_F = O(f_\pi)$. In this case

$$\langle 0 | \mathcal{L}_{\text{break}} | 0 \rangle = O\left(-\frac{f_\pi^2}{2} m_D^2\right) \quad (2.23)$$

Substituting (2.22) and (2.23) into the consistency condition (2.21), and using Eq. (2.8) for $\langle 0 | V | 0 \rangle$ we get

$$\frac{\mu^2 v^2}{4} = O\left(\frac{f_\pi^2 m_D^2}{2}\right) \quad (2.24)$$

Using (2.2) and (2.9) this yields

$$m_H^2 = O(4\sqrt{2} G_F m_D^2 f_\pi^2) \quad (2.25)$$

With $f_\pi \simeq m_\pi$ and assuming $m_D \approx 2 \text{ GeV}$ this formula gives

$$m_H = O(2) \text{ MeV} \quad (2.26)$$

*) This would not be true if there were yet another spontaneous symmetry breakdown, as for example if there was a higher symmetry broken down to $SU(2) \times U(1)$ by the Higgs mechanism.

Several remarks about this speculative estimate are in order :

- A) personally, we find this argument interesting rather than persuasive ;
- B) the argument needs $\langle 0 | \mathcal{L}_{\text{sym}} | 0 \rangle \neq 0$ in an essential way, for it is evident from (2.8) and (2.20) that $\langle 0 | V | 0 \rangle$ and $\langle 0 | \mathcal{L}_{\text{break}} | 0 \rangle$ are both < 0 . This is expected to be a general feature of spontaneous breakdown. Indeed it is a theorem³¹⁾ that if a field theory has two possible vacua, it will always choose the lower one, whatever the potential barrier between them. Hence, either this theorem must be circumvented, or the strong and weak spontaneous breakdowns are not independent, and the asymmetric vacuum must have the same level as the symmetric one. We have been unable to construct a theory which has such a property, respects theorems^{32),33)} ensuring that parity and hadronic symmetry breakdown occur only in $O(\alpha)$, and involves fundamental quark fields.

If the vacuum theorem³¹⁾ were not circumvented, and the strong and weak spontaneous breakdowns were independent, then the cosmological argument would require both the strong and weak breakdowns separately to leave $\langle 0 | \mathcal{L} | 0 \rangle = 0$. This is possible for the weak breakdown if the theory's Higgs content is changed¹⁸⁾.

- C) It is possible that hadrons have an approximate symmetry higher than $SU(4)$. For example, several authors³⁴⁾ have suggested there may be at least six quarks. In order for the associated "hidden supercharm" vector mesons to have escaped detection in e^+e^- experiments, they would need masses $\gtrsim 8$ GeV and the associated "supercharmed" pseudo-scalar analogues of the conjectured D and F mesons would probably have masses $\gtrsim 5$ GeV. The estimate (2.26) is then modified to $m_H \gtrsim O(7)$ MeV. This is just one example of the sensitivity of (2.26) to the presence of additional structure in the world Lagrangian.

Regardless of the above arguments, a low mass for the Higgs boson is no less likely than a high mass, and it is worth while trying systematically to improve the lower mass limits from nuclear physics. Ways of doing this are studied in the next section.

3. - PRODUCTION OF THE HIGGS BOSON

In this section we discuss how the Higgs boson might be produced in various reactions, mainly involving hadrons. Typical hadronic mass scales $\geq O(\text{few hundred})$ MeV can be used either to exclude possible mass ranges for the Higgs, or perhaps to see it. We start by proving a theorem about Higgs couplings, relating them to matrix elements of the energy momentum tensor, and then go on to consider production both directly and via decays of other particles.

3.1. - The Higgs Boson as a Dilaton ²⁰⁾

In the discussion of Section 2.1 it was apparent that in the simplest version of the Weinberg-Salam model the couplings of the Higgs boson to fundamental fermion and boson fields via their masses (2.4), (2.6), (2.11) mean that if the renormalization due to interactions is ignored, the Higgs boson couples to other particles via the trace of the energy momentum tensor

$$\Theta_{\mu}^{\mu \text{ unren.}} = \sum_f m_f \bar{f}f + 2m_W^2 W^+W^- + 2M_Z^2 Z^0Z^0 \quad (3.1)$$

This observation can be extended in the presence of interactions to give the following theorem : if A and B are states not containing Higgs bosons, then to the lowest relevant order in the semi-weak coupling constant g, the amplitude for $A \rightarrow B + H$ is given by

$$\langle B, H | A \rangle = -\frac{ig}{2m_W} \langle B | \Theta_{\mu}^{\mu} | A \rangle \quad (3.2)$$

where Θ_{μ}^{μ} is the trace of the renormalized improved ³⁵⁾ energy momentum tensor.

To prove (3.2), consider first the Weinberg-Salam model, including strong interactions via a quark-gluon coupling, in the unitary gauge ³⁶⁾ where no unphysical Higgs bosons appear. The Lagrangian is of the form

$$\mathcal{L}^u = \mathcal{L}'(H+u) - V(H) \quad (3.3)$$

where

$$V(H) = \frac{1}{2} m_H^2 \left[H^2 + \frac{1}{v} H^3 + \frac{1}{4v^2} H^4 \right] \quad (3.4)$$

and the vacuum expectation value v is the only dimensional parameter appearing in $\mathcal{L}'$. Since v has the dimension of mass, the trace of the improved energy-momentum tensor³⁵⁾ is determined by

$$\begin{aligned} \Theta_{\mu}^{\mu} &= (\mathcal{D}-4) \mathcal{L}' \\ &= -v \frac{\partial \mathcal{L}'(H+v)}{\partial v} - (\mathcal{D}-4)V \\ &= -v \frac{\partial \mathcal{L}'(v)}{\partial v} + O(H) \end{aligned} \quad (3.5)$$

where $\mathcal{D}$ is an operator which effectively multiplies each term in $\mathcal{L}'$ by its dimension. The linear couplings of the Higgs boson are determined by

$$\begin{aligned} \mathcal{L}_H^v &= H \left. \frac{\partial \mathcal{L}'(H+v)}{\partial H} \right|_{H=0} + O(H^2) \\ &= H \frac{\partial \mathcal{L}'(v)}{\partial v} + O(H^2) \end{aligned} \quad (3.6)$$

Comparing (3.5) and (3.6), we see we can write the part of $\mathcal{L}'$ containing Higgs fields in the form

$$\mathcal{L}_H^v = -\frac{1}{v} H \Theta_{\mu}^{\mu} + O(H^2) \quad (3.7)$$

Then, for processes in which higher order couplings in H can be neglected, the general amplitude for $A \rightarrow B + H$ is given by (3.2).

This result has been obtained by formal manipulations in the unitary gauge. In the general renormalizable gauge³⁷⁾ R_{ξ} , the gauge conditions have the form

$$\partial_{\mu} \tilde{W}^{\mu} + i \left(m_W / \xi \right) H = 0 \quad (3.8)$$

where the $\underline{H}$ are unphysical Higgs scalars. Condition (3.8) is not scale invariant, and the definition of Θ_{μ}^{μ} is not straightforward. However, if we choose the "R gauge" defined by $\xi \rightarrow 0$, the gauge condition $\partial_{\mu} W^{\mu} = 0$ is invariant and we may proceed as before. One finds that Eq. (3.8) is replaced by

$$\mathcal{L}_H^R = -\frac{1}{v} H \Theta_{\mu}^{\mu} + \delta \mathcal{L}_H + O(H^2) \quad (3.9)$$

where

$$\delta \mathcal{L}_H = -\frac{g m_H^2}{2 m_W} H \left(\tilde{H}^2 + H^{\dagger} H \right) + \frac{g}{2} \partial^{\mu} H \left\{ i (W_{\mu}^{-} H^{+} - W_{\mu}^{+} H^{-}) + m_Z^2 / m_W Z_{\mu}^0 \tilde{H} \right\} \quad (3.10)$$

and if $\mathcal{L}^R$ is the complete Lagrangian in the R gauge

$$\Theta_{\mu}^{\mu} \equiv (\mathcal{D}-4) \mathcal{L}^R \quad (3.11)$$

Now consider a transition between quarks $q_{\alpha} \rightarrow q_{\beta} + H$. According to Eq. (3.7) the amplitude is given by

$$\langle q_{\beta}, H | q_{\alpha} \rangle = \frac{-ig}{2m_W} \langle q_{\beta} | \Theta_{\mu}^{\mu} | q_{\alpha} \rangle \quad (3.12)$$

Using the properties :

- (a) that $\Theta^{\mu\nu}$ is a symmetric, divergenceless tensor,
- (b) that the space-integrated operator $\int d^3 \underline{x} \Theta^{0\mu} \equiv P^{\mu}$ is the total four-momentum,
- (c) that in the limit of vanishing quark mass $m_{\alpha} \rightarrow 0$, only the left-handed component of q_{α} is coupled, and
- (d) CP invariance and crossing symmetry,

the matrix element (3.12) can be shown to take the form

$$\begin{aligned} \langle q_{\beta} | \Theta_{\mu}^{\mu}(k) | q_{\alpha} \rangle &= m_{\alpha}^2 \delta_{\alpha\beta} + k^2 A (m_{\alpha} R + m_{\beta} L) \\ &+ (m_{\alpha}^2 - m_{\beta}^2) [B (m_{\alpha} L - m_{\beta} R) + m_{\alpha} m_{\beta} C (m_{\alpha} R - m_{\beta} L)] \end{aligned} \quad (3.13)$$

where $k = p_\alpha - p_\beta$, $L = \frac{1}{2}(1 - \gamma_5)$ and $R = \frac{1}{2}(1 + \gamma_5)$, and A , B and C are functions of the momenta, arbitrary except that they contain no singularities for $k \rightarrow 0$.

The matrix element of Θ_L^μ is not in itself a physical observable and may be gauge dependent, except in the limit $k \rightarrow 0$, where it is determined by condition (b) above. However, the matrix element for $q_\alpha \rightarrow q_\beta + H$ with all particles on the mass-shell must be gauge invariant. Therefore the contribution of $\delta \mathcal{L}_H$ (3.10) in Eq. (3.9) must be of the form of (3.13) without the zero momentum transfer term $\delta_{\alpha\beta} m_\alpha^2$.

Of the amplitudes we will consider, weak and electromagnetic couplings are significant only for those with $|\Delta I| = 1$ and $|\Delta S| = 1$. Then the diagrams which can contribute are those of Fig. 4. We have found by explicit calculation of their contribution to the quark amplitudes ^{*)}

$$\begin{aligned} \langle u | H | u \rangle &= \langle d | H | d \rangle & (\Delta I = 1) \\ \langle d | H | s \rangle & & (\Delta S = 1) \end{aligned}$$

that the contribution of $\delta \mathcal{L}_H$ is of the required form to lowest order in the weak coupling and in m_q^2/m_w^2 . We therefore conclude that the formal result obtained in the unitary gauge is valid. We will later use the properties (a) to (d) above, and the theorem (3.2) to estimate various physical amplitudes, both for production (in subsequent parts of this section) and for decays (in Section 4).

3.2. - Production in Hadronic Collisions

In this section we discuss production of H in various hadronic reactions (πp , pp collisions) and photoproduction in several kinematic conditions.

^{*)} Where necessary, we denote the usual quarks by u , d and s , and the proton and neutron by p and n .

3.2.1. - Low energies

We first consider the possibility that $m_H = O(m_\pi)$, in the range $15 \text{ MeV} < m_H < 2m_\pi$ say, and examine production in πp and γp collisions via the reactions $\pi^- p \rightarrow Hn$, $\gamma p \rightarrow Hp$ close to threshold. We construct a simple model in which the Higgs boson interacts with π , p and n via ^{*)}

$$\begin{aligned} \mathcal{L}_{int} = & \frac{M}{v} [\bar{p} p H + \bar{n} n H] \\ & + \frac{1}{2v} (m_H^2 + 2p_\pi^2 + 2p_\pi^2 - 2m_\pi^2) \pi_1 \cdot \pi_2 H \end{aligned} \quad (3.14)$$

where we have denoted $m_p \simeq m_n \equiv M$. The Feynman diagrams relevant to $\pi^- p \rightarrow Hn$ are shown in Fig. 5. The contributions of these graphs to the differential cross-section are found to be

$$\frac{d\sigma}{dt}(\pi^- p \rightarrow Hn) = \frac{\pi m_p m_n}{s |p_\pi^{c.m.}|^2} \left(\frac{1}{4\pi v^2} \right) \left(\frac{G^2}{4\pi} \right) |M|^2 \quad (3.15)$$

where G is the usual $\bar{p}n\pi^+$ coupling constant and

$$|M|^2 = \begin{cases} \frac{-t}{(t-m_\pi^2)^2} \left(\frac{m_H^2 + 2t}{2M} \right)^2 & (3.16a) \end{cases}$$

$$= \begin{cases} \frac{M^2 m_\pi^2}{(s-M^2)^2} & (3.16b) \end{cases}$$

$$= \begin{cases} \frac{M^2 m_\pi^2}{(u-M^2)^2} & (3.16c) \end{cases}$$

for diagrams 5a, b and c, respectively. We do not believe the precise numerical value of the cross-section given by (3.15) and (3.16), but we do believe it is likely to be of the right order of magnitude. In each of the cases (3.16a, b, c)

$$|M|^2 = O\left(\frac{m_\pi^2}{M^2}\right) \quad (3.17)$$

^{*)} The couplings to p and n follow from the dilaton theorem of Section 3.1 while motivation for the form of the $Hn\pi$ coupling is discussed in Section 4.2.

Substituting this into (3.15) and using

$$\frac{1}{f_\pi^2} = \sqrt{2} G_F$$

and the known values of G and f_π , we find :

$$\frac{d\sigma}{dt}(\pi^- p \rightarrow Hn) \simeq \frac{G_F m_\pi^2}{S |p_\pi^{c.m.}|^2} \simeq \frac{10^{-7}}{S |p_\pi^{c.m.}|^2} \quad (3.18)$$

This value may seem very small, but there is a proposal ^{21),38)} to look at the reaction $\pi^- p \rightarrow \pi^0 n$ near threshold with sufficient sensitivity to see the decay $\pi^0 \rightarrow e^+ e^-$ if it has a branching ratio $O(10^{-8})$. This experiment then has a fair chance of seeing $\pi^- p \rightarrow Hn$, $H \rightarrow e^+ e^-$ if $m_H \lesssim O(m_\pi)$.

Another possibility is to study $\gamma p \rightarrow H p$ ^{*}). In this case

$$\frac{d\sigma}{dt}(\gamma p \rightarrow H p) \simeq \left(\frac{\alpha}{\pi}\right) \frac{G_F m_\pi^2}{S |p_\gamma^{c.m.}|^2} \quad (3.19)$$

which is very small. Nevertheless the availability of high intensity low energy photon beams may make the reaction accessible, especially if we recall that the reaction could be studied below π production threshold to reduce background and look for a Higgs boson less massive than the pion.

3.2.2. - Intermediate energies

We now go to higher energies, say of the order of a few GeV. One candidate reaction is again $\pi^- p \rightarrow Hn$, where one looks for production via one pion exchange in the forward direction. If we now compare (3.15) and (3.16a) with the corresponding formula for $\pi^- p \rightarrow \rho^0 n$ via one pion exchange ^{**)}, we find

$$\frac{\frac{d\sigma}{dt}(\pi^- p \rightarrow Hn)}{\frac{d\sigma}{dt}(\pi^- p \rightarrow \rho^0 n)} \simeq \frac{(m_H^2 + 2t)^2 \sqrt{2} G_F m_\rho^2}{[t - (m_\rho - m_\pi)^2][t - (m_\rho + m_\pi)^2] g_{\rho\pi\pi}^2} \quad (3.20)$$

^{*}) We thank T. Ericson for suggesting to us this reaction.

^{**)} Absorptive corrections to one pion exchange may well cancel out in the ratio of cross-sections.

so that if $m_H^2 = O(m_\rho^2)$ and we consider $|t| = O(m_\pi^2)$ then

$$\frac{\frac{d\sigma}{dt}(\pi^- p \rightarrow H n)}{\frac{d\sigma}{dt}(\pi^- p \rightarrow \rho^0 n)} \approx \frac{m_H^4}{m_\rho^4} \sqrt{2} G_F f_\pi^2 \quad (3.21)$$

if we assume the KSFR relation $g_{\rho\pi\pi} \approx m_\rho/f_\pi$. Hence we again get

$$\frac{\frac{d\sigma}{dt}(\pi^- p \rightarrow H n)}{\frac{d\sigma}{dt}(\pi^- p \rightarrow \rho^0 n)} = O(G_F f_\pi^2) = O(10^{-7}) \quad (3.22)$$

This process therefore seems to have an unappealingly low cross-section, but for future reference we point out that $(G_F f_\pi^2)$ seems to be a typical ratio for Higgs boson production relative to the dominant hadronic reactions in similar kinematical conditions.

3.2.3. - High energies

We now consider H production in higher energy reactions, typically inclusive reactions at the higher CERN PS/BNL, Fermilab, ISR and CERN SPS energies. First we consider $pp \rightarrow H + X$ in a case where $m_H = O(m_\rho)$. In this case the lowest order contribution to the matrix element is expected from the theorem (3.2) to be

$$\langle X, H | pp \rangle = \frac{-i}{v} \langle X | \Theta_\mu^H | pp \rangle \quad (3.23)$$

In this mass range matrix elements of Θ_μ^H may ²⁰⁾ be dominated by the $\epsilon(700?)$ meson. If so, then previous analyses ²⁰⁾ suggest $\langle \Theta | \Theta_\mu^H | \epsilon \rangle = f_\epsilon m_\epsilon^2$ with $f_\epsilon = O(f_\pi)$. We would therefore have

$$\frac{\langle X, H | pp \rangle}{\langle X, \epsilon | pp \rangle} \approx \frac{f_\epsilon}{v} \quad (3.24)$$

and thence

$$\frac{\sigma(pp \rightarrow H + X)}{\sigma(pp \rightarrow \epsilon + X)} = O(G_F f_\pi^2) \quad (3.25)$$

This ratio accords with the production ratio (3.22) obtained previously by different methods in a different kinematic region, so we may be prepared to believe it independently of the somewhat questionable derivation. If we guess $\sigma(pp \rightarrow e+X) = O(1 \text{ to } 10) \text{ mb}$ at ISR energies by analogy with $\sigma(pp \rightarrow \rho+X)$ then we find $\sigma(pp \rightarrow H+X) = O(0.1 \text{ to } 1) \text{ nanobarns}$. One way of expressing this result is to observe that since the branching ratio for $\rho \rightarrow \mu^+ \mu^-$ is $O(5 \times 10^{-5})$ and we have estimated $(\sigma(pp \rightarrow H+X) / \sigma(pp \rightarrow \rho+X)) = O(10^{-7})$ then the Higgs meson is $500 \times (\Gamma(H \rightarrow \text{all}) / \Gamma(H \rightarrow \mu^+ \mu^-))$ more difficult to see than the ρ via its $(\mu^+ \mu^-)$ decay mode.

Let us now consider $pp \rightarrow H+X$ in the case where $m_H \gtrsim O(1) \text{ GeV}$, and we may use Drell-Yan ³⁹⁾ scaling arguments to estimate the production cross-section. Assuming that the standard quark parton-antiparton annihilation graphs of Fig. 6 dominate the production of H and of the $(\mu^+ \mu^-)$ continuum background, we find

$$\frac{\sigma(pp \rightarrow H+X)}{\sigma(pp \rightarrow (\mu^+ \mu^-)+X)} = \frac{[\sum_L M_q^2 + p^2 \sum_H M_q^2] \sigma(e^+ e^- \rightarrow H)}{M_e^2 [\frac{2}{3} + p^2 \sum_H Q_q^2] \sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} \quad (3.26)$$

In deriving (3.26) we have assumed the form $(m_q/v) \bar{q}qH$ for the couplings of H to quark partons with charges Q_q , have grouped partons into "light" ($L=u,d,s$) and "heavy" ($H=c,c' \dots ?$), and have assumed production at rest in the centre-of-mass from "sea" partons and anti-partons with distributions u_q such that

$$\left. \begin{aligned} u_u = u_{\bar{u}} = u_d = u_{\bar{d}} \approx u_s = u_{\bar{s}} \\ u_c = u_{\bar{c}} \approx u_{c'} = u_{\bar{c}'} \approx \dots \end{aligned} \right\} \text{ and } u_c/u_u \equiv p \quad (3.27)$$

If we now use

$$\sigma(e^+ e^- \rightarrow H) = \frac{\pi \sqrt{2}}{3} G_F M_e^2 \delta(q^2 - m_H^2)$$

$$\sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = \frac{4\pi}{3} \frac{\alpha^2}{q^2}$$

and integrate numerator and denominator in (3.26) over a range ΔQ in $Q \equiv \sqrt{q^2}$, then we find

$$\frac{\sigma(pp \rightarrow H \xrightarrow{\mu^+\mu^-} + X)}{\sigma(pp \rightarrow (\mu^+\mu^-) + X)} = \left[\frac{\sum_L m_q^2 + \rho^2 \sum_H m_q^2}{\frac{2}{3} + \rho^2 \sum_H Q_q^2} \right] \frac{G_F}{4\sqrt{2}\alpha^2} \left(\frac{Q}{\Delta Q} \right) \frac{\Gamma(H \rightarrow \mu^+\mu^-)}{\Gamma(H \rightarrow all)} \quad (3.28)$$

Then if an experimental $(\mu^+\mu^-)$ pair spectrum can be binned in mass ranges ΔQ , Eq. (3.28) will enable one to estimate the Higgs boson signal to background ratio. The numerical value of (3.28) is very sensitive to the value assumed for the parameter ρ , because $m_H \gg m_L$. If we suppose $\rho = 0$ [an SU(3) symmetric "sea"] or that there is one new quark c with $m_c \simeq 2$ GeV, and $\rho = 1$ [SU(4) symmetric "sea"] then we find

$$\frac{\sigma(pp \rightarrow H \xrightarrow{\mu^+\mu^-} + X)}{\sigma(pp \rightarrow (\mu^+\mu^-) + X)} \simeq \frac{\Gamma(H \rightarrow \mu^+\mu^-)}{\Gamma(H \rightarrow all)} \frac{Q}{\Delta Q} \times \begin{cases} 10^{-3}: \text{SU(3) case} \\ 10^{-1}: \text{SU(4) case} \end{cases} \quad (3.29)$$

The sophistication of present experiments is clearly insufficient to see $H \rightarrow \mu^+\mu^-$ if formula (3.29) is accepted. However, it is not impossible that if the SU(4) [or still better SU(6) or higher] symmetry estimate is correct, then the Higgs boson may show up *) in future $pp \rightarrow (\mu^+\mu^-) + X$ experiments - at least as long as $m_H < 4$ GeV so that, as will be discussed in Section 4, the $H \rightarrow \mu^+\mu^-$ branching ratio may not be completely negligible.

3.3. - Production via Decays of Other Particles

Since the Higgs coupling is semi-weak, one might a priori expect branching ratios into decay modes involving the Higgs boson to be $O(10^{-5})$. Such rates might not be inaccessible, for example in η decay where $\eta \rightarrow \mu^+\mu^-$ has been observed with a branching ratio of 10^{-5} , and particularly in K decay where branching ratios of $O(10^{-7})$ ($K^+ \rightarrow \pi^+ e^+ e^-$) and $O(10^{-8})$ ($K_L \rightarrow \mu^+\mu^-$) have been observed. Unfortunately the dilaton rôle (3.2) of the Higgs boson has the effect of suppressing these decay modes more strongly.

*) We will see in Section 4 that the branching ratio for $H \rightarrow e^+e^-$ is expected to be minute for $m_H > 2m_\mu$, so that a $pp \rightarrow (e^+e^-) + X$ experiment should not be vulnerable to the Higgs boson.

We shall see that the decay $K^+ \rightarrow \pi^+ + H$ might be accessible to experiment, but that the most favourable source of low-mass Higgs particles may be the newly discovered ²⁾ heavy vector mesons, where the competing modes are suppressed by Zweig's rule ²²⁾.

3.3.1. - $\Delta I = 1$ transitions : η and Σ decays

There are a priori two ways in which the decays

$$\eta \rightarrow \pi^0 + H, \quad \Sigma^0 \rightarrow \Lambda + H \quad (3.30)$$

might proceed. First, suppose ⁴⁰⁾ there is a tadpole term $\epsilon_3 u_3$ in the chiral symmetry breaking part $\mathcal{L}_{\text{break}}$ (2.19) of the strong Lagrangian, corresponding in the quark model to a u-d quark mass difference. Then the conventional scheme (2.11) for coupling the Higgs to hadrons is

$$\mathcal{L}_H = \mathcal{L}_{\text{break}} \left(\frac{v+H}{v} \right) \quad (3.31)$$

which gives a coupling $\epsilon_3 u_3 H/v$. There is then a contribution to η and Σ^0 decays with Higgs bosons of the form

$$\frac{1}{v} \langle \text{final} | \mathcal{L}_{\text{break}} | \text{initial} \rangle \quad (3.32)$$

The other contributions to η and Σ^0 decays come from higher order weak and electromagnetic effects.

However, it is clear that the contributions of (3.32) to $\eta \rightarrow \pi^0 + H$ and $\Sigma^0 \rightarrow \Lambda + H$ in fact vanish. This is because the matrix elements of $\mathcal{L}_{\text{break}}$ between physical states which are eigenstates of the strong Hamiltonian must vanish. Consider, for example, a phenomenological Lagrangian model for $\mathcal{L}_{\text{break}}$, including a u_3 term as in the work of Osborne and Wallace ⁴¹⁾. Introducing an octet P_i of pseudoscalar fields, $\mathcal{L}_{\text{break}}$ receives contributions

$$-\frac{1}{2} m_3^2 P_3^2 - \frac{1}{2} m_8^2 P_8^2 - \chi P_3 P_8 \quad (3.33)$$

where the P_3^2 and P_8^2 terms come from the $SU(3)$ and $SU(3) \times SU(3)$ breaking terms u_0 and cu_8 in $\mathcal{L}_{\text{break}}$, and the $P_3 P_8$ term comes from the conjectured ^{40),41)} tadpole I spin violating term $e_3 u_3$. The mass terms (3.33) must then be diagonalized to get the physical π^0 and η masses :

$$-\frac{1}{2} M_{\pi^0}^2 \pi^0{}^2 - \frac{1}{2} M_{\eta}^2 \eta^2 \quad (3.34)$$

The Higgs couplings (3.31), being proportional to (3.33), will also be proportional to (3.34) and hence diagonalized away. Thus the $e_3 u_3$ contribution to $\eta \rightarrow \pi^0 + H$ vanishes, and a similar argument applies to $\Sigma^0 \rightarrow \Lambda + H$.

For the other, higher order weak and electromagnetic, contributions to these decays, we will invoke the more general dilaton-like nature (3.2) of the Higgs boson. The effective $\Delta I=1$ quark operator which can induce the decays (3.30) is represented diagrammatically in Fig. 7. By virtue of Eqs. (3.2) and (3.13) this operator may be written in the form

$$\mathcal{L}_{\text{eff}}(\Delta I=1) = \frac{-ig}{2m_W} \bar{q} \frac{\tau_3}{2} q H (m_u - m_d + a m_H^2 m_q) : q = \begin{pmatrix} u \\ d \end{pmatrix} \quad (3.35)$$

The term proportional to $m_u - m_d$ is just the u_3 term discussed above. The remaining term represents the radiative corrections and must be $O(\alpha)$, so that we can neglect the (u-d) mass difference, as was done in writing this part of (3.35). The parameter a has dimension $[\text{mass}]^{-2}$, and allowing for strong interaction effects a reasonable guess is

$$a = O\left(\frac{\alpha}{m_e^2}\right) \quad (3.36)$$

where $m_e = O(700)$ MeV. Then we obtain for $\eta \rightarrow \pi^0 + H$, for example :

$$\begin{aligned} \langle \pi^0, H | \eta \rangle &\simeq -\frac{ig\alpha}{2m_W} \frac{m_H^2}{m_e^2} \langle \pi^0 | m_q \bar{q} \frac{\tau_3}{2} q | \eta \rangle \\ &= -\frac{ig\alpha}{2m_W} \frac{m_\pi^2}{\sqrt{3}} \frac{m_H^2}{m_e^2} \end{aligned} \quad (3.37)$$

yielding a branching ratio

$$R_\eta = \frac{\Gamma(\eta \rightarrow \pi^0 + H)}{\Gamma(\eta \rightarrow \text{all})} = O(10^{-8}) \quad (3.38)$$

for $m_H^2 \approx m_\pi^2$. However, as photon exchange is not a characteristically short-range phenomenon, the relevance of the quark operator (3.35) is not obvious. More generally, the requirements (a) to (d) of Section 3.2 allow a contribution of the form

$$\langle \pi^0, H | \gamma \rangle = \frac{-ig}{2m_W} \langle \pi | \Theta_\mu^\mu | \gamma \rangle \approx \frac{-ig\alpha}{2m_W} \frac{(m_\gamma^2 - m_\pi^2)^2}{m_\pi^2} \quad (3.39)$$

yielding

$$R_\gamma = O(5 \times 10^{-7}) \quad (3.40)$$

for $m_H^2 \approx m_\pi^2$. The somewhat different estimates (3.38) and (3.40) bracket our expectations, indicating a small branching ratio for $\eta \rightarrow \pi + H$.

For $\Sigma^0 \rightarrow \Lambda + H$ the branching ratio is even lower since the dominant amplitude $\Sigma^0 \rightarrow \Lambda + \gamma$ is only first order in e . Under assumptions analogous to those leading to (3.38) and (3.40) we estimate

$$R_{\Sigma^0} = \frac{\Gamma(\Sigma^0 \rightarrow \Lambda + H)}{\Gamma(\Sigma^0 \rightarrow \text{all})} < 10^{-9} \quad (3.41)$$

where we have taken ⁴²⁾ $\tau_{\Sigma^0} = (0.6 \pm 0.3) \times 10^{-19}$ sec. The amplitude for $\Sigma^0 \rightarrow \Lambda + H + \gamma$ is of lower order in e , and has a $1/k$ dependence in the H momentum, but the decay is inhibited by three-body phase space. From the diagrams of Fig. 8 we find the model independent result

$$\frac{\Gamma(\Sigma^0 \rightarrow \Lambda + H + \gamma)}{\Gamma(\Sigma^0 \rightarrow \Lambda + \gamma)} \approx 2.6 \times 10^{-8} F\left(\frac{2m_H m_\Lambda}{m_\Sigma^2 - m_\Lambda^2}\right) \quad (3.42)$$

where

$$F(x) = -\ln x - \frac{11}{6} + 3x - \frac{3}{2}x^2 + \frac{x^3}{3}$$

For a Higgs boson mass in the range $m_H = (20 \text{ to } 60) \text{ MeV}$ we find a branching ratio in the range $(6 \times 10^{-9} \text{ to } 3 \times 10^{-11})$.

3.3.2. - $|\Delta S|=1$ transitions : K decay

The $\Delta Q=0$, $|\Delta S|=1$ decay $K^+ \rightarrow \pi^+ e^+ e^-$ has now ²³⁾ been seen with a branching ratio $O(10^{-6}$ to $10^{-7})$, comparable with predictions ⁴³⁾ made using gauge theories and charm ^{1),3)}. Reference to Fig. 1 shows ^{*)} that over a range in $m_H < 2m_\mu$, $H \rightarrow e^+ e^-$ is a substantial decay mode. Hence it is interesting to use the gauge theory and charm ^{1),3)} framework to estimate $K^+ \rightarrow \pi^+ + H$, and see whether present or future experiments on K decay can eliminate a range of m_H .

Unfortunately, we are again hurt by the dilaton property (3.2) of the Higgs boson. Consider first the one-body quark operator derived from the diagrams of Fig. 9. Reference to Eq. (3.13) tells us that the effective transition operator for $s \rightarrow d + H$ is of the form

$$\mathcal{L}_{\text{eff}}(\Delta S=1) = -\frac{ig^2}{2m_W^2} m_s \bar{d} R s H [b m_s^2 + c m_H^2] \quad (3.43)$$

[For simplicity, in this section we will assume ^{**) m_u \simeq m_d \simeq 0} : R was defined after Eq. (3.13).] Moreover, the Glashow-Iliopoulos-Maiani mechanism ²⁾ requires that b and c $\rightarrow 0$ in the limit of quark mass degeneracy ($m_c \rightarrow 0$). The relevant question is whether this leads to an additional strong suppression

$$b, c \simeq \frac{m_c^2}{m_W^2} \quad (3.44)$$

as for $K_L \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \bar{\nu} \nu$, or whether the dependence on the exchanged quark mass is only logarithmic as in $K^+ \rightarrow \pi^+ e^+ e^-$ ⁴³⁾. To answer this question we calculated explicitly the diagrams of Fig. 9 in the 't Hooft-Feynman gauge and found in fact that the operator (3.43) vanishes to lowest order in $1/m_W$ so that

$$\mathcal{L}_{\text{eff}}(\Delta S=1) = O\left(g^3 \frac{m_{q,H}^5}{m_W^3}\right) \quad (3.45)$$

^{*)} Decay modes of H are discussed in more detail in Section 4.

^{**) The topic of quark masses is taken up in Section 4.}

Then taking the $K-\pi$ matrix element of the quark operator

$$m_s \bar{d} R_s = i \partial_\mu (\bar{d} \gamma^\mu L_s)$$

[R and L were defined after Eq. (3.13)] which is the divergence of the weak current, we find a contribution to the decay amplitude of order

$$\langle \pi^+, H | K^+ \rangle \leq \frac{-ig}{4m_W} G_F m_s^2 m_K^2 \frac{m_c^2}{m_W^2} \ln \left(\frac{m_W^2}{m_c^2} \right) \quad (3.46)$$

Could strong interactions modify the strong suppression (3.46) ? It seems unlikely that corrections to the short-distance behaviour are important : the cancellation holds in the presence of gluon exchange as long as all diagrams are effectively cut off at the same momenta, and the situation here is similar to that for $K_L \rightarrow \mu^+ \mu^-$ (*). To account for low energy effects, one might correct the $\bar{c}cH$ vertex by (**)

$$m_c \rightarrow m_c \left(1 + \frac{k^2}{m_{\epsilon_c}^2} \right)^{-1}$$

where ϵ_c is a $0^{++}(\bar{c}c)$ bound state presumably of mass ~ 3 GeV. Then the cancellation is destroyed for $k^2 \neq 0$, and we estimate

$$\langle \pi^+, H | K^+ \rangle \approx \frac{-ig}{4m_W} G_F m_s^2 m_K^2 \frac{m_H^2}{m_{\epsilon_c}^2} \quad (3.47)$$

probably with an extra factor of $1/\pi^2$ from the loop integration.

There will also be contributions from two-body operators as illustrated in Fig. 10. The effective operator arising from the diagram of Fig. 10a is simply related to the usual non-leptonic $|\Delta S|=1$ transition operator :

*) The behaviour of the $\bar{s}dH$ vertex is similar to that of the $\bar{s}d\phi_2$ vertex discussed by Gaillard, Lee and Schrock (44). However, opinion is not unanimous - see Ref. 45).

**) This is consistent only if the contributions involving this vertex are separately gauge invariant. We have not checked this.

$$\begin{aligned}
 \mathcal{L}_{\text{eff}}^{10a} &= \frac{-ig^3}{2m_w^3} H (\bar{u} \delta_\mu L d) (\bar{S} \delta^\mu L u) + (\text{h.c.}) + O\left(\frac{m_q^2}{m_w^5}, \frac{m_H^2}{m_w^5}\right) \\
 &= \frac{g}{2m_w} H \mathcal{L}_{\text{eff}}^{\text{Nonleptonic}} + O\left(\frac{m^2}{m_w^5}\right)
 \end{aligned} \tag{3.48}$$

Diagrams of the type in Fig. 10b yield non-local operators which induce, among others, the pole diagrams of Fig. 11. A simple estimate, consistent with the dilaton-like coupling of the Higgs boson, is to sum the contributions of Figs. 10 and 11. The result vanishes for $m_H \rightarrow 0$: defining

$$h(k^2) = \langle \pi^+(p-k) | \mathcal{L}_{\text{eff}}^{\text{Nonleptonic}} | K^+(p) \rangle \simeq h \left(1 - \frac{k^2}{m_K^2}\right)$$

where $\mathcal{K}$ is a conjectured scalar πK resonance with $m_{\mathcal{K}} = O(1)$ GeV, and using broken scale invariance ²⁰⁾ [see Section 4, especially Eq. (4.5)] to estimate the k^2 dependence of the $\pi\pi H$ and $K\bar{K}H$ vertices, we find

$$\langle \pi^+, H | K^+ \rangle \simeq \frac{-ig m_H^2 h}{2m_w} \frac{1}{m_{\mathcal{K}}^2} \tag{3.49}$$

The parameter h is related through soft pion theorems to the $K_S \rightarrow 2\pi$ decay amplitude. Then using either (3.47) or (3.49) we obtain a branching ratio

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + H)}{\Gamma(K^+ \rightarrow \text{all})} \simeq 10^{-7} \quad \text{for } m_H^2 \simeq m_\pi^2 \tag{3.50}$$

i.e., not far below the observed level ²³⁾ for $K^+ \rightarrow \pi^+ e^+ e^-$. We should observe that a priori an amplitude like (3.47) or (3.49) but with m_H^2 replaced by $(m_K^2 - m_\pi^2)^2 / m_h^2$ where $m_h = O(1)$ GeV is also possible. Such an amplitude would raise the branching ratio (3.50) above the experimental ²³⁾ rate for $K^+ \rightarrow \pi^+ e^+ e^-$, but we have been unable to construct a phenomenological model for such a term.

A similar estimate to (3.50) would hold for $K_L \rightarrow \pi^0 + H$. The process $K^0 \rightarrow 2H$ might be thought competitive with $K^0 \rightarrow \mu^+ \mu^-$, but by CP invariance contributes only to K_S decay.

3.3.3. - Heavy particle decay

We conclude this section with the more optimistic possibility that the decay of one of the new ²⁾ narrow vector resonances $3.7 \rightarrow 3.1 + H$ may be observable. We estimate the branching ratio by comparison with the decays $\rho'(1600) \rightarrow \rho + \pi + \pi$ and $3.7 \rightarrow 3.1 + \pi + \pi$, where we assume the $(\pi\pi)$ system is dominated by the $\epsilon(700?)$ meson. Zweig's rule ²²⁾ tells us that the hadronic mass scale associated with the amplitude for $3.7 \rightarrow 3.1 + H$ is $O(3 \text{ GeV})$, and since we must have $m_H \lesssim 600 \text{ MeV}$ for the process to occur, we write

$$\langle 3.1, H | 3.7 \rangle = \frac{-ig}{2m_w} \left[\langle 3.1 | \theta_\mu^\mu | 3.7 \rangle \Big|_{k^2=0} + O\left(\frac{m_H^2}{(3.7 \text{ GeV})^2}\right) \right] \quad (3.51)$$

Now consider the decay $\rho' \rightarrow \rho + \pi + \pi$. The assumption of ϵ dominance of the $\rho' - \rho$ coupling to θ_μ^μ determines the $\rho' \rho \epsilon$ coupling by

$$\langle \rho | \theta_\mu^\mu | \rho' \rangle \approx g_{\epsilon \rho' \rho} f_\epsilon \quad (3.52)$$

where $\langle 0 | \theta_\mu^\mu | \epsilon \rangle = f_\epsilon m_\epsilon^2$ with ²⁰⁾ $f_\epsilon = O(m_\pi)$. Defining

$$g_{\theta \rho' \rho} = \langle \rho | \theta_\mu^\mu | \rho' \rangle \Big|_{k^2=0}$$

we obtain

$$\frac{\Gamma(3.7 \rightarrow 3.1 + H)}{\Gamma(\rho' \rightarrow \rho + \epsilon)} \approx \sqrt{2} g_F f_\epsilon^2 \left(\frac{g_{\theta, 3.7, 3.1}}{g_{\theta \rho' \rho}} \right)^2 \frac{p_H}{p_\epsilon} \frac{m_{\rho'}^2}{(3.7 \text{ GeV})^2} \quad (3.53)$$

where p_H and p_ϵ are the momenta of the Higgs and ϵ in the respective decays.

The divergence conditions for $\theta^{\mu\nu}$ tell us that the matrix elements (3.52) must be at least quadratic in the momentum transfer k :

$$g_{\theta \rho' \rho} = a_\nu [(p+p') \cdot k]^2 \epsilon \cdot \epsilon' + b_\nu (\epsilon \cdot k)(\epsilon \cdot k') + \dots \quad (3.54)$$

where ϵ and ϵ' are the polarization vectors of V and V' . Since the polarization vector ϵ will be averaged over, the effective momentum dependence can only be through the variable $(p+p') \cdot k = m_V^2 - m_V'^2$. So, without loss of generality we retain only the first term on the right-hand side of (3.54) getting :

$$\frac{\Gamma(3.7 \rightarrow 3.1 + H)}{\Gamma(\rho' \rightarrow \rho + \epsilon)} \simeq 4.6 \times 10^{-7} \frac{p_H}{p_\epsilon} \left(\frac{a_{3.1}}{a_\rho} \right)^2 \quad (3.55)$$

In the $SU(4)$ limit, $a_{3.1} = a_\rho$: but a_V has dimension $[\text{mass}]^{-2}$, and we are faced with the usual problem of how to parametrize in the broken symmetry case. If p_H is a few hundred MeV, we find for example

$$\frac{\Gamma(3.7 \rightarrow 3.1 + H)}{\Gamma(3.7 \rightarrow \text{all})} = \text{a few} \times \begin{cases} 2 \times 10^{-4} : a_{3.1} = a_\rho \\ 0.8 \times 10^{-5} : \frac{a_{3.1}}{a_\rho} = \frac{m_{\rho'}^2}{(3.7 \text{ GeV})^2} \end{cases} \quad (3.56)$$

An alternative estimate is obtained by starting from the decay $3.7 \rightarrow 3.1 + \pi + \pi$ and assuming

$$g_{3.7, 3.1, \epsilon} = Z g_{3.7, 3.1, \epsilon_c} \quad (3.57)$$

where Z is a Zweig²²⁾ suppression factor. Next we determine the ϵ_c coupling by assuming ϵ_c dominance of the Θ_{μ}^{μ} matrix element, getting

$$g_{\Theta, 3.7, 3.1} = f_{\epsilon_c} g_{\epsilon_c, 3.7, 3.1} \quad (3.58)$$

where we have introduced f_{ϵ_c} by analogy with f_ϵ :

$$\langle 0 | \Theta_{\mu}^{\mu} | \epsilon_c \rangle = f_{\epsilon_c} m_{\epsilon_c}^2$$

Combining (3.57) and (3.58) we can then write

$$\frac{\Gamma(3.7 \rightarrow 3.1 + H)}{\Gamma(3.7 \rightarrow 3.1 + \epsilon)} = \frac{\sqrt{2} G_F f_{\epsilon_c}^2}{Z^2} \frac{p_H}{p_\epsilon} \quad (3.59)$$

where p_e is the effective e momentum, determined by the phase space integral weighted by the resonance factor to be $p_e \simeq 60$ MeV. Then for $p_H = 0$ (few hundred MeV) and $Z^2 \simeq 10^{-3}$ we find

$$\frac{\Gamma(3.7 \rightarrow 3.1 + H)}{\Gamma(3.7 \rightarrow \text{all})} \simeq \text{a few} \times \begin{cases} 1.4 \times 10^{-4} & : f_{e_c}^2 = f_e^2 \\ 2.3 \times 10^{-5} & : m_{e_c} f_{e_c}^2 = m_e^2 f_e^2 \end{cases} \quad (3.60)$$

The second assumption for f_{e_c} is made in analogy with the observed suppression of the photon coupling to the 3.1 vector meson relative to the $\gamma - \rho^0$ coupling.

We could, of course, choose mass factors so as to suppress further the estimates (3.56) and (3.60). However, the important point is that the Zweig²²⁾ rule, by suppressing the total hadronic width of the 3.7 state, is helping to overcome the effective Higgs coupling $O(g_{F_e}^2) = O(10^{-7})$ so that a branching ratio for $3.7 \rightarrow 3.1 + H$ of $O(10^{-4})$ is not unthinkable.

3.4. - Production by Bremsstrahlung

Just as conservation of the electromagnetic current leads to low energy theorems for photon emission, energy momentum conservation leads to low energy theorems for Higgs boson emission. Following Low⁴⁶⁾, we define the general amplitude^{*)} $\mathcal{M}^{\mu\nu}(p_1, \dots, p_n, k)$ illustrated in Fig. 12a, and expand in powers of k . The momenta p_i are taken on their mass shells $p_i^2 = m_i^2$, and $\Theta_{\mu\nu}(k)$ is the Fourier transform of the improved³⁵⁾ energy momentum tensor. Then $\mathcal{M}^{\mu\nu}$ may be expressed in terms of two contributions :

$$\mathcal{M}^{\mu\nu}(p_1, \dots, p_n, k) = \sum_i \mathcal{M}(p_1, \dots, p_i - k, \dots, p_n) i \Delta^i(p_i - k) Q_i^{\mu\nu}(p_i, k) + \mathcal{M}_0^{\mu\nu}(p_1, \dots, p_n, k) \quad (3.61)$$

where Δ^i is the propagator for the i 'th particle, $Q_i^{\mu\nu}$ is the vertex function of Fig. 12b, and $\mathcal{M}$ is the n point function of Fig. 12c. By definition the amplitude $\mathcal{M}_0^{\mu\nu}$ has no singularities as $k \rightarrow 0$.

*) The low energy theorem for matrix elements of Θ_{μ}^{μ} has been given by Mack⁴⁷⁾ in a slightly different form.

The vertex function $Q_i^{\mu\nu}$ obeys the Ward identity ⁴⁸⁾

$$k_\mu Q_i^{\mu\nu}(p_i, k) = i \left[\Delta_i(p_i - k) \right]^{-1} \left\{ p_i^\nu - \frac{1}{2} k_\mu \Sigma_i^{\mu\nu} \right\} \quad (3.62)$$

when $p_i^2 = m_i^2$, where $\Sigma_i^{\mu\nu}$ is the covariant spin matrix for the i 'th particle. The divergence condition for the full amplitude (3.61) then gives

$$\begin{aligned} k_\mu \mathcal{M}^{\mu\nu}(p_1, \dots, p_n, k) &= 0 \\ &= - \sum_i \mathcal{M}(p_1, \dots, p_i - k, \dots, p_n) \left\{ p_i^\nu - \frac{1}{2} k_\mu \Sigma_i^{\mu\nu} \right\} + k_\mu \mathcal{M}_0^{\mu\nu}(p_1, \dots, p_n, k) \end{aligned} \quad (3.63)$$

For a symmetric amplitude $\Gamma^{\mu\nu}(k)$ with no singularities, the divergence condition $k_\mu \Gamma^{\mu\nu}(k) = k_\mu \Gamma^{\nu\mu}(k) = \Gamma^\nu(k)$ determines $\Gamma^{\mu\nu}$ to $O(k)$:

$$\Gamma^{\mu\nu}(k) = \frac{1}{2} \left[\partial^\mu \Gamma^\nu(k) + \partial^\nu \Gamma^\mu(k) - k_\rho \partial^\mu \partial^\nu \Gamma^\rho(k) \right] + O(k^2) : \partial^\mu \equiv \frac{\partial}{\partial k_\mu}$$

Therefore we may use Eqs. (3.61) and (3.62) to determine $Q^{\mu\nu}$ and $\mathcal{M}_0^{\mu\nu}$, respectively, to $O(k)$. Since $\Delta_i(p_i - k) \sim 1/k$, the full amplitude (3.61) is thus determined to $O(k^0)$. Expanding the n point function

$$\mathcal{M}(p_1, \dots, p_i - k, \dots, p_n) = \mathcal{M}(p_1, \dots, p_i, \dots, p_n) - k^\mu \frac{\partial}{\partial p_i^\mu} \mathcal{M}(p_1, \dots, p_i, \dots, p_n) + \dots$$

and using conservation of total four-momentum : $\sum_{i=1}^n p_i = k$ and of total angular momentum :

$$\sum_{i=1}^n \Sigma_i^{\mu\nu} = - \sum_{i=1}^n L_i^{\mu\nu} + k^\mu \frac{\partial}{\partial k^\nu} - k^\nu \frac{\partial}{\partial k^\mu}$$

where

$$L_i^{\mu\nu} \equiv p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}}$$

is the orbital angular momentum, the amplitude for Higgs boson emission

$$\langle i+1, \dots, n, H | 1, 2, \dots, i \rangle = \frac{-ig}{2m_w} \mathcal{M}_\mu^{\mu\nu}(p_1, \dots, p_n, k)$$

may be cast in the form (for spins ≤ 1 : α and β are indices in vector meson spin space with $g_{\beta}^{\alpha} = \delta_{\alpha\beta}$) :

$$\begin{aligned} & \langle i+1, \dots, n, H | 1, 2, \dots, i \rangle \\ &= \frac{ig}{2m_W} \left\{ \sum_{i=1}^n \left[\frac{2m_i^2}{2p_i \cdot k - m_H^2} - \left(\frac{m_i^2 k^\mu}{2p_i \cdot k - m_H^2} - p_i^\mu \right) \frac{\partial}{\partial p_i^\mu} \right] + 4 - n_B - \frac{3}{2} n_f \right\} \mathcal{M}(p_1, \dots, p_n) \\ & - \frac{ig}{2m_W} \mathcal{M}(p_1, \dots, p_n) \left\{ \sum_{i,s=\frac{1}{2}}^n \frac{k m_i}{2p_i \cdot k - m_H^2} + \sum_{i,s=1}^n \frac{p_i^\alpha k_\beta}{2p_i \cdot k - m_H^2} \right\} + O(k) \end{aligned} \quad (3.64)$$

where n_B and n_f are the total numbers of bosons and fermions, respectively, and the momenta p_i are defined to be flowing inwards. As in the case of photon emission, derivatives of $\mathcal{M}$ with respect to the external masses cancel out, as do unphysical amplitudes which may appear for spin > 0 . The terms which are singular as $k \rightarrow 0$ are just those expected for the bremsstrahlung emission of a Higgs boson from an external line with coupling proportional to $2m_i^2(m_i)$ for bosons (fermions). The non-pole terms depend on the dimension n_{eff} of the effective coupling constant associated with the amplitude $\mathcal{M}$. It is readily verified that they vanish for $n_{\text{eff}} = 4$ as for effective couplings of the type

$$\mathcal{L}_{\text{eff}} = g \phi^4, \quad g \phi \bar{\psi} \psi, \quad g V_\mu \phi_1 \overleftrightarrow{\partial}^\mu \phi_2, \dots$$

Now let us consider the leading terms in Eq. (3.64), i.e., those of order $1/k$. If we are interested in the Higgs bremsstrahlung associated with a single particle in an inclusive reaction :

$$A+B \rightarrow C(p) + H(k) + X \quad : \quad p^2 = m_C^2$$

the bremsstrahlung cross-section is

$$d\sigma_C^H(p, k, \theta) = \sqrt{2} g_F \frac{m_C^4}{2\pi^2} \frac{k dk_0 d(\cos\theta)}{(2p \cdot k + m_H^2)^2} d\sigma_C(p) + O(k^0) \quad (3.65)$$

where

$$d\sigma_C(p) = d\sigma(A+B \rightarrow C(p) + X)$$

and θ is the angle between $\underline{p}$ and $\underline{k}$. In the rest frame of C , (3.65) becomes

$$d\sigma^H(p, k) \approx \frac{k dk_0}{k_0^2} \sqrt{2} g_F \frac{m_c^2}{4\pi^2} d\sigma_c(p) \quad (3.66)$$

Formula (3.66) yields, for example, bremsstrahlung associated with the 3.1 resonance at the low level of

$$d\sigma_{3.1}^H(p, k) \approx \frac{k dk_0}{k_0^2} (4 \times 10^{-6}) d\sigma_{3.1}(p) \quad (3.67)$$

The ideal sources of Higgs bosons are, of course, the heavy intermediate bosons for which the Higgs couplings (2.4), (2.6) become strong. For $m_W = 0(100) \text{ GeV}$:

$$d\sigma_W^H(p, k) \approx \frac{k dk_0}{k_0^2} (4 \times 10^{-3}) d\sigma_W(p) \quad (3.68)$$

The specific examples of (3.67) and (3.68) are taken up in detail for e^+e^- annihilation in Section 3.5. For the moment we make some general remarks about the formulae (3.65) and (3.66).

- (i) For a relativistic particle with $E^2 \gg m^2$, the associated Higgs emission is peaked at an energy $k_0 = Em_H/m$ and at an angle $\theta < m/E$.
- (ii) Note that for a system of particles interacting at rest the bremsstrahlung emission from initial and final particles interferes destructively :

$$d\sigma^H(p_i \approx 0) \sqrt{2} g_F \frac{k dk_0}{k_0^2} \left(\sum_{\text{out}} m_i - \sum_{\text{in}} m_i \right)^2 d\sigma(p_i \approx 0) \quad (3.69)$$

Thus bremsstrahlung associated with elastic scattering off a heavy nucleus for example is suppressed, as is coherent photoproduction $\gamma + Z \rightarrow Z + H$.

- (iii) On the other hand, the k independent term in (3.64) presents the amusing possibility of enhancing Higgs production in multiparticle reactions. Consider, for example, $p\bar{p}$ annihilation at rest : $p\bar{p} \rightarrow n\pi$ where n is odd. For sufficiently large n the final state will be in an s wave. Then $\mathcal{M}(p_i)$ is momentum independent and

$$\frac{\sigma(\bar{p}p \rightarrow n\pi + H)}{\sigma(\bar{p}p \rightarrow n\pi)} \approx \frac{(n-1)! k_{\max}^2}{8\pi^2} \sqrt{2} G_F \quad (3.70)$$

if $k_{\max}^2$ is not so large as to appreciably reduce $(n\pi)$ phase space. For $k_{\max} \sim 300$ MeV and $n=9$ we find $\sigma(H)/\sigma \sim 10^{-6}$: still discouragingly small.

3.5. - Production in e^+e^- Collisions

There are at least three ways the Higgs boson could be produced in e^+e^- collisions : one of them is indirectly via the production and decay of the 3.7 resonance discussed in Section 3.3.3. Two other natural ways to consider are the following.

3.5.1. - Direct production $e^+e^- \rightarrow H \rightarrow \text{all}$ ⁴⁹⁾

The situation here is far worse than in pp collisions, because of the small coupling of the Higgs boson to electrons. For $e^+e^- \rightarrow H \rightarrow \text{all}$ we have

$$\sigma_{\text{peak}}(e^+e^- \rightarrow H \rightarrow \text{all}) = \frac{2\pi^2}{m_H^2} \frac{\Gamma(H \rightarrow e^+e^-)}{\Delta E} \quad (3.71)$$

where ΔE is the beam-beam centre-of-mass energy resolution, and we have assumed $\Gamma(H \rightarrow \text{all}) \ll \Delta E$. Then by comparison with the 3.1 resonance :

$$\frac{\sigma_{\text{peak}}(e^+e^- \rightarrow H \rightarrow \text{all})}{\sigma_{\text{peak}}(e^+e^- \rightarrow 3.1 \rightarrow \text{all})} = \frac{\Gamma(H \rightarrow e^+e^-)}{\Gamma(3.1 \rightarrow e^+e^-)} \times \frac{1}{3} \times \frac{m_{3.1}^2}{m_H^2} \quad (3.72)$$

Taking $\Gamma(3.1 \rightarrow e^+e^-) \approx 5$ keV and presuming $\Gamma(H \rightarrow e^+e^-) \approx 1.7 \times 10^{-13} m_H$ (see Section 4) we get

$$\frac{\sigma_{\text{peak}}(H)}{\sigma_{\text{peak}}(3.1)} \approx \frac{10^{-7}}{m_H} \quad (3.73)$$

when m_H is expressed in units of 1 GeV. Thus the prospects of seeing the Higgs boson produced directly in e^+e^- collisions seem negligible.

3.5.2. - Production by $e^+e^- \rightarrow Z^0 + H$ or $3.1 + H$

These processes would proceed by the bremsstrahlung mechanism studied in Section 3.4, and come from the diagrams of Fig. 13. We have calculated these two ways : from first principles and using the general bremsstrahlung formula (3.66). We find that for $e^+e^- \rightarrow Z^0 \rightarrow Z^0 + H$ in the Weinberg-Salam ¹¹⁾ model : $\sigma_{\mu^+\mu^-}$ is the pure QED $\sigma(e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-)$

$$\frac{\sigma_Z^H}{\sigma_{\mu^+\mu^-}} = \frac{\sqrt{2} G_F m_Z^2}{4 e^2 \sqrt{m_H^2 + \frac{\Gamma^2}{4}}} \approx \frac{17}{\sqrt{m_H^2 + \frac{\Gamma^2}{4}}} \quad (3.74)$$

where m_H is expressed in units of GeV, and we have allowed for the effects of a finite Z^0 decay width Γ , expected to be $O(1)$ GeV. The cross-section ratio (3.74) has been evaluated at the optimum centre-of-mass energy, which is $\sqrt{s} = m_{Z^0} + \sqrt{2} \sqrt{m_H^2 + \Gamma^2/4}$ in the Weinberg-Salam model with the experimentally favoured value of θ_w , $m_{Z^0} \simeq 75$ GeV.

A similar application of the bremsstrahlung formula (3.66) can be made to $e^+e^- \rightarrow V \rightarrow V + H$, where V is some narrow hadronic vector meson (e.g., the 3.1 resonance ²⁾). We find

$$\frac{\sigma_V^H}{\sigma_{\mu^+\mu^-}} = (0.048) \frac{m_V^2}{m_\rho^2} \frac{\Gamma(V \rightarrow e^+e^-)}{m_H} \quad (3.75)$$

For the 3.1 resonance with $\Gamma(3.1 \rightarrow e^+e^-) \simeq 5$ keV, Eq. (3.75) yields

$$\frac{\sigma_{3.1}^H}{\sigma_{\mu^+\mu^-}} \simeq \frac{1.6 \times 10^{-3}}{m_H} \quad (3.76)$$

where m_H is expressed in units of MeV, and the cross-section has been evaluated at the optimal energy $\sqrt{s} \simeq 3.1 + \sqrt{2} m_H$.

There are two other ways one might imagine looking for the Higgs boson in e^+e^- collisions. One is bremsstrahlung from a heavy hadron or lepton pair $L^\pm$: $e^+e^- \rightarrow L^\pm L^\mp H$. The basic bremsstrahlung formula (3.66) indicates this would be small for $m_{L^\pm} = O(2 \text{ GeV})$. Another possibility is the two-photon process $\gamma\gamma \rightarrow H$, where the $\gamma\gamma$ pair comes either from colliding e^+e^- rings, or in the Primakoff effect. Anticipating the $H \rightarrow \gamma\gamma$ decay rate calculation of Section 4, we find that characteristically, for $m_H \simeq m_\pi$ to 3 GeV, the ratio of $\Gamma(H \rightarrow \gamma\gamma)$ to the $\gamma\gamma$ decay width of a typical hadron h of similar mass, is

$$\frac{\Gamma(H \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)} \approx O(10^{-5})$$

Thus Primakoff production of a low mass Higgs boson is barely conceivable : the $\gamma\gamma$ process looks hopeless otherwise.

4. - DECAYS OF THE HIGGS BOSON

In the simplest Weinberg-Salam ¹¹⁾ scheme of Section 2, the Higgs boson was directly responsible for giving e , μ and quarks their masses, and had couplings to fundamental fermions $g_{H\bar{f}f} = 1/v m_f$ where $v = (\sqrt{2} G_F)^{-1/2}$. Many decays of the Higgs boson are then easy to calculate.

4.1. - Decays to e^+e^- , $\mu^+\mu^-$

These have been calculated ¹²⁾ as

$$\Gamma(H \rightarrow e^+e^-) = \frac{G_F m_e^2}{4\sqrt{2}\pi} m_H \left[1 - \frac{4m_e^2}{m_H^2} \right]^{3/2} \quad (4.1)$$

with $\Gamma(H \rightarrow \mu^+\mu^-)$ given by replacing $m_e \rightarrow m_\mu$ in (4.1). Thus

$$\begin{aligned} \Gamma(H \rightarrow e^+e^-) &= 1.7 \times 10^{-13} m_H \left[1 - \frac{4m_e^2}{m_H^2} \right]^{3/2} \\ \Gamma(H \rightarrow \mu^+\mu^-) &= 7.3 \times 10^{-9} m_H \left[1 - \frac{4m_\mu^2}{m_H^2} \right]^{3/2} \end{aligned} \quad (4.2)$$

and the decay to $\mu^+\mu^-$ already dominates the decay to e^+e^- for m_H just above $2m_\mu$.

4.2. - Decays to Hadrons

The inclusive decays to hadrons can be estimated using the quark parton model analogously to $e^+e^- \rightarrow \text{hadrons}$. Assuming the usual three coloured quarks, for $m_H \gg m_\mu, m_u, m_d$:

$$\frac{\Gamma(H \rightarrow \text{non-strange hadrons})}{\Gamma(H \rightarrow \mu^+ \mu^-)} \approx \frac{3(m_u^2 + m_d^2)}{m_\mu^2} \quad (4.3)$$

Estimates of m_u and m_d vary between $0(10) \text{ MeV}$ ²⁴⁾ and $0(300) \text{ MeV}$ ²⁵⁾, the former corresponding more to the "current" quark picture used in deriving (4.3). For this range of quark masses, the ratio (4.3) varies between about 0.06 and about 50. We do not know at what value of m_H the formula (4.3) should start being applicable, but estimates of exclusive contributions to $\Gamma(H \rightarrow \text{non-strange hadrons})$ favour a large rate in the region $m_H < 2 \text{ GeV}$.

Consider $H \rightarrow \pi\pi$ close to threshold : to estimate this we use the dilaton theorem (3.2) :

$$\langle \pi^+ \pi^- | H \rangle = \frac{-i}{f} \langle \pi^+ \pi^- | \theta_\mu^\mu | 0 \rangle \quad (4.4)$$

and then use a broken scale invariance ²⁰⁾ estimate of $\langle \pi^+ \pi^- | \theta_\mu^\mu | 0 \rangle$:

$$\langle \pi^+(p) \pi^-(q) | \theta_\mu^\mu(k) | 0 \rangle \approx 2(p^2 + q^2 - m_\pi^2) + k^2 + \dots \quad (4.5)$$

as a power series in p^2, q^2 and k^2 ^{*}). Evaluating (4.5) at the physical point $p^2 = q^2 = m_\pi^2$ and $k^2 = m_H^2$, and substituting into (4.4) we get

$$\langle \pi^+ \pi^- | H \rangle \approx \frac{-i}{f} (m_H^2 + 2m_\pi^2)$$

and so, close to the $\pi\pi$ threshold

$$\Gamma(H \rightarrow \pi\pi) \approx \frac{3}{16\sqrt{2}\pi} G_F^2 m_H^3 \sqrt{1 - \frac{4m_\pi^2}{m_H^2}} \left(1 + \frac{2m_\pi^2}{m_H^2}\right)^2 \quad (4.6)$$

^{*}) This estimate was used for the $\pi\pi H$ vertex in Section 3.2.

Comparing the decay rates (4.1) and (4.6) it is evident that the decay to $\pi\pi$ dominates the decay to $\mu^+\mu^-$ already for m_H not far above $\pi\pi$ threshold. For higher $(\pi\pi)$ energies we might expect the $H \rightarrow \pi\pi$ decay to proceed via an intermediate $e(700?)$ meson if it exists, as indicated in Fig. 14. The dilaton theorem (3.2) implies a transition

$$\langle e | H \rangle = -\frac{i}{v} f_e m_e^2 \quad (4.7)$$

and we find

$$\Gamma(H \rightarrow \pi\pi) \simeq \frac{3}{4\sqrt{2}\pi} G_F^2 m_H^3 \frac{f_e^2 g_{e\pi\pi}^2}{(m_H^2 - m_e^2)^2 + m_e^2 \Gamma_e^2} \sqrt{1 - \frac{4m_\pi^2}{m_H^2}} \quad (4.8)$$

for $m_H \simeq m_e$. The formula (4.8) again indicates that $H \rightarrow \pi\pi$ strongly dominates over $H \rightarrow \mu^+\mu^-$ in this mass range. The upper $(\pi\pi)$ branching ratio curve in Fig. 1 is obtained from the formulae (4.6) and (4.8) with a reasonable interpolation between them and an extrapolation to $m_H \simeq 1$ GeV. The lower $(\pi\pi)$ branching ratio curve in Fig. 1 was obtained by arbitrarily reducing these calculations by a factor of 10 to 100, so as to connect up more naturally with the light quark value for the branching ratio (4.3) at higher values of m_H . We are rather sceptical about a very low value for (4.3) in the region up to $m_H = 2$ GeV, as estimates of contributions like $H \rightarrow \rho\rho$ also give a large branching ratio.

For $m_H \gtrsim 1$ GeV, the coupling to strange quarks presumably becomes dominant : asymptotically

$$\frac{\Gamma(H \rightarrow \text{strange hadrons})}{\Gamma(H \rightarrow \mu^+\mu^-)} \simeq \frac{3m_s^2}{m_\mu^2} \quad (4.9)$$

with estimates of m_s varying between $O(150)$ MeV²⁴⁾ and $O(500)$ MeV²⁵⁾ yielding values for (4.9) between 6 and 60. The corresponding branching ratios (4.9) are plotted in Fig. 1 : an estimate of the $K\bar{K}$ contribution lies in the same ball-park close to threshold.

At $m_H \approx 4$ GeV, the couplings to new hadronic (charm ?) or leptonic (heavy lepton ?) degrees of freedom presumably take over. In the simplest SU(4) charm picture we would have

$$\frac{\Gamma(H \rightarrow \text{charmed hadrons})}{\Gamma(H \rightarrow \mu^+ \mu^-)} \approx \frac{3m_c^2}{m_\mu^2} \approx 600 \quad (4.10)$$

for $m_c \approx 1.5$ GeV. We conclude that in view of the invisibility of charmed particles to date, the detection of a Higgs boson with mass $m_H > 4$ GeV may be non-trivial.

4.3. - Decays to $\gamma\gamma$

Since the photon is massless, these decays do not proceed directly, but via virtual intermediate states, such as electrons, muons, hadrons and intermediate bosons. ¹²⁾ The decay rate is

$$\Gamma(H \rightarrow 2\gamma) = \frac{\alpha^2}{8\sqrt{2}\pi^3} G_F m_H^3 |I|^2 \quad (4.11)$$

where $I = I_e + I_\mu + I_{\text{hadrons}} + I_W + \dots$ is calculated from the diagrams of Fig. 15. The contributions I_ℓ of leptons (e, μ , heavy lepton ?) are known ¹²⁾: the function $I_\ell(m_\ell^2/m_H^2)$ is plotted in Fig. 16. To estimate the contribution I_{hadrons} we use the dilaton theorem (3.2) and broken scale invariance arguments ^{20), 50)}. In Ref. 50) it was shown that the matrix element $\langle \gamma\gamma | \theta_\mu^\mu(k) | 0 \rangle$ for k^2 small was determined by a scale invariance anomaly related to the high energy behaviour of $e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}$. In the notation of (4.11) this corresponds to

$$I_{\text{hadrons}} = \frac{R}{2} : R = \lim_{s \rightarrow \infty} \frac{\sigma(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-)} \quad (4.12)$$

In the simplest SU(4) charm model we would get $I_{\text{hadrons}} = 10/6$. This estimate is supposed to work for $m_H^2 \lesssim$ a typical hadronic (mass)² scale. For large values of m_H^2 , I_{hadrons} presumably $\rightarrow 0$ in a way similar to the behaviour of I_ℓ shown in Fig. 16.

Finally we must include the intermediate vector boson loop contribution to the $\gamma\gamma$ decay mode. This involves a long but straightforward calculation of the Feynman diagrams of Fig. 17, where $H^\pm$ is an

unphysical Higgs boson and $\phi^\pm$ a Fadeev-Popov ghost. The relevant Feynman rules have been extracted from Appendix C.2 of Fujikawa et al.³⁷⁾, and are displayed in Fig. 18, where the 't Hooft-Feynman⁵¹⁾ gauge has been adopted. There is an extra minus sign for each closed loop of the ghost field $\phi^\pm$. To handle divergent integrals we use dimensional regularization. After Feynman parametrization, the relevant integrals are of the form⁵²⁾ (our metric is $g_{\mu\nu} = +---$) :

$$\int d_n p \frac{a + b^\mu p_\mu}{(p^2 - 2k \cdot p - m^2)^\alpha} = \frac{(-1)^\alpha i \pi^{n/2}}{(m^2 + k^2)^{\alpha - n/2}} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} [a + b^\mu k_\mu] \quad (4.13)$$

$$\int d_n p \frac{c p^2 + d^{\mu\nu} p_\mu p_\nu}{(p^2 - 2k \cdot p - m^2)^\alpha} = \frac{(-1)^\alpha i \pi^{n/2}}{(m^2 + k^2)^{\alpha - n/2}} \frac{1}{\Gamma(\alpha)} \times$$

$$\left\{ \Gamma(\alpha - n/2) (c k^2 + d^{\mu\nu} k_\mu k_\nu) - \Gamma(\alpha - 1 - n/2) (m^2 + k^2) \left(\frac{n}{2} c + \frac{1}{2} d^{\mu\nu} g_{\mu\nu} \right) \right\} \quad (4.14)$$

where n is the dimension of space-time, and

$$x^{-z} \Gamma(z) = \frac{\Gamma(z+1)}{z} e^{-z \ln x} \xrightarrow{z \rightarrow 0} \frac{1}{z} - \ln x + O(z) \quad (4.15)$$

To extract the finite terms in the limit $n \rightarrow 4$, we retain only the lowest relevant order in $(k_1 \times k_2)/m_W^2$. Then, after integration over the Feynman parameters, each diagram of Fig. 17 gives a contribution of the form :

$$i g m_W \frac{e^2 \pi^2}{(2\pi)^4} \left\{ A \Gamma(2 - n/2) g_{\mu\nu} + B g_{\mu\nu} \frac{m_H^2}{2m_W^2} + C \frac{k_{2\mu} k_{1\nu}}{m_W^2} \right\} + O(n-4) \quad (4.16)$$

where we have dropped terms which vanish on the photon mass shell and set $k_1 \cdot k_2 = m_H^2/2$. In order that the result be finite and gauge invariant the summed coefficients must obey

$$\sum A \propto (n-4), \quad \sum C = -\sum B - \frac{2m_W^2}{m_H^2} \frac{2}{4-n} \sum A \quad (4.17)$$

The contributions to A, B and C from each graph of Fig. 17 are listed in the Table. Their sum is seen to obey the conditions (4.17). Thus while the individual contributions are neither finite nor gauge invariant, they sum to give a finite gauge invariant result :

$$\frac{i\alpha}{v\pi} \frac{7}{2} (-k_{1\mu} k_{2\nu} + \frac{m_H^2}{2} g_{\mu\nu}) \quad (4.18)$$

In the notation of Eq. (4.11), the result (4.18) corresponds to

$$I_W = \frac{7}{4} \quad (4.19)$$

The diagrams of Fig. 19 involving a closed $H^\pm$ loop are separately finite and gauge invariant, as they must be by virtue of the renormalizability of scalar electrodynamics. Their contribution is of order m_H^2/m_W^2 relative to the amplitude (4.18) ^{*)}. We conclude from (4.19) that the contribution of the W boson loop to $H \rightarrow \gamma\gamma$ may be very important. We have not calculated corrections to I_W as m_H approaches the W^+W^- threshold : we expect the behaviour to be similar to that for I_ℓ , Fig. 16.

A calculation of $\Gamma(H \rightarrow \gamma\gamma)$ using (4.11), I_ℓ from Fig. 16, I_{hadrons} from (4.12) and I_W from (4.19) indicates that the $\gamma\gamma$ decay mode could well be significant for certain ranges of m_H ^{**)} . In particular, we note from Fig. 1 that $\Gamma(H \rightarrow \gamma\gamma)$ is dominant for $80 \text{ MeV} < m_H < 2m_\mu$. It should also be remembered that the $H \rightarrow \gamma\gamma$ rate could be increased by the addition of heavy leptons L ⁹⁾ (for $m_H \lesssim 2m_L$), more hadronic degrees of freedom ³⁴⁾, and even more so by the addition of extra charged W bosons.

^{*)} We have also repeated the fermion loop calculation of Resnick et al. ¹²⁾ in order to determine the relative signs of the amplitudes. This serves as a further check on the over-all normalization of (4.19).

^{**)} However, the value of $\Gamma(H \rightarrow \gamma\gamma)$ is $O(10^{-5}) \times \Gamma(h \rightarrow \gamma\gamma)$ for a hadron h of comparable mass, as was mentioned in Section 3.5.1.

5. - DISCUSSION

Let us now discuss the observability of the Higgs boson in the light of the calculations of its production (Section 3) and decays (Section 4). Previous limits on the Higgs boson mass, together with our proposals for improving these limits, are tabulated in Fig. 3.

If the Higgs boson has a mass $\lesssim 500$ MeV, then we could identify three possible places to look for it : production at threshold in hadronic collisions (Section 3.2.1.) in the decay $K \rightarrow \pi + H$ (Section 3.3.2.) and in a decay of the 3.7 resonance to $3.1 + H$ (Section 3.3.3.). Higgs bosons with masses $\lesssim 2m_\mu$ decay predominantly into e^+e^- and $\gamma\gamma$ (see Fig. 1), and have lifetimes $\gtrsim 2 \times 10^{-12}$ seconds (see Fig. 2). Hence one could try looking for narrow bumps in these channels, possibly coming from a particle with a decay path of delectably non-zero length. One possible experiment ³⁸⁾ would be π^- capture at rest in a light nucleus such as Be^9 or C^{12} , in which the production of π^0 is forbidden. From the cross-section (3.18) the ratio of Higgs production to the dominant process $\pi^- p \rightarrow \gamma n$ is ^{*)}

$$\frac{\Gamma(\pi^- p \rightarrow H n)}{\Gamma(\pi^- p \rightarrow \gamma n)} \approx O(10^{-6})$$

for Higgs bosons with masses $O(15 \text{ to } 100)$ MeV. Detection of a Higgs boson with a mass in the lower part of this range would be aided by its lifetime of $O(10^{-10})$ sec which means that it travels several centimeters before decaying. A Higgs boson in the higher part of this mass range would live $O(10^{-11})$ sec and not travel very far, but the background γ spectrum would be lower.

For $m_H \gtrsim 2m_\mu$ the e^+e^- and $\gamma\gamma$ decay modes are disfavoured with respect to $H \rightarrow \mu^+\mu^-$, and $H \rightarrow \pi^+\pi^-$ should dominate above $(\pi\pi)$ threshold, but are beset by severe background problems in 3.7 decays from the dominant cascade $3.7 \rightarrow 3.1 + \pi + \pi$. Also, the lifetime would be too short for the Higgs boson to leave a discernible track. One might have expected ⁵³⁾ K or η decays to have already ruled out light Higgs bosons, but this is not so. Reference to Section 3.3.2. reveals the estimate

^{*)} We particularly thank John Bailey for giving us this number.

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + H)}{\Gamma(K^+ \rightarrow \text{all})} \approx 10^{-7}$$

while experiment ²³⁾ sees

$$\frac{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)}{\Gamma(K^+ \rightarrow \text{all})} = (2.6 \pm 0.5) \times 10^{-7}$$

For the range $m_\pi < m_{e^+e^-} < m_K - m_\pi$ to which the experiment was sensitive to e^+e^- pairs, we find from Fig. 1 that $(\Gamma(H \rightarrow e^+e^-))/(\Gamma(H \rightarrow \text{all}))$ decreases from $\sim 30\%$ at $m_H = m_\pi$ to $\sim 12\%$ at $m_H = 2m_\mu$, and is negligible for $m_H > 2m_\mu$. Hence we would estimate

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + H, H \rightarrow e^+e^-)}{\Gamma(K^+ \rightarrow \text{all})} \approx 0(1 \text{ to } 3) \times 10^{-8}$$

for $2m_\mu > m_H > m_\pi$, and the published ²³⁾ data seem unable to rule out a narrow peak in $m_{e^+e^-}$ produced with such a branching ratio. However, future experiments on $\Delta S = 1$ neutral currents in K decays might well be sensitive to Higgs boson production if its mass is appropriately small.

If the Higgs boson has a mass in the range $500 \text{ MeV} < m_H < 1500 \text{ MeV}$, then its detection may be very difficult. We estimated in Sections 3.2.2. and 3.2.3. that its production cross-section would be $O(10^{-7})$ of ρ production in comparable kinematic conditions. The Higgs boson would be most easily detectable via its decay to $\mu^+\mu^-$, which Fig. 1 shows to have a branching ratio $O(1 \text{ to } 30)\%$ in this mass range. Since $\rho \rightarrow \mu^+\mu^-$ has a branching ratio $\sim 5 \times 10^{-5}$, this means that

$$\frac{\sigma(e) \frac{\Gamma(\rho \rightarrow \mu^+\mu^-)}{\Gamma(\rho \rightarrow \text{all})}}{\sigma(H) \frac{\Gamma(H \rightarrow \mu^+\mu^-)}{\Gamma(H \rightarrow \text{all})}} \approx O(10^3 - 10^5) \quad (5.1)$$

Thus it would have to be a very high statistics experiment indeed which could see a small narrow Higgs bump above the tail of the ρ meson ^{*)}. In fact there seems little advantage in looking for a Higgs boson with a

^{*)} For this reason and because of its low e^+e^- branching ratio, there seems little likelihood that the Higgs boson could be the mysterious source of directly produced leptons sought by Lederman and White ⁵⁴⁾.

mass in the range 500 MeV to 1500 MeV at a very high energy accelerator. An experiment at moderate energies, where the dominant production mechanism would be one pion exchange (as discussed in Section 3.2.2.) and the backgrounds might be smaller, might be an equally good possibility.

For $1.5 \text{ GeV} < m_H < 4 \text{ GeV}$, production at very high energies and detection as a small bump sitting on top of the Drell-Yan ³⁹⁾ $(\mu^+ \mu^-)$ continuum seems the only possibility ^{*}). Combining the cross-section estimate (3.29) with the branching ratios of Fig. 1, we find

$$\frac{\sigma(pp \rightarrow H + X) \xrightarrow{\mu^+ \mu^-}}{\sigma(pp \rightarrow (\mu^+ \mu^-) + X)} \approx \frac{Q}{\Delta Q} \times O(3 \times 10^{-2} \text{ to } 10^{-5}) \quad (5.2)$$

which is not encouraging. Even if such a bump were seen, how would one know it was a Higgs boson, and not some random ²⁾ hadronic vector meson? There would be two distinctive features of the Higgs boson: it would not be seen in $(e^+ e^-)$ if it had such a mass, and it would be much narrower than some vector mesons, though not visibly narrower than others ²⁾.

For $m_H > 4 \text{ GeV}$ the Higgs boson's production cross-section by any mechanism we have been able to think of is minute, and it decays predominantly to as yet conjectural ³⁾⁻⁵⁾ massive new particles.

We should perhaps finish with an apology and a caution. We apologize to experimentalists for having no idea what is the mass of the Higgs boson, unlike the case with charm ^{3),4)} and for not being sure of its couplings to other particles, except that they are probably all very small. For these reasons we do not want to encourage big experimental searches for the Higgs boson, but we do feel that people performing experiments vulnerable to the Higgs boson should know how it may turn up.

^{*}) At least until intermediate vector bosons are discovered, and the bremsstrahlung production of Section 3.4 becomes thinkable !

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Graph	A	B	C
a + crossed	$3(n-1)$	$37/6$	$-23/6$
b	$-2(n-1)$	-2	0
c + d + crossed	$-\frac{1}{2}(n-1)$	$19/12$	$-23/12$
e + crossed	0	$2m_W^2/m_H^2 + 1/6$	0
f + crossed	0	1	0
g + h + crossed	1	$1/6$	$-7/6$
i + crossed	-2	0	0
$2 \times j$ + crossed	$-\frac{1}{2}$	$-1/12$	$-1/12$
SUM	$\frac{n}{2}-2$	$2m_W^2/m_H^2 + 7$	-7

TABLE : Contributions to the $H \rightarrow \gamma\gamma$ amplitude from the graphs of Fig. 17.

REFERENCES

- 1) J. Iliopoulos - Proceedings of the XVIIth International Conference on High Energy Physics, London 1974, ed. by J.R. Smith, p. III-89 (S.R.C., Rutherford Laboratory, 1974) ;
B.W. Lee - Invited talk presented at the International Symposium on Lepton and Photon Interactions at High Energies, Stanford 1975, Fermilab-Conf-75/2-THY (1975).
- 2) J.J. Aubert et al. - Phys.Rev.Letters 33, 1404 (1974) ;
J.-E. Augustin et al. - Phys.Rev.Letters 33, 1406 (1974) ;
G.S. Abrams et al. - Phys.Rev.Letters 33, 1453 (1974).
- 3) Y. Hara - Phys.Rev. 134, B701 (1964) ;
D. Amati, H. Bacry, J. Nuyts and J. Prentki - Phys.Letters 11, 190 (1964) ;
B.J. Björken and S.L. Glashow - Phys.Letters 11, 255 (1964) ;
Z. Maki and Y. Ohnuki - Progr.Theor.Phys.(Kyoto) 32, 144 (1964) ;
S.L. Glashow, J. Iliopoulos and L. Maiani - Phys.Rev. D2, 1285 (1970).
- 4) M.K. Gaillard, B.W. Lee and J.L. Rosner - Revs.Modern Phys. 47, 277 (1975).
- 5) H. Georgi and S.L. Glashow - Phys.Rev.Letters 28, 1494 (1972), for example.
- 6) P.W. Higgs - Phys.Rev.Letters 12, 132 (1964) ;
F. Englert and R. Brout - Phys.Rev.Letters 13, 321 (1964) ;
P.W. Higgs - Phys.Rev.Letters 13, 508 (1965) ;
G.S. Guralnik, C.R. Hagen and T.W.B. Kibble - Phys.Rev.Letters 13, 585 (1965) ;
P.W. Higgs - Phys.Rev. 145, 1156 (1966) ;
T.W.B. Kibble - Phys.Rev. 155, 1554 (1967).
- 7) B.C. Barish - Proceedings of the XVIIth International Conference on High Energy Physics, London 1974, ed. by J.R. Smith, p. IV-111 (S.R.C., Rutherford Laboratory, 1974).
- 8) S. Orito et al. - Phys.Letters 48B, 165 (1974).
- 9) M.L. Perl et al. - SLAC-PUB-1626 (1975).
- 10) S. Weinberg - "Implications of Dynamical Symmetry Breaking", Harvard Preprint (1975) and references therein.
- 11) S. Weinberg - Phys.Rev.Letters 19, 1264 (1967) ;
A. Salam - Proceedings of the 8th Nobel Symposium, Stockholm 1968, ed. by N. Svartholm, p. 367 (Almqvist and Wiksells, Stockholm, 1968).
- 12) M.K. Sundareshan and P.J.S. Watson - Phys.Rev.Letters 29, 15 (1972) ;
L. Resnick, M.K. Sundareshan and P.J.S. Watson - Phys.Rev. D8, 172 (1973).

- 13) D. Kohler, B.A. Watson and J.A. Becker - Phys.Rev.Letters 33, 1628 (1974).
- 14) R. Barbieri and T.E.O. Ericson - Phys.Letters 57B 270 (1975).
- 15) R. Jackiw and S. Weinberg - Phys.Rev. D5, 2396 (1972).
- 16) R. Rafelski, B. Muller, G. Soff and W. Greiner - Ann.Phys.(Leipzig) 88, 419 (1974) ;
P.J.S. Watson and M.K. Sundaresan - Can.J.Phys. 52, 2037 (1974) ;
S.L. Adler, R.F. Dashen and S.B. Treiman - Phys.Rev. D10, 3728 (1974).
- 17) A.D. Linde - Zh.Eksper.Teor.Fiz.Pis.Red. 19, 320 (1974), JETP Letters 19, 183 (1974) ;
J. Dreitlein - Phys.Rev.Letters 33, 1243 (1974) ;
M. Veltman - Phys.Rev.Letters 34, 777 (1975).
- 18) S.P. Rosen - Phys.Rev.Letters 33, 614 (1974) ;
D.A. Ross and M. Veltman - Nuclear Phys. B95, 135 (1975).
- 19) See, for example, the Proceedings of the Colloque International sur la Physique du Neutrino à Haute Energie, Paris, 1975, p. 183 (C.N.R.S., Paris, 1975), and the Lepton-Photon Symposium, Stanford 1975.
- 20) P. Carruthers - Phys.Rev. D2, 2265 (1970), and Phys.Reports 1C, 1 (1971) ;
R.J. Crewther - Phys.Letters 33B, 305 (1970), and Phys.Rev. D3, 3152 (1971) ;
J. Ellis - Nuclear Phys. B22, 478 (1970), and "Broken Scale Invariance and the Light Cone", ed. by M. Dal Cin, G.J. Iverson and A. Perlmutter, p. 77 (Gordon and Breach, New York, 1971).
- 21) The Omicron Collaboration - Proposal Ph.III-75/15 to the CERN Physics III Experiments Committee (1975).
- 22) G. Zweig - CERN Preprints TH. 401 and 412 (1964) (unpublished) and many other authors.
- 23) P. Bloch et al. - Phys.Letters 56B, 201 (1975).
- 24) H. Leutwyler - Phys.Letters 48B, 431 (1974) ;
J. Gasser and H. Leutwyler - Nuclear Phys. B94, 269 (1975).
- 25) J.L. Rosner - Phys.Reports 11C, 189 (1974), and references therein.
- 26) J.D. Bjorken and C.H. Llewellyn Smith - Phys.Rev. D7, 887 (1973).
- 27) T. Appelquist and J. Carrazone - Phys.Rev. D11, 2856 (1975).
- 28) A. Bertin et al. - Phys.Letters 55B, 411 (1975) ;
L. Tauscher et al. - Phys.Rev.Letters 35, 410 (1975).
- 29) R. Barbieri - Phys.Letters 56B, 266 (1975).
- 30) M. Gell-Mann, R.J. Oakes and B. Renner - Phys.Rev. 175, 2195 (1968).

- 31) Folklore communicated to us by S. Coleman and B. Zumino.
- 32) S. Weinberg - Phys.Rev.Letters 31, 494 (1973) ;
D.V. Nanopoulos - Nuovo Cimento Letters 8, 873 (1973).
- 33) B. de Wit - Nuclear Phys. B51, 237 (1973) ;
S.D. Joglekar - Phys.Rev. D11, 2490 (1975).
- 34) H. Harari - Invited talk presented at the International Symposium
on Lepton and Photon Interactions at High Energies, Stanford
1975, Weizmann Institute Preprint WIS-75/40 Ph (1975), and
references therein.
- 35) C.G. Callan, S. Coleman and R. Jackiw - Ann.Phys.(N.Y.) 59, 42 (1970).
- 36) E.S. Abers and B.W. Lee - Phys.Reports 9C, 1 (1973).
- 37) K. Fujikawa, B.W. Lee and A.I. Sanda - Phys.Rev. D10, 2923 (1972).
- 38) Another proposal is in preparation : J. Bailey, J. Davies, D. Frame
and C. Wilkin (private communication). We thank them for
discussions about the experimental possibilities.
- 39) S.D. Drell and T.-M. Yan - Phys.Rev.Letters 25, 316 (1970).
- 40) S. Coleman and S.L. Glashow - Phys.Rev. 134B, 671 (1964).
- 41) H. Osborn and D.J. Wallace - Nuclear Phys. B20, 23 (1970).
- 42) F. Dydak et al. - "Measurement of the Σ^0 Lifetime", Paper submitted
to the Palermo Conference (1975).
- 43) M.K. Gaillard and B. W. Lee - Phys.Rev. D10, 897 (1974) ;
D.V. Nanopoulos and G.G. Ross - Phys.Letters 56B, 279 (1975).
- 44) M.K. Gaillard, B.W. Lee and R.E. Schrock - CERN Preprint TH. 2066
and Fermilab Pub-75/68-THY (1975).
- 45) E.B. Bogomolny, V.A. Novikov and M.A. Shifman - Moscow Preprint ITEP-
42 (1975) ;
A.I. Vainshtein, V.I. Zakharov, V.A. Novikov and M.A. Shifman -
Moscow Preprint ITEP-44 (1975).
- 46) F.E. Low - Phys.Rev. 110, 947 (1958).
- 47) G. Mack - Nuclear Phys. B5, 499 (1966).
- 48) S. Coleman and R. Jackiw - Ann.Phys.(N.Y.) 67, 522 (1971).
- 49) T. Goldman and P. Vinciarelli - Phys.Rev.Letters 33, 246 (1974).
- 50) R.J. Crewther - Phys.Rev.Letters 28, 1421 (1972) ;
M.S. Chanowitz and J. Ellis - Phys.Letters 40B, 397 (1972) and Phys.
Rev. D7, 2490 (1973).
- 51) G. 't Hooft - Nuclear Phys. B35, 167 (1971).

- 52) G. 't Hooft and M. Veltman - "Diagrammar", CERN Yellow Report 73-9 (1973).
- 53) J. Iliopoulos and others - Private communications.
- 54) L. Lederman and S. White - Columbia University Preprint (1975).

FIGURE CAPTIONS

- Figure 1 Branching ratios of the Higgs boson for different values of its mass. The curves are calculated from the decay rates of Section 4.
- Figure 2 The total lifetime of the Higgs boson for different values of its mass. The curves are calculated from the decay rates of Section 4.
- Figure 3 Present and possible future limits on the Higgs boson mass.
- Figure 4 Contributions of $\delta \mathcal{L}_H$ (3.10) to Eq. (3.13).
- Figure 5 Diagrams contributing to the process $\pi^- p \rightarrow H n$.
- Figure 6 Drell-Yan contributions to $pp \rightarrow H + X$ and $pp \rightarrow (\mu^+ \mu^-) + X$.
- Figure 7 The effective $\Delta I = 1$ quark operator.
- Figure 8 Diagrams for $\Sigma^0 \rightarrow \Lambda + H + \gamma$.
- Figure 9 Contributions to the one-body $|\Delta S| = 1$ quark operator.
- Figure 10 Contributions to the two-body $|\Delta S| = 1$ quark operator.
- Figure 11 Pole diagrams from the graphs of Fig. 10b.
- Figure 12 Graphical definition of quantities appearing in Eq. (3.61).
- Figure 13 The dominant bremsstrahlung diagrams (a) for $e^+ e^- \rightarrow Z^0 + H$, (b) for $e^+ e^- \rightarrow J/\psi + H$.
- Figure 14 The decay $H \rightarrow \pi\pi$ via an intermediate $\epsilon(700?)$ meson.
- Figure 15 Contributions to the quantity I of Eq. (4.11).
- Figure 16 A graph of the real and imaginary parts of I_ℓ .
- Figure 17 Feynman diagrams for I_w .

Figure 18 Feynman rules for the graphs of Fig. 17.

Figure 19 Contributions to I_w from unphysical Higgs bosons.

Higgs Boson Mass (MeV)

FIG.1

FIG. 2

FIG. 3

FIG.4

FIG.5

FIG.6

FIG.7

FIG.8

FIG. 9

FIG.10

FIG.11

FIG. 12

FIG. 13

FIG. 14

FIG. 15

FIG.16

$$\mu \text{---} \text{wavy} \text{---} \nu \quad W(p) = \frac{-ig_{\mu\nu} + ip_\mu p_\nu}{p^2 - m_w^2}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H^-(p) = \frac{i}{p^2 - m_w^2}$$

$$\text{.....} \text{---} \text{.....} \quad \Phi^\pm(p) = \frac{i}{p^2 - m_w^2}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H(q) \rightarrow H^-(p) \quad W^- = \frac{ig}{2} (p+q)_\mu$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H(q) \rightarrow H^-(p) \quad W^- = \frac{ig}{2} (p-q)_\mu$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H(q) \rightarrow W^- \quad W^- = ig \ln_w g_{\mu\nu}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H(q) \rightarrow H^- \quad H^- = \frac{-ig}{2m_w} m_H^2$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H(q) \rightarrow W^- \quad W^- = -\frac{ie}{2} g g_{\mu\nu}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H(q) \rightarrow \Phi^\pm \quad \Phi^\pm = -ig \frac{m_w}{2}$$

$$\nu \text{---} \text{wavy} \text{---} \rho \quad W^-(p') \quad \gamma(k) \quad W^-(p) = ie [g_{\mu\nu} (p+p')_\rho - p_\nu g_{\mu\rho} - p'_\mu g_{\nu\rho} - g_{\mu\rho} k_\nu + g_{\nu\rho} k_\mu]$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H^-(p') \rightarrow H^-(p) \quad \gamma = -ie (p+p')_\mu$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H^- \rightarrow W^- \quad W^- = -iem_w g_{\mu\nu}$$

$$\nu \text{---} \text{wavy} \text{---} \rho \quad W^- \quad \gamma \quad W^- \quad \gamma \quad \sigma = -ie^2 [2g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho} - g_{\mu\rho} g_{\nu\sigma}]$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad H^- \rightarrow H^- \quad \gamma = 2ie^2 g_{\mu\nu}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad \Phi^\pm(p) \rightarrow \Phi^\pm \quad \gamma(k) = \pm ie p_\mu$$

FIG. 18

FIG. 19